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Minority Governments, Minimal Winning Coalitions and Surplus Majorities in Parliamentary Systems

by

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Abstract

This paper studies government formation in parliamentary systems. A formal model is presented to explain the emergence of minority governments, minimal winning coalitions and surplus majorities, as a function of the largest party's share of the seats in the legislature and its location in the policy space. I conclude that as the largest party becomes larger and more central, the government changes from a surplus majority to a minimal winning coalition and from a minimal winning coalition to a minority government. I find empirical support for these conclusions in a study of government formation in eleven parliamentary democracies.

1 Introduction

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Cabinet governments rely on a parliamentary majority for their formation and continuation. If no single party has an absolute majority in the legislature, i.e., in minority situations, three types of government can be formed: minority governments, minimal winning coalitions and surplus majorities. In minority governments, as opposed to majority governments, the parties that hold cabinet ministries do not control a majority of the seats in the legislature, but their government is supported by parties without government portfolios. Surplus majorities are governments in which at least one party holding portfolios could depart from the government and still leave it a majority. The other majority governments are minimal winning coalitions. Several studies provide data on the frequency of the different types of government.¹ Surplus majorities account for almost one fourth of the governments in minority situations in post-World War II parliamentary democracies. More than a third of the governments are minority governments.

Surprisingly, formal theorists have paid little attention to minority governments and surplus majorities, focusing instead on minimal winning coalitions. This lack of attention is often considered a concession of the inability of formal theory to explain these phenomena. In contrast, other scholars have shown an abundant interest in minority governments and surplus majorities. This paper is a modest attempt to bridge this gap by developing a rigorous, theoretical model that provides insight into the conditions leading to the different types of government. The model yields refutable predictions, and explains the formation of minority governments, minimal winning coalitions and surplus majorities, as a function of the largest party's share of the seats in the legislature and its location in the policy space. I conclude that as

¹See for example Herman and Pope (1973), Laver and Schofield (1990), Lijphart (1984), Luebbert (1986), and Strom (1990).

the largest party becomes larger and more central, the government changes from a surplus majority to a minimal winning coalition and from a minimal winning coalition to a minority government. I find empirical support for these conclusions in a study of eleven countries.

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In my theory, minority governments are signs of the largest party's strength. When the largest party is large and centrally-located, it has a good bargaining position. It is likely to be selected to form the government, and the government policy is close to its ideal policy, even if it is not selected to form the government. The other parties then have a bad bargaining position. Thus, the largest party need not offer them portfolios to obtain support for its government, and it can form a minority government.

Surplus majorities arise because I incorporate the possibility of a motion of noconfidence into the model. "The ability of a government to win votes of confidence is ... the key to its ability to remain in office" (Laver and Schofield 1990, 66). How the possibility of a no-confidence vote might motivate a party to form a surplus majority is explained succinctly by Luebbert (1986, 79):

A minimum winning coalition would contain no excess parties, and the withdrawal of one party would bring down the government. This situation permits a kind of blackmail of the dominant party (and all other parties) by a single dissatisfied party; for a party can leave the government at will, and thus compel the dominant party to choose between making concessions or renegotiating the entire government agreement. The leaders of the dominant party can avoid this dilemma if they can form a government that includes one or more unnecessary parties, none of which can bring down the government by itself.

Other models (implicitly) assume the game to be over once the government formed after the election passes the investiture vote. In this model a coalition party can withdraw its support and call for a vote of no-confidence. If the government loses this vote, a new government has to be formed. I assume there is more uncertainty as to which party will then be selected to form a government than after the election. The increased opportunity to form a government then might give one of the largest party's coalition parties an incentive to leave the coalition. There is no such incentive in a surplus majority, since I assume that the largest party can reshuffle the government with its remaining coalition parties to survive the no-confidence motion. With a surplus majority the largest party can thus react first to a party's withdrawal and credibly threaten to reshuffle the government with its remaining coalition party. Its coalition parties are thus in a prisoner's dilemma, which works to the advantage of the largest party.

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Surplus majorities are signs of the largest party's weakness. As the largest party becomes smaller and less central, its bargaining position worsens. It is less likely to be selected to form the government, and the government policy is farther from its policy position. The other parties then have a better bargaining position and a stronger incentive to leave a minimal winning coalition. To assure its coalition party's continued support for a minimal winning coalition, the largest party thus has to offer its coalition party a higher utility as it becomes smaller and less central. Since in a surplus majority the largest party's coalition parties do not have an incentive to withdraw their support, the largest party forms a surplus majority when it is small and not centrally-located.

The key institutional features of the model are the possibility of a no-confidence motion, the largest party's right to reshuffle the government with its remaining coalition party before a no-confidence vote, and its declining selection probability in the government formation process. Other specific assumptions concerning the party system and the government formation process render the model analytically more tractable but are not crucial for the results. The theory does not refer to countryspecific institutional characteristics. Nonetheless, such characteristics are important, and hence suitable for inclusion in future models.

In the following section I briefly review the literature on government formation, and in the third section I develop the model. The empirical tests are discussed in the fourth section, and in the fifth section I take a closer look at the types of government in four countries. I draw conclusions in the final section, and provide proofs in the appendix.

2 Literature Review

Parties are generally considered the relevant unit of analysis in game theoretical models of government formation, and they are assumed to be office-seeking or policyseeking. Laver and Schofield (1990) present an elaborate discussion of these two assumptions. The assumption that parties are motivated by holding office originated in the seminal work on coalition theory by Riker (1962). Using cooperative game theory and an institutionless setting, he predicted winning coalitions of minimum size. It is difficult, however, to explain the frequent occurrence of minority governments and surplus majorities with a purely office-seeking theory. Parties that do not have government portfolios prefer at least weakly to oppose the government rather than to support it. Moreover, parties prefer to keep more portfolios for themselves, rather than to give them to parties that are not needed to obtain a majority in the legislature.

