Proton-neutron pairing correlations in the self-conjugate nucleus ^{38}K probed via a direct measurement of the isomer shift

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(Dated: May 30, 2014)

A marked difference in the nuclear charge radius was observed between the $I^\pi = 3^+$ ground state and the $I^\pi = 0^+$ isomer of ^{38}K and is schematically explained using an intuitive picture of proton-neutron pairing. In a high-precision measurement of the isomer shift using bunched-beam collinear laser spectroscopy at CERN-ISOLDE, a change in the mean-square charge radius of $\left\langle r_c^2 \right\rangle \left(^{38}\mathrm{K}^m\right) - \left\langle r_c^2 \right\rangle \left(^{38}\mathrm{K}^g\right) = 0.100(6)$ fm^2 was obtained. This is an order of magnitude more accurate than the result of a previous indirect measurement from which it was concluded that both long-lived states in ^{38}K have similar charge radii. Our observation leads to a substantially different understanding since the difference in charge radius is moreover opposite in sign to previously reported theoretical predictions. It is demonstrated that the observed isomer shift can be reproduced by large-scale shell-model calculations including proton and neutron excitations across the $N, Z = 20$ shell gaps, confirming the significance of cross-shell correlations in the region of ^{40}Ca.

Since the early days of nuclear physics it has been known that protons and neutrons favour the formation of $I = 0$ pairs with anti-aligned spins. In 1935 the first parametrisation of the Bethe-Weizsäcker mass formula [1] included a term that took $\pi\pi$ and $\nu\nu$ pairs into account by enhancing the binding in even-even nuclei. Only one year later an analysis of nucleon-nucleon scattering data demonstrated that within experimental errors the strong interaction between nucleons can be considered *charge independent* [2]. This discovery led directly to the application [3] of Heisenberg's concept of "isotopic spin" (isospin) to finite nuclei. As a direct consequence $\pi\nu$ pairs with $T = 1$, $T_z = 0$ should be treated on an equal footing to the $T = 1$ $\pi\pi$ and $\nu\nu$ pairs with $T_z = +1, -1$ respectively. Whilst this charge independent treatment of isovector pairing remains a standard feature of modern shell model calculations and is under development in mean-field approaches [4, 5], the experimental investigation of the $\pi\nu$ pairing interaction remains an active area of interest [6, 7]. Specifically theoretical debate [8–11] on the role of $\pi\nu$ pairing in the microscopic origins of the nuclear symmetry energy has wide ranging consequences not only for nuclear structure but also nuclear astrophysics [12–14]. Consequently an experimental constraint on the strength of isovector pairing correlations is vital. Here we demonstrate that when $\pi\nu$ pairing correlations are considered on an equal footing to $\pi\pi$ and $\nu\nu$ correlations the mean square charge radius of $^{38}\mathrm{K}^m$ can be fully accounted for. Furthermore the isomer shift is shown to provide a highly sensitive test of the strength of isovector pairing employed in modern nuclear structure calculations.

The setup for collinear laser spectroscopy [15] located at CERN-ISOLDE was used to obtain the hyperfine spectra (hfs) of $^{38,39,42,44,46\text{-}51}$K isotopes. During the experiment protons bombarded a $\mathrm{UC_x}$ target producing a wide range of radioactive nuclei. The ions of interest were cooled and bunched in a helium filled radio-frequency quadrupole ISCOOL [16] after mass separation by the HRS (High-Resolution Mass Separator). Resonant excitation of the $4s\,^2S_{1/2} \rightarrow 4p\,^2P_{1/2}$ atomic transition was obtained using a cw Ti:sapphire laser. With the laser frequency kept constant, Doppler-tuning of the ions was performed by applying an additional voltage to the potassium neutralization cell located in front of the detection region. The background from scattered light was reduced by only accepting signals from the detectors (4 photomultiplier tubes) when bunches of K atoms arrived in the detection region. More detailed descriptions of the experimental setup may be found in Papuga *et al.* [17] and Kreim *et al.* [18] reporting respectively on spins and magnetic moments of ^{49,51}K and charge radii of the K isotopes between $N = 27$ and $N = 32$.