Theories based on purely policy-seeking parties cannot distinguish between the three different types of government, although neither do they rule out any type of government. In a non-cooperative bargaining model, Baron (1991) finds that a centrally located party is likely to have the government policy located near its ideal policy, and gives an example of all political parties supporting the government proposed by the centrally located party. Baron (1993) provides a theory of government in which the parties' policy positions are endogenous, and points out that minority governments are consistent with the theory. Schofield (1993) concludes that large parties whose policy positions are located in the "core" of the policy space, i.e., such that no policy is preferred to their policy positions by a simple majority, tend to form minority governments and surplus majorities.

Between purely office-seeking and purely policy-seeking theories lies a range of theories that take account of both office and policy aspects. Most often one is considered to be a means of achieving the other. Using non-cooperative game theory, Austen-Smith and Banks (1988) model a political party as choosing a policy platform to enhance its chances of being in office after the election. Minority governments are consistent with their theory, although they do not discuss this possibility.

Laver and Shepsle (1990) as well as Austen-Smith and Banks (1990) develop a portfolio allocation model, using cooperative game theory. Parties have preferences over portfolio allocations only because the party holding a portfolio can choose the government policy in a set of dimensions. Governments need not be minimum winning coalitions, but no formal characterization is given of situations that lead to other types of government.

Using a model based on cooperative game theory, Axelrod (1970) predicts that parties will form "minimal connected winning coalitions." In connected coalitions, the coalition parties, i.e. parties that hold government portfolios, are adjacent to each other in a one-dimensional policy space. Minimal connected winning coalitions include no more parties than necessary to be connected and winning. A minimal connected winning coalition is not necessarily a minimal winning coalition, however. It can include small parties located between lager parties that themselves could form an unconnected majority coalition. De Swaan (1973) predicts the formation of a minimal connected winning coalition with the smallest ideological range.

Minority governments are often regarded as deviations from a norm, as signs of

crisis or systemic instability. I find the opposite; minority governments are a sign of the largest party's strengths of two types: 1) size and 2) location. Herman and Pope (1973) identify five situations as often leading to the formation of a minority government. Since their approach focuses on instability, it does not explain the predominance of minority governments in some political systems.

Strom (1990) argues that minority governments emerge in political systems with many institutional opportunities for the opposition to influence policy, since these decrease the benefits of government participation. He also points to the costs of holding government portfolios. Some parties prefer to remain in the opposition if joining a coalition government would harm their future electoral success, and hence, their prospects of holding office in the future. Minority governments are thus more likely in political systems in which voters are responsive to government performance and government formation is responsive to election results. Strom assumes that there is a negative incumbency effect, and that voters do not penalize opposition parties that support the government to the same extent as they penalize coalition parties.

Bergman (1993) also considers institutional characteristics. He finds that minority governments occur less often in political systems in which the government must obtain the parliament's support before it assumes office than in political systems in which the government must only be tolerated by the parliament and can stay in office as long as the parliament does not pass a no-confidence vote against it.

Luebbert (1986) expects minority governments to occur in "consensual democracies" and in "conflictual democracies." Consensual democracies are unpolarized political systems with "well-developed consensus-building mechanisms" outside the government. These consensus-building mechanisms reduce the parties' motivation for government participation. Conflictual democracies are the opposite of consensual democracies. They are polarized, non-corporatist political systems. The high polarization and the absence of consensus-building mechanisms hinder the formation of a majority government. Luebbert also claims that surplus majorities tend to occur in "dominated-competitive democracies." These are unpolarized, non-corporatist political systems dominated by one party. The absence of consensus-building mechanisms outside the government motivates the parties to participate in the government, and the low polarization facilitates policy agreement. The dominant party is the party without which no government can be formed, and is usually centrally located. It forms surplus majorities in order not to rely on a single party for support. In contrast I find that large, centrally located parties form minority governments, and that surplus majorities are signs of the largest party's weakness.

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Powell (1982) sees the formation of minority governments as a sign of high polarization, or "intense legislative conflict." He also argues that surplus majorities arise when the party that is forming a coalition cannot obtain a majority with parties that have policy views close to its own policy, and so needs the support of a party that is located more distantly. It will then try to include in the coalition the parties whose policy positions lie in between its own position and the position of the distant party, to obtain a government policy closer to its own ideal point.

Lijphart (1977) finds grand coalitions — not necessarily the same as surplus majorities — to be common in "consociational societies." These are pluralist societies, divided by "segmental cleavages," but characterized by political cooperation among segmental elites. Austria, Belgium and the Netherlands are examples of consociational societies.

Laver and Schofield (1990) mention three commonly cited reasons for the formation of surplus majorities: the requirement of special majorities for certain kind of legislation, as for constitutional amendments in Belgium, low intra-party cohesion, and a generally perceived need for a national unity government, as was the case in many European countries immediately after World War II. They admit, however, that many surplus majorities cannot be explained on these grounds. This brief literature review identifies the need for a theory that explains the formation of minority governments and surplus majorities more systematically and rigorously. The theory presented in this paper explains the type of government as a function of the largest party's size and location in the policy space. It allows for cross-country analysis as well as longitudinal analysis, which is less common in the literature. Parties are assumed to be driven by the desire to hold office as well as by the desire to influence government policy. The office-seeking component enables me to distinguish among the different types of government; the policy-seeking assumption makes it possible to explain them.

3 The Model

I assume there are three political parties motivated by both office and policy. Thus, two considerations enter the government formation process: cabinet portfolios must be distributed among parties, and a government policy p must be determined. Party *i*'s utility is assumed to be linear in the non-negative value s_i of its government portfolios. The parties are assumed to value the portfolios equally, with the total value of the cabinet portfolios equal to S, so $\sum_{i=1}^{3} s_i \leq S$. Party *i* has an ideal policy position \hat{p}_i in an n-dimensional policy space $P \subset \mathbb{R}^n$. I normalize the distance $d_{1,2}$ between the policy positions of parties 1 and 2, i.e. $d_{1,2} = 1$; and I assume the distance between the policy positions of parties 1 and 3 to be twice that distance, i.e. $d_{1,3} = 2$. Figure 1 shows such a configuration of the parties' policy positions.

Party *i* has Euclidean preferences over the government policy. Its utility U_i is thus linear in the value s_i of its government portfolios, and quadratic in the distance $d_{i.g}$ between its policy position and the government policy, as in Austen-Smith and Banks (1988):²

²The first terms of the parties' utility functions can be thought of as the product of the value of

Figure 1: Example of the Parties' Policy Positions.



$$U_i(s_i, p) = s_i - d_{i,g}^2$$

The government formation process is shown in Figures 2 and 3. In the first stage in Figure 2 the party *i* with the largest share π_i of the seats in the unicameral legislature gets the chance to form a government, as a reward for its election victory. I order the parties such that party 1 has the largest share of the seats and party 3 the smallest share. Moreover, I assume that party 1 does not have an absolute majority in the legislature, and to simplify the analysis I assume party 2 has one third of the seats. Party 1 thus makes a proposal in the first stage of the government formation process. The proposal specifies a distribution of government portfolios and a government policy. Party 1 can propose a surplus majority, a minimal winning coalition or a minority government. Figure 2 shows the subsequent stages in the government or a minimal winning coalition, and Figure 3 shows the subsequent stages if party 1 proposes a surplus majority.

Suppose party 1 proposes a surplus majority in the first stage. In the second stage in Figure 3 parties 2 and 3 then vote on party 1's proposal. If both parties accept the parties' portfolios and a parameter measuring how much the parties value the portfolios relative to the government policy. Since I assume all parties value the portfolios equally, this parameter can be omitted and S is the total value of the portfolios relative to the government policy.



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Figure 3: The Formation of a Surplus Majority.

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the proposal, the proposed government takes office.

After the government takes office, the larger party $k \neq 1$ that supported the government in the second stage can withdraw its support and call for a vote of noconfidence in the third stage.³ If it continues its support, the government stays in office until the end of the legislative period. If it withdraws its support, party 1 can reshuffle the government in the fourth stage, before the legislature votes on the motion of no-confidence. Party 1 can thus make the first move after coalition partner k withdraws its support. It remains in control at least until the vote of no-confidence. In particular, it can alter the distribution of portfolios among the remaining coalition partner.

The assumptions that party 1 can reshuffle the government to avoid its collapse and that it can do so only with its remaining coalition partner are important for explaining the formation of surplus majorities in this model. If any party could be included in a reshuffle or the two smaller parties could together form a government before party 1 can reshuffle its government, party 1 might not be able credibly to threaten party i that it will form a government with the other party if party i withdraws its support. This threat creates a prisoner's dilemma and plays an important role in the formation of surplus majorities.

In the fifth stage the legislature L votes on the motion of no-confidence. The government needs the support of an absolute majority including all parties that receive portfolios to survive the motion. Since I assume strict party discipline and no single party has an absolute majority in the legislature, a proposal needs the support of at least two parties. If the government survives, it continues to govern until the end

³For simplicity I assume that only the larger party can withdraw its support. Allowing both parties to withdraw their support does not change the results, unless they can withdraw their support simultaneously.

of the legislative period. If the government collapses, a new government has to be formed. In the sixth stage nature N selects the party to make a final proposal in the seventh stage. Party *i*'s selection probability is assumed to be equal to its share π_i of the seats in the legislature. Party 1 is thus no longer sure to retain control over the government formation process after the collapse of the government. In the first stage it was given the chance to form a government as a reward for its election victory and in the fourth stage it was given the right to reshuffle the government to survive a vote of no-confidence. If its government collapses, however, all parties can have a chance.

In the final stage the legislature votes on the final proposal. If the final proposal obtains an absolute majority, the proposed government takes office and remains in power until the end of the legislative period. If the new proposal is rejected, a caretaker government is formed. A caretaker government is asumed to give all parties a zero-utility.⁴ ⁵

If party 1's initial proposal is rejected by at least one party in the second stage, as in the three right branches of Figure 3, it is not certain to be given another chance to form a government.⁶ It is likely to retain control over the government formation process, but other parties might be given a chance too. In particular, the parties'

⁴The implications of this assumption are discussed in the appendix.

⁵The government policy that minimizes the sum $\sum_{i=1}^{3} d_{i,g}^2$ of the three parties' disutilities due to the distance between their policy positions and the government policy, is the average of the three parties' policy positions, i.e. the centroid. To enable a caretaker government to give all parties a zero-utility the value S of the government portfolios thus has to be greater than or equal to $\sum_{i=1}^{3} d_{i,c}^2$, where $d_{i,c}$ is the distance between party *i*'s policy position and the centroid. The maximum value of $\sum_{i=1}^{3} d_{i,c}^2$ is $\frac{14}{3}$. Therefore I assume that $S \ge \frac{14}{3}$.

⁶Since party 1 is not certain to be selected again, it cannot credibly threaten to exclude from the government a party that does not support its initial proposal. If it could credibly make this threat, party 1 need not give parties 2 and 3 portfolios to attract their support. Thus, party 1 would not propose surplus majorities.

selection probabilities are assumed to be the average of their selection probabilities in the first and seventh stages. If the new proposal obtains an absolute majority in the fifth stage, the proposed government takes office and remains in power until the end of the legislative period. If the new proposal is rejected, a caretaker government is formed.