Here we will focus only on the discussion of the iso-

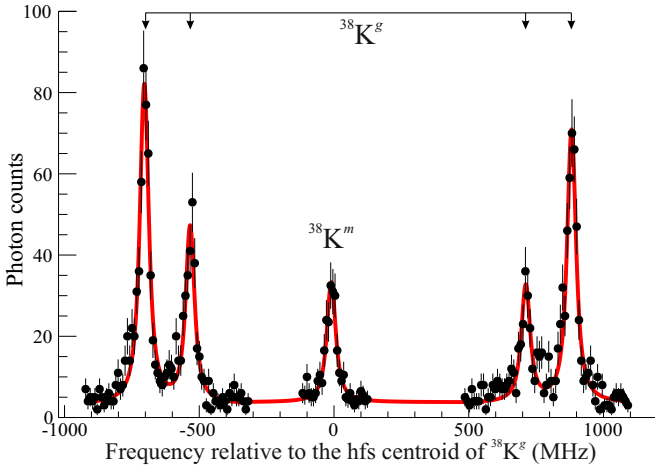

FIG. 1. Observed hyperfine spectra of $^{38}K^{g,m}$. Four peaks are obtained for $I = 3$ (ground state) and only one for $I = 0$ (isomer).

mer shift between the two observed states in ^{38}K. As the mass difference between the two states is small, systematic uncertainties on the kinematic shift arising from an imperfect knowledge of the ion beam energy [19] are negligible. An example frequency spectrum containing both the ground state and isomer of ^{38}K is presented in Fig. 1. Fitting was performed using a χ^2 minimization procedure in which the asymmetric line shapes associated with collisional ion energy loss [20] were accounted for.

The difference in hyperfine structure centroid of two isotopes or nuclear states $\delta\nu^{A,A'} = \nu^{A'} - \nu^{A}$ may be related to the difference in mean square charge radii $\delta\langle r_c^2\rangle^{A,A'} = \langle r_c^2\rangle^{A'} - \langle r_c^2\rangle^{A}$ via

$$\delta\langle r_c^2\rangle^{A,A'} = \frac{1}{F}(\delta\nu^{A,A'} - K_{\mathrm{MS}}\frac{m_{A'} - m_A}{m_{A'}m_A}) \qquad (1)$$

where m_A and $m_{A'}$ are the masses of relevant isotopes or nuclear states taken from Wang *et al.* [21]. K_{MS} is the total mass shift factor given by the sum $K_{\mathrm{MS}} = K_{\mathrm{SMS}} + K_{\mathrm{NMS}}$, in which the specific mass shift $K_{\mathrm{SMS}} = -15.4(38)$ GHz u from Martensson-Pendrill *et al.* [22] and the normal mass shift $K_{\mathrm{NMS}} = \nu^A m_e = 213.55$ GHz u. The electronic factor $F = -110(3)$ MHz fm^{-2} was also taken from reference [22].

Behr *et al.* [24] measured the isotope shift $\delta\nu^{38m,39}$ and combined this with $\delta\nu^{38g,39}$ measured by Touchard *et al.* [23]. From this approach they found that within errors the ground and isomeric state had the same charge radii. In table I excellent agreement can be seen between the ^{38m,39}K isotope shift measured in this work and that of Behr *et al.* and agreement within errors between our ^{38g,39}K isotope shift and that of Touchard *et al.*. It is only possible to conclude that the cumulative effect of the relatively large uncertainties in [24] and [23] led to the conclusion that the ground state and isomer are of the same size. Here the advantages of performing a *direct* isomer shift measurement are most clearly visible.

TABLE I. Isomer and isotope shifts determined in this work compared with literature values. The second uncertainty contained within the square brackets corresponds to the systematic contribution associated with the atomic parameters K_{SMS} and F.

A'	A	$\delta\nu^{A,A'}$ (MHz)	$\delta\langle r_c^2\rangle^{A,A'}$ (fm^2)	Ref.
38g	39	−123.4 (10)	−0.089 (9)[23]	*this work*
		−127.0 (53)	−0.057 (48)[23]	[23]
38m	39	−134.5 (11)	0.011 (10)[23]	*this work*
		−132 (3)	−0.02 (3)[2] [a]	[24]
38m	38g	−11.03 (56)	0.100 (5)[3]	*this work*
		−4 (6)	0.04 (6)	[23, 24]

[a] Measured in the $4s\,^2S_{1/2} \rightarrow 4p\,^2P_{3/2}$ transition assuming the same F and K_{SMS} as in the $4s\,^2S_{1/2} \rightarrow 4p\,^2P_{1/2}$ transition.

The authors of [24] also performed a detailed calculation of the difference in charge radius of the two states. Their Hartree-Fock calculation constrained by shell model orbital occupancies concluded that the ground state should be larger than the isomer by 0.014 fm^2. This evaluation clearly contradicts the measurement reported in this work in both magnitude and sign.