It is important in this model that party 1's selection probability declines as it fails to form a government and as its government collapses. Since party 1's selection probability declines, its bargaining position worsens. In minority governments and minimal winning coalitions, the party $k \neq 1$ that supports the government thus achieves a higher expected utility by withdrawing from the government in the third stage than by rejecting the proposed government in the second stage. In a surplus majority, however, party 1 can reshuffle the government with its remaining coalition party if a coalition party withdraws its support. Party 1 can thus avoid the collapse of the government and is in a stronger bargaining position. Thus, party 1 would not form a surplus majority if its bargaining position improved during the government formation process.

Suppose party 1 proposes a minimal winning coalition or a minority government in the first stage, as in the right branch of Figure 2. The government formation process is then similar to the process described above. However, if the party $k \neq 1$ that accepted the government party 1 proposed in the second stage withdraws its support in the third stage, the government collapses. Party 1 cannot reshuffle the government in the fourth stage, since there is no coalition partner left.

The model assumes complete information. The parties know each other's preferences and the sequential structure of the model. At any stage in the model they have perfect information about the actions taken in prior stages. The equilibrium concept is subgame perfect Nash. The following proposition presents the main results. It explains the type of government as a function of the largest party's size and its location in the policy space relative to the other parties.⁷

Proposition: As the largest party becomes larger and more central, the government changes from a surplus majority to a minimal winning coalition, and from a minimal winning coalition to a minority government.

The proof of this proposition is presented in the appendix. Here, I provide some intuition. Suppose party 1 seeks the support of only 1 other party i for a minimal winning coalition or a minority government in the first stage. By rejecting the proposed government in the second stage party i obtains its expected utility v_i^f after the failure (f) of party 1's initial formation attempt. By withdrawing its support in the third stage party i obtains its expected utility v_i^c after the collapse (c) of the government, since party i's withdrawal of support leads to the collapse of the government. Party i's expected utility v_i^c after the collapse of the government is higher than its expected utility v_i^f after the failure of party 1's initial formation attempt, because its selection probability is greater after the collapse of the government. Thus, to obtain party i's support for a lasting government, party 1 has to offer party i its expected utility v_i^c after the collapse of the government. Since it is smaller and farther from party 1 than is party 2, party 3's expected utility v_3^c after the collapse of the government is lower than party 2's expected utility v_2^c after the collapse of the government. Therefore, party 1 seeks party 3's support in the first stage, even though party 1 is closer to party 2.

As party 1 becomes larger, party 3 becomes smaller and is thus less likely to be selected to form a government in the seventh stage. As party 1 becomes more central, party 3 is more distant from party 2 and the government policy is farther from its

⁷I measure a party's centrality by 1 minus its distance from the centroid as a proportion of the sum of all parties' distances from the centroid, i.e. $1 - d_{i.c} / \sum_{i=1}^{3} d_{i.c}$.

policy position, whether or not it is selected to form a government in the seventh stage. Thus, party 3's expected utility v_3^c decreases as party 1 becomes larger and more central. As party 3's expected utility v_3^c decreases, party 1 need not offer party 3 as many government portfolios. Party 1's utility thus increases as it becomes larger and more central. If party 1 is sufficiently large and central, it is optimal for party 1 to attract party 3's support without offering party 3 any portfolios and to form a minority government.⁸ A minority government is thus a sign of the largest party's strength, rather than a sign of instability or crisis.

Suppose party 1 seeks the support of both the other parties for a surplus majority in the first stage. If the legislature rejects the proposed government in the second stage, party *i* obtains its expected utility v_i^f after the failure of party 1's initial formation attempt. By withdrawing its support in the third stage party *i* obtains a negative utility v_i^r , since party 1 successfully reshuffles (*r*) the government and continues to govern with its remaining coalition partner.

Party *i*'s expected utility v_i^f after the failure of the initial formation attempt in the second stage is higher than its utility v_i^r after it withdraws from the government, because after the failure of the initial formation attempt party *i* can become a coalition party. Thus, for a lasting surplus majority, party 1 only has to offer the other parties their expected utilities v_i^f after the failure of the initial formation attempt. In a surplus majority party 1's coalition partners are in a prisoner's dilemma. Even though coalition partner *i*'s expected utility v_i^e after the collapse of the government is higher than its utility in the government, it does not withdraw its support, because party 1 would successfully reshuffle the government and continue to govern with the remaining party. Party *i* would then receive a negative utility. Each coalition partner prefers

⁸How central and large party 1 needs to be for a minority government to be optimal depends on the total value S of the government portfolios. As parties value the portfolios more relative to the government policy (S increases), party 1 needs to be larger and more central.

that the other party leave the government, because its bargaining power and hence its expected utility would be higher. Both thus stay which causes their bargaining power to be lower.

As seen above, party 1's utility in a minimal winning coalition with party 3 decreases as it becomes smaller and less central. If party 1 is sufficiently small and out of the center, party 1 prefers to form a surplus majority by offering both the other parties their expected utilities v_i^f after the failure of the initial formation attempt. Therefore, the government changes from a minimal winning coalition to a surplus majority as party 1 becomes smaller and more out of the center.

The types of governments the theory predicts are shown in Table 3. If party 1 is large and centrally-located, the theory predicts a minority government (MG). If the largest party is small and located out of the center, surplus majorities (SM) are expected. Minimal winning coalitions (MWC) are formed in the intermediate cases. Figure 4 shows the types of government as a function of the largest party's size and centrality — 1 stands for maximum centrality — with the total value of the portfolios $S = 15.^9$ In the upper right corner of Figure 4 party 1 is large and centrally-located. Thus, it forms minority governments. In the lower left corner it is small and located out of the central, and forms surplus majorities. Elsewhere it forms minimal winning coalitions.

4 Empirical Tests

To test this theory, I examine data on government formation since World War II in minority situations in eleven parliamentary democracies: Austria, Belgium, Canada,

⁹The measure of centrality used in Figure 4 is different from the measure used elsewhere in the paper. It is party 2's coordinate x on the X-axis, with party 1 being located at the origin and party 3 having coordinate -2x on the X-axis.

Table 1: Prediction of Government Type.

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| Dominant Party's | Dominant Party's Position | | | |
|------------------|---------------------------|---------|--|--|
| Size | Out of Center | Central | | |
| Large | MWC | MG | | |
| Small | SM | MWC | | |

Figure 4: Types of Government with S = 15.



Denmark, Germany, Ireland, Italy, Luxembourg, the Netherlands, Norway, and Sweden.¹⁰ This results in a sample of 184 governments.¹¹

Minority governments account for about forty percent of this sample, surplus majorities for slightly under twenty percent, and minimal winning coalitions for forty percent. These proportions roughly correspond to the data I reported in the introduction. The respective percentages for the governments included in this study can be found in Table 2.

The selection of countries was based solely on availability of data. In particular, the need for data on parties' policy positions during election campaigns has limited the number of countries included in this study.¹² The rules for counting governments

¹⁰More precisely, the governments included are those formed in minority situations in the following countries and periods: Austria 1945-1983, Belgium 1946-1985, Canada 1945-1992, Denmark 1945-1984, Germany 1949-1990, Ireland 1944-1987, Italy 1946-1987, Luxembourg 1947-1984, Netherlands 1946-1982, Norway 1945-1989, Sweden 1948-1985. Austria, Belgium, Canada, Germany, Ireland and Luxembourg (used to) have three party systems. I restrict my attention to minority situations. This does not bias the results of the empirical analysis in favor of the theory developed in this paper. When applied to majority situations, the theory predicts that the majority party will form the government without offering any other party government portfolios; since the majority party behaves as a unitary actor in this model, it is certain to receive support for its government from all its members in the investiture vote as well as in a no-confidence vote. Empirically almost all governments formed in majority situations have been single-party majority governments. Consequently, adding majority situations to the data set would greatly improve the empirical performance of the theory. I have preferred not to do so and instead concentrate on minority situations.

¹¹I also subjected smaller samples of governments to the tests. These samples were obtained by removing from the full sample transitional governments, governments that consisted of the same parties as the previous government and were not formed after an election, or governments that did not include the largest party. The conclusions concerning sign and significance of the estimated coefficients of the variables of the model were the same as those reached for the full sample. Hence, I do not report those results here.

¹²I used the data on parties' policy positions gathered by Budge, et al. (1987) for the "manifesto-

| | MG | MWC | SM | Total |
|-------------|------|------|------|-------|
| Austria | 1 | 9 | 0 | 10 |
| Belgium | 3 | 17 | 5 | 25 |
| Canada | 7 | 0 | 0 | 7 |
| Denmark | 20 | 3 | 0 | 23 |
| Germany | 0 | 14 | 1 | 15 |
| Ireland | 6 | 3 | 0 | 9 |
| Italy | 15 | 4 | 18 | 37 |
| Luxembourg | 0 | 11 | 0 | 11 |
| Netherlands | 3 | 5 | 9 | 17 |
| Norway | 12 | 3 | 0 | 15 |
| Sweden | 10 | 5 | 0 | 15 |
| Total | 77 | 74 | 33 | 184 |
| Percentages | 41.8 | 40.2 | 17.9 | 100 |

 Table 2: Governments Formed in Minority Situations, 1945-1992.

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are those that were followed by Strom (1990), except that governments that did not pass the initial investiture vote were excluded.¹³

Using this sample, I estimated ordered probit equations with the type of government formed as the dependent variable, taking the following values: -1 for a minority government, 0 for a MWC, and 1 for a surplus majority.¹⁴ First, only the two variables emerging from the theory in this paper as explanatory were included as independent variables: the share of the seats obtained by the dominant party, and a measure for the non-centrality of that party.

Data on the first independent variable were found in Mackie and Rose (1990). To calculate a measure of non-centrality I used data on party policy positions gathered from manifestos by Budge, et al. (1987).¹⁵ Although the coding of manifesto data might be questioned, it is the most thorough, available source of information on project." In that project parties' policy positions for post-World War II elections in twenty democracies are coded on a large number of dimensions based on the parties' election programs. Countries with first-past-the-post systems which have (almost) never had a minority situation and countries with political systems which usually lead to pre-electoral coalition formation, such as the two-step French electoral system were omitted from this study for obvious reasons.

¹³These governments are: in Belgium, Spaak I (March 1946); in Italy, De Gasperi XI (July 1953), Fanfani I (January 1954), Andreotti I (February 1972), Andreotti V (March 1979) and Fanfani VI (April 1987).

¹⁴Some readers may consider an unordered logit a more appropriate estimation technique than probit for these data. At issue is whether it is reasonable to consider minority governments, minimal winning coalitions and surplus majorities "ordered," or whether they are merely different types of government. As there are reasonable arguments both ways, I estimated both probit and logit models, and found that results were essentially the same. The logit analysis is available upon request.

¹⁵In particular, I used the raw data, i.e. the percentages of (parts of) sentences in the manifestos that dealt with certain issues. The issues I selected for each country were chosen based on the frequency with which they occurred in the policy manifestos in that country and on their correlation with the two main policy dimensions that were the result of a factor analysis executed by Budge, et al. (1987).

what parties stand for. In addition, these data allow me to obtain a measure of non-centrality that varies across elections within the same country.