To develop an understanding of the origins of the difference in size of these two nuclear states, we begin by comparing the mean square radii with neighbouring nuclei in Fig. 2. Clearly the increase in size of the isomeric state is larger than the normal ground-state odd-even staggering by about a factor of 2. An alternative view of this effect can be obtained by plotting the differences in charge radii as a function of A along the line of $N = Z$. The results of this process are shown in Fig. 3. Here it is seen that the larger charge radius of the isomer is consistent with a smooth increase in size along the $N = Z$ line whilst the ground state is somewhat smaller than the average of its two neighbours.

To interpret these observations one may begin by considering the origins of the ubiquitous normal odd-even staggering (OES) in nuclear charge radii. This phenomenon may be readily explained by considering $\pi\pi$ or $\nu\nu$ $I = 0$ pairs scattering to a large number of states near the Fermi surface in the even N or Z nuclei. The addition of an odd proton or neutron 'blocks' a specific orbit, thus reducing the scattering of pairs. As less bound orbitals naturally have a larger spacial extent, $\langle r_c^2\rangle$ of odd N or Z nuclei are consistently found to be smaller than the average of their even neighbours. In the case of isotopic OES an increase in neutron orbital correlations translates into a change in charge radius either by a global broadening of the proton distribution in mean-field calculations or via a direct enhancement of the scattering of valence $\pi\pi$ pairs. Although such pairing arguments are typically employed when considering pairs of protons or neutrons, *charge in-*

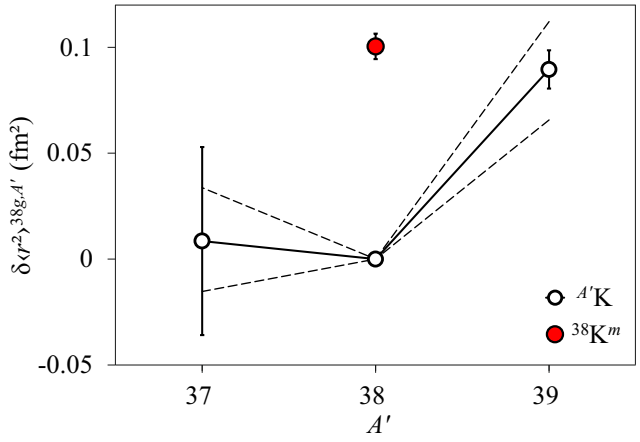

FIG. 2. Changes in mean square charge radius referenced to ^{38}K. The systematic uncertainty related to the atomic specific mass shift is represented by the two dotted lines. Datum for ^{37}K taken from [24].

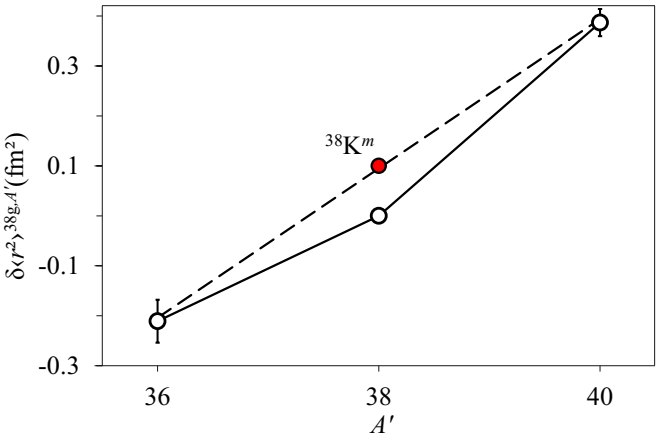

FIG. 3. Changes in mean square charge radii between the self-conjugate nuclei ^{36}Ar, ^{38}K and ^{40}Ca from this work and [30].

dependence of the nucleon-nucleon interaction effectively results in the possibility of $\pi\nu$ pairing correlations.

Indeed in ^{38}K it would appear that for the $T = 1$ isomer the $\pi\nu$ pair coupled to $I = 0$ is free to scatter into a range of orbitals. Conversely the $\pi\nu$ pair making up the $T = 0$, $I^\pi = 3^+$ ground state is heavily restricted in the number of states with which it can mix, significantly reducing the proton occupancy in the fp shell.