From these data I determined party positions on two dimensions. This was done for every party, for every election, in the eleven countries mentioned above. In general, the two dimensions are an economic left/right dimension, and a social progressive/conservative dimension. Subsequently, I calculated a measure for the dominant party's non-centrality, by dividing the squared distance between this party's position and the centroid of all parties' positions by the sum of the squared distances of all parties' positions from this centroid.¹⁶

Next, in a second regression, I expanded the set of independent variables to include those most frequently broached as determinants of type of government. I limited my attention to those relevant characteristics of party systems that can be measured relatively uncontroversially. Also, I did not attempt to include institutional elements which might enhance the formation of a particular type of government.¹⁷

My additional variables, accordingly, are a measure for fractionalization, and a measure for polarization of the political landscape. As a measure of fractionalization, I used Rae's index of fractionalization of seat shares (Rae 1971): $1 - \sum_{i=1}^{n} S_i^2$, where n is the number of parties represented in the legislature and S_i is the share of the seats in the legislature won by party i. To measure the extent to which a legislature is polarized, I summed the seat shares of parties usually regarded in the literature as belonging neither to the left nor to the right and substracted this from one.¹⁸

¹⁶The centroid is the average party position on every dimension.

¹⁷Opportunities for the opposition to influence policy in parliamentary committees, for example, might enhance the formation of minority governments, as claimed by Strom (1990).

¹⁸These parties are: in Austria: FPÖ; in Belgium: CVP/PSC, FDF, RW, VU; in Canada: none; in Denmark: RF, RV, V; in Germany: FDP; in Ireland: none; in Italy: PRI, PSDI; in Luxembourg: DP; in the Netherlands: ARP, CDA, CHU, KVP; in Norway: DNF/DLF, V; in Sweden: CP, FP. My measure of polarization is different from the approach taken by Powell (1982) and Strom (1990).

The results are shown in Table 3. They give strong and consistent support to the theory developed in this paper. In both sets of estimates the coefficients of the two variables of the model, share and non-centrality, have the predicted sign, and are significant at the .01 level.¹⁹ Although the percentages correctly predicted might not look particularly high, it is well to remember that the dependent variable can take three different values. Hence, prediction rates are naturally lower than in the more usual dichotomous probit. Fractionalization is also found significantly to enhance the formation of minority governments, whereas polarization seems to have no effect. The significance of the fractionalization variable suggests that there are other important elements which play a role in government formation, and should be included into future research. Fractionalization might improve the largest party's bargaining position and thus enhance the formation of minority governments. The empirical results do indicate that this theory has identified important variables in determining what type of government is formed.

They measure polarization by the percentage of votes or seats won by "extremist" (non-democratic, separatist or protest) parties. However, using their classification of parties this measure reaches high values in only one of the countries studied here: in Italy, where the Communist Party is classified as an extremist party. Apart from possible arguments that the former Italian Communist Party was not more leftist than many socialist parties in other European countries, the problem is that this variable would behave like a dummy variable for Italy. Rather than reclassifying some parties I have preferred to redefine the notion of polarization in a way that better reflects the extent to which a political system is divided into two blocs. However, using this definition high polarization does not necessarily imply "large" ideological differences within the party system. Therefore, this variable might also be called "bifurcation."

¹⁹To see whether these findings are due largely to an outlying country. I estimated eleven models including a dummy variable for one of the countries involved. In all instances the conclusions concerning the two variables of the theory remained unaltered.

| | Equat | ion 1 | Equation 2 | | |
|---------------------|-------------|-------------|-------------|-------------|--|
| | Coefficient | t-statistic | Coefficient | t-statistic | |
| Constant | 2.09 | 3.43 | 8.47 | 3.28 | |
| Share | -5.43 | -3.75 | -10.75 | -3.94 | |
| Non-centrality | 0.45 | 3.91 | 0.38 | 3.26 | |
| Fractionalization | | | -5.73 | -2.55 | |
| Polarization | | | -0.12 | -0.22 | |
| Threshold | 1.24 | 10.02 | 1.25 | 10.07 | |
| Correctly predicted | 47.28 % | | 55.44 % | | |

Table 3: Probit Estimates.

5 Country-Specific Discussion

To illustrate the theory I take a closer look at four countries, one in which MWC's are predominant, one in which minority governments as well as surplus majorities occur frequently, one in which surplus majorities seem to be the rule, and one in which there has usually been a minority government.

5.1 Belgium

For the purposes of this study Belgian political history since World War II can be divided into two periods: before and after 1958. Before 1958 Belgian politics were dominated by the "Royal Question" and the "Schools War." These two crises split the country along a progressive/conservative line, which in Belgium is more appropriately labeled anti-clerical/clerical. The political landscape consisted of two blocs. One was formed by the christian democrats, with an absolute majority in the North (Flanders). They either obtained a majority in the legislature or narrowly failed to do so. The other was composed of the socialists, who had an absolute majority in the South (Wallonia), and the much smaller liberals and communists. These parties managed to keep the christian democrats out of office for about five years during this period.

All six governments formed in minority situations between the 1946 and 1958 general elections were MWC's. Our theory predicts this type of government well. The christian democrats obtained more than 44 percent of the vote in all the elections between 1946 and 1958, but the party's extreme location in the policy space did not allow it to find external support for a minority government.

The "Royal Question" and the "Schools War" were both resolved in a compromise worked out under christian democratic, one-party governments, in 1950 and 1958, respectively. The two major points of tension along the anti-clerical/clerical line were thus removed. Attention then shifted to economic policy and growing problems between Flemish and Walloons. The christian democrats evolved into a center party. However, they remained unable to find external support for a minority government. MWC's were still the predominant type of government. In 1973 the first surplus majority government was formed, and this type of government has become common. This can be explained by the christian democrats' electoral decline which started after 1958 and which has lasted almost continuously since, except for a few years in the 1970's under the leadership of the popular conservative prime minister Leo Tindemans. Nonetheless, the Belgian christian democrats have been in power uninterruptedly since 1958.

It is surprising how well the two independent variables in the theory, the strength and the location of the dominant party in a left/right, progressive/conservative space, can account for the types of government formed in Belgium, even without including any consideration of the Flemish/Walloon dimension, which has dominated political life for the past three decades, and has split all political parties along linguistic lines.

5.2 Italy

Until 1992 political life in Italy was also dominated by the christian democrats. They were in government without interruption throughout the entire postwar period. Very few of these governments, however, were MWC's. Between 1953 and 1963, two thirds of the governments formed were minority governments, and none was a surplus majority. More than two thirds of the governments formed between 1963 and 1992, and all those formed between 1980 and 1992 were surplus majorities, with only one MWC in the whole period. From 1974 until 1980, there was a relatively long period of minority government. I now look for an explanation for the frequent occurrence of minority governments and surplus majorities in the same political system.

As in Belgium, the christian democrats suffered a gradual erosion of their electoral support, from about 50 percent down to 30 percent in 1992.²⁰ These losses occurred in four stages. The christian democrats lost around five percent in each of the elections of 1953, 1963, 1983, and 1992. Their support remained remarkably stable in between these four election defeats.

In addition to this electoral decline, the christian democrats' location in the Italian political landscape changed over time. They were generally regarded as a center or center-right party. However, in Italian politics there were few or no important parties to their right. In the early postwar years, there were three such parties: the liberals (on economic issues), the monarchists, and the fascists. From 1953 until 1976, these three parties polled between 10 and 15 percent in all general elections. The monarchists, however, disappeared as an independent political party after the 1968 general election, whereas the liberals, who had obtained between 3 and 12 percent of the vote until 1976, won only about 2 percent in all four subsequent general elections. The fascists' share of the vote remained around 5 percent. Hence, the two remaining

²⁰The post-1992 period is not studied in this paper.

parties right of the christian democrats represented less than 10 percent of the voters. Thus, the christian democrats found themselves at the right rather than in the center of Italian politics, even though they might have been considered a center party, when compared to the right in other countries, and even though their policy positions might not have changed over time.

These two simultaneous evolutions, the christian democrats' decline and their de facto move to the right, explain Italy's transition from minority government to surplus majority government. As a large center party they were able to form minority governments. After heavy electoral losses and the near-disappearance of rightwing parties, they preferred surplus majorities. Although the theory does not include intra-party factional politics, which are extremely important in Italy, it predicts the evolution well.

5.3 The Netherlands

Although it is customary to characterize Dutch postwar politics as having been dominated by christian democrats, as in Belgium and Italy, such a characterization requires elaboration. Until the seventies there was no single christian democratic party, but instead three independent religious parties: one catholic, and two protestant. Religious issues could not be represented by a single dimension. In addition to a clerical/anti-clerical, there was a catholic/protestant dimension. Some degree of cooperation between the religious parties existed, but they often did not enter or leave a coalition government together. The largest of the three, the catholic party, usually polled about 30 percent of the vote until the late sixties. It was the largest Dutch political party, except for two elections in the fifties, when the socialists outpolled it.

Whether the catholics or the socialists were the largest party, they were still quite small. No single election after World War II resulted in a party obtaining more than 35 percent of the vote. As for policy positions, the religious parties were centrally located on economic issues, but each defended the interests of its own church. Thus, like the socialists, they were all located some distance from the center. Hence, it is no surprise that until 1977 most governments were surplus majorities.

By 1972 the catholic party had lost almost half of its support, and the vote shares of the other two religious parties were also declining. The socialists had become the major political party. The three religious parties merged to form a united christian democratic party, which is now the largest political party in the Netherlands. Subsequently, MWC's have become predominant. All governments since 1982 have been MWC's with christian democratic prime ministers. Until 1994 the christian democratic party won about 35 percent of the vote and was located closer to the center on religious issues than the catholic party used to be. Hence, the theory predicts well these evolutions in the types of government.

5.4 Sweden

Sweden had social democratic, one-party governments for most of the postwar period. Except for 1968, however, they did not obtain an absolute majority. It is no surprise that social democratic minority governments were predominant, since they obtained more than 43 percent of the seats in every election until the 1991 general election. They took a policy position close to the center, even though their position might have been considered quite far to the left of the center in many other democracies. In this respect the Swedish social democrats found themselves in a situation similar to that of the Italian christian democrats before the sixties: consistently winning around 45 percent of the vote, and with parties to their right as well as to their left that could give them a majority. Until 1991, however, Swedish politics, remained quite stable. The electoral support for the social democrats, the combination of the three major center-right parties, and the communists did not fluctuate, and the parties' policy positions did not change significantly.

6 Conclusion

This paper formally explains the formation of minority governments and surplus majorities in parliamentary systems. It presents a theoretical model to provide insight into the conditions favoring the formation of the different types of government, and concludes that as the largest party becomes larger and more central, the government changes from a surplus majority to a minimal winning coalition and then from a minimal winning coalition to a minority government.

If the largest party is large and centrally located, it has a favorable bargaining position. Since it is large, it is likely to keep control of the government formation process. Moreover, government policy is close to its policy position, because it is centrally located. Therefore, it does not have to offer other parties government portfolios to attract support for a government, and it can form a minority government. A minority government thus indicates a strong largest party.

If the largest party is small and not centrally located, it does not have such a favorable position. It is more likely to lose control of the government formation process, and in that case government policy is farther from its policy position. Thus, it has to offer the other parties portfolios to attract their support. In a surplus majority, however, the largest party's coalition parties are in a prisoner's dilemma, since the largest party can reshuffle the government with its remaining coalition partner if one coalition party withdraws its support from the government. The largest party thus offers portfolios to more parties than necessary to obtain a majority in order not to be too dependent on a single coalition party for its majority. Therefore, surplus majorities are to be expected if the largest party is small and not centrally located.