With this simple picture it is immediately apparent why the isomer shift should be larger than the normal isotopic OES. As the even N ^{37,39}K isotopes have proton distributions which remain blocked by the single $\pi d_{3/2}^{-1}$, the scale of the OES remains somewhat smaller than in the neighbouring even Z isotopic chains. The formation of a $I = 0$ $\pi\nu$ pair in ^{38}K^m effectively removes this blocking, thus enhancing the πfp occupancy. Under the assumption of charge independence, this $\pi\nu$ pair should be free to scatter on an equal basis to $\pi\pi$, $\nu\nu$ pairs removing the OES along the line of $N = Z$. Precisely this behaviour is observed in the experimental $\delta\langle r_c^2\rangle^{A,A'}$ presented in Fig. 3.

As the observed radii differences can be understood with a simple intuitive model it remains to investigate how the previous detailed theoretical calculation failed to predict both the sign and magnitude of the isomer shift. It was shown by Caurier *et al.* [25] that the isotope shifts in Ca could be reproduced with a reasonable accuracy if one calculates the occupancy of the πfp shell as a function of A. In this work the expression

$$\delta\langle r_c^2\rangle^{A,A'} = \frac{1}{Z}\Delta n_{fp}^{\pi}(A,A')b^2 \qquad (2)$$

was used, where b is the oscillator parameter and n_{fp}^{π} refers to the number of protons lifted across the $Z = 20$ shell closure. The choice of oscillator parameter b remains a subject of much interest. Whilst a number of approaches exist within the literature [26–29], it should be noted that all produce values of b within a few percent of each other. In the following we assume $b^2(^{38}\mathrm{K}) = 3.944\,\mathrm{fm}^2$ as determined by the equation of Duflo and Zuker [28]. With this value and Eq. 2 it is immediately obvious that for a successful reproduction of the isomer shift one would require the isomer to have an average πfp orbital occupancy, n_{fp}^{π}, of the order of 0.48 protons larger than the corresponding ground state occupancy.

TABLE II. Proton occupancies of the $f_{7/2}p_{3/2}$ orbitals, and the difference in charge radii between 0^+ isomer and 3^+ ground state, calculated within the shell model. See text for details.

	$n_{fp}^{\pi}(38m)$	$n_{fp}^{\pi}(38g)$	$\delta\left\langle r_c^2\right\rangle^{38g,38m}$ (fm^2)
ZBM2	0.86	0.50	0.075
ZBM2 modified	0.82	0.41	0.085
Experiment			0.100(6)

To test the validity of this conclusion shell model calculations were performed in the model space comprising $1s_{1/2}, 1d_{3/2}, 0f_{7/2}, 1p_{3/2}$ orbitals for neutrons and protons, with the ZBM2 interaction from Ref. [25]. Full space diagonalization in this model space has been achieved using the shell model code ANTOINE [31]. The results of the calculations are reported in table II, where the summed pf-shell proton occupancies are listed for ground and isomeric states along with the corresponding $\delta\left\langle r^2\right\rangle^{38g,38m}$ obtained from Eq. 2. As can be seen, the ZBM2 interaction gives a fair agreement with the experimental value, confirming the realistic character of the wave functions obtained. However, as demonstrated in Fig. 4 it fails to produce a correct order of the $T = 1$ versus $T = 0$ states. This problem can be traced back to the uncertainty on the adjustments of $d_{3/2} - d_{3/2}$ monopole matrix elements: since ^{28}O is unbound, the strength of

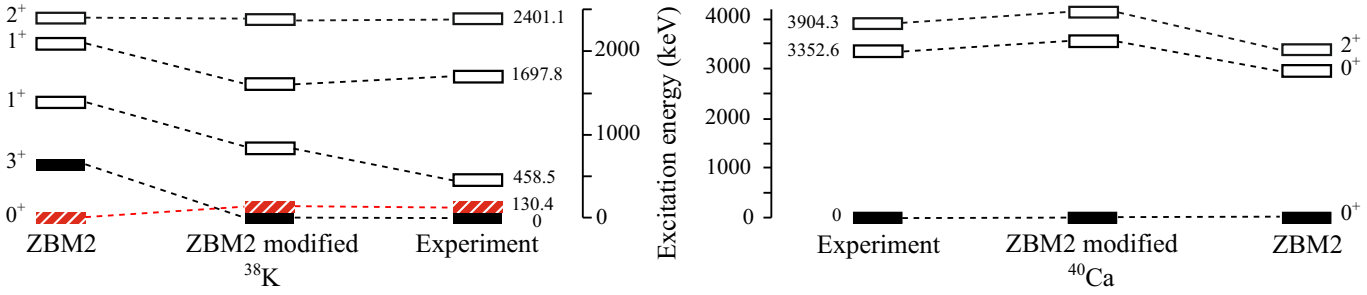

FIG. 4. Low energy excitation spectra of ^{38}K (left) and ^{40}Ca (right) calculated with the ZBM2 interaction before and after modification of the $V^{0,1}_{d_{3/2},d_{3/2}}$ centroids (see text for details.)

the $T = 1$ monopole cannot be precisely determined in a purely neutron system. As appears here, the $T = 1$ matrix elements of the ZBM2 interaction are too strong with respect to the $T = 0$ elements producing an inversion of the 0^+ and 3^+ levels.

In the isospin formalism, the centroids V_{ij} enter the monopole Hamiltonian through the coefficients a_{ij} and b_{ij}, where $a_{ij} = \frac{1}{4}(3V^{T=1}_{ij} + V^{T=0}_{ij})$ and $b_{ij} = V^{T=1}_{ij} - V^{T=0}_{ij}$. While the b_{ij} coefficient fixes the position of the configurations with a given isospin T value, the a_{ij} component fixes the position of configurations dependent on the particle number involved. It is thus always possible to modify the position of the $T = 0$ versus $T = 1$ states via the b_{ij} parameter, leaving the position of particle-hole excited states versus zero-particle–zero-hole configuration unchanged. In our case, we have modified the $V^{0,1}_{d_{3/2},d_{3/2}}$ centroids to reproduce the spectrum of ^{38}K in the ZBM2 calculation as shown in Fig. 4. This modification leads to a proper behaviour of the two-neutron separation energies along the oxygen chain. At the same time the a_{ij} value is left invariant, preserving the description of the isotope shifts in calcium and other results from the original ZBM2 interaction as illustrated for ^{40}Ca in Fig. 4. From table II it can be seen that ZBM2 and the modified version of the interaction provide a similar composition of the ground and isomeric state wave functions, the latter giving $\delta\left\langle r_c^2\right\rangle^{38g,38m} = 0.085$ fm^2, even closer to the experimental value of 0.100(6) fm^2.

Finally, we have quantified the role of pairing correlations in the description of the ground and isomeric states by taking the expectation value of the $T = 1$ pairing Hamiltonian in the wave functions of the 3^+ and 0^+ states obtained from the shell-model diagonalization. The absolute value of the pairing contribution to the 0^+ state is 5 MeV larger than to the 3^+. On the contrary, taking the pairing Hamiltonian restricted to proton-proton and neutron-neutron pairs only, one obtains a small and nearly equal expectation value for both states. Therefore, these are indeed the $T = 1, I = 0$ proton-neutron pairs that constitute the essential difference in the building of the 0^+ and 3^+ states.

Returning to Behr *et al.*'s shell-model occupancy constrained spherical Hartree-Fock calculations it is likely that the exclusion of cross-shell correlations and the resulting small difference between the orbital occupancies of the two states is the origin for the incorrect sign of the isomer shift.

It is intriguing to note that the only other odd-odd self-conjugate isomer shift measured to date [32] in ^{50}Mn [33] results in a $\delta\left\langle r^2\right\rangle$ which is identical in magnitude and opposite in sign to that measured here. The sign can be easily understood when one recalls that the $T = 0$ and $T = 1$ levels are inverted after $A = 40$ for all known cases with the exception of ^{58}Cu. Whilst it is too early to draw direct conclusions from this similarity, the measurement of other isomer shifts in odd-odd self-conjugate nuclei such as ^{26}Al and ^{42}Sc could map the evolution of proton-neutron pairing correlations along the line of $N = Z$.

To conclude, both the direction and magnitude of the isomer shift in ^{38}K^m can be described phenomenologically when isovector proton-neutron pairing correlations are considered. Furthermore a detailed study within the shell model framework has demonstrated good agreement with the observed isomer shift, confirming the significance of these cross-shell correlations in the region of ^{40}Ca.

This work was supported by the IAP-projects P6/23, P7/12, the FWO-Vlaanderen, NSF grant PHY-1068217, BMBF (05 P12 RDCIC), Max-Planck Society and EU FP7 via ENSAR (no. 262010). We would like to thank to the ISOLDE technical group for their support and assistance. H.N. and K.B. acknowledge support from the Helmholtz Association through the Nuclear Astrophysics Virtual Institute NAVI (VH-VI-417).

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