Surplus majorities are thus signs of the largest party's weakness. In intermediate cases minimum winning coalitions form.

In an empirical study of government formation in eleven post-World War II parliamentary democracies I find strong and consistent support for the predictions of the theory.

Appendix

Equilibrium in Stages 6 through 8

Suppose the initially formed government — whatever its type is — collapses. In the sixth stage of the government formation process, as shown in Figure 2 and Figure 3, nature then selects a party i to make a final proposal, and parties vote whether to accept or reject this proposal in the last stage of the government formation process. If the proposal obtains an absolute majority in the legislature, the new government takes office. If a majority rejects the proposal, a caretaker government is formed. This caretaker government gives all parties a zero-utility. Party j thus votes in favor of the proposal if it obtains a non-negative utility under the proposal.

In the seventh stage party *i* gets a chance to form a government with probability equal to its share π_i of the seats in the legislature. Suppose party *i* seeks party *j*'s support. To obtain party *j*'s support, party *i* has to offer party *j* a non-negative utility. Party *j*'s utility is linear in the value of its government portfolios s_j and quadratic in the distance $d_{j,g}$ between its policy position \hat{p}_j and the government policy *p*. Since a party's utility is strictly increasing in its government portfolios, party *j* receives all the portfolios that party *i* does not keep for itself: i.e., $s_j = S - s_i$. Party *i*'s maximization problem thus is:

 $\max_{s_i,p} \ U_i = s_i - d_{i,g}^2$

s.t.
$$S - s_i - d_{j,g}^2 \ge 0$$

 $s_i \ge 0$
 $S - s_i \ge 0.$

The first constraint ensures party j a non-negative utility. The other two constraints state that the value of the portfolios allocated to either party must be nonnegative.

Party *i* proposes the midpoint between the policy positions of parties *i* and *j* as government policy.²¹ Party *j* is offered just enough portfolios to obtain a zeroutility: i.e., $s_j = d_{j,ij}^2$, where $d_{j,ij}$ is the Euclidean distance between party *j*'s policy position and the midpoint between the policy positions of parties *i* and *j*. Party *i* keeps portfolios worth $s_i = S - d_{j,ij}^2$ for itself and incurs a disutility equal to $d_{i,ij}^2$. Its utility thus is: $U_i = S - 2d_{i,ij}^2$. Party *i* is willing to make this proposal because it receives a non-negative utility: i.e. $S - 2d_{i,ij}^2 \ge 0.^{22}$ The government formed is a minimal winning coalition. The third party *k* obtains a negative utility, since it does not receive any portfolios.

Since party *i*'s utility is decreasing in the distance between its own policy position and party *j*'s policy position, it seeks the support of the party closer to itself. Party 1 is selected to form a government with probability $\pi_1 \in [\frac{1}{3}, \frac{1}{2}]$. It seeks the support of party 2, since party 2 is closer than party 3: i.e., $d_{1,2} = 1 < d_{1,3} = 2$. Thus, if it is selected to form a government, party 1 obtains a utility $U_1 = S - 2d_{1,12}^2 =$ $S - \frac{1}{2}$. Party 2 receives a zero-utility, since party 1 offers party 2 its utility under a caretaker government. Party 3 receives a negative utility $U_3 = -d_{3,12}^2$, since it does

²¹This result was also obtained by Austen-Smith and Banks (1988).

²²The distance between parties 2 and 3 has a maximum value of 3. The maximum value of $d_{i,ij}^2$ is thus $\frac{9}{4}$. The total value of the government portfolios was assumed to be greater than or equal to $\frac{14}{3}$ to enable a caretaker government to give all parties a zero-utility. Party *i*'s utility is thus non-negative.

not receive government portfolios but incurs a disutility due to the difference between the government policy and its policy position.

Party 2 is selected with probability $\pi_2 = \frac{1}{3}$. It seeks party 1's support, since party 1 is closer than party 3: i.e., $d_{1,2} = 1 \le d_{2,3}$. It then obtains a utility $U_2 = S - 2d_{2,12}^2 = S - \frac{1}{2}$. Party 1 receives a zero-utility and party 3 a negative utility $U_3 = -d_{3,12}^2$.

Party 3 is selected with probability $\pi_3 = \frac{2}{3} - \pi_1$. It seeks party 2's support, if it is closer to party 2 than to party 1: i.e., if $d_{2,3} < d_{1,3} = 2$. If it is selected to form a government, it obtains a utility $U_3 = S - 2d_{3,23}^2$. Party 2 then receives a zero-utility and party 1 a negative utility $U_1 = -d_{1,23}^2$. The parties' expected utilities v_i^c after the collapse of a government then are:

$$\begin{split} v_1^c &= \pi_1 (S - \frac{1}{2}) - (\frac{2}{3} - \pi_1) d_{1.23}^2, \\ v_2^c &= \frac{1}{3} (S - \frac{1}{2}), \\ v_3^c &= -(\frac{1}{3} + \pi_1) d_{3.12}^2 + (\frac{2}{3} - \pi_1) (S - 2d_{3.23}^2). \end{split}$$

Party 2's expected utility is larger than party 3's expected utility, since its selection probability is higher and it is closer to party 1.

Party 3 seeks party 1's support, if it is selected and closer to party 1 than to party 2: i.e., if $d_{2,3} \ge d_{1,3} = 2$. If it is selected to form a government, it obtains a utility $U_3 = S - 2d_{3,13}^2 = S - 2$. Party 1 then receives a zero-utility and party 2 a negative utility $U_2 = -d_{2,13}^2$. The parties' expected utilities v_i^c after the collapse of a government then are:

$$\begin{aligned} v_1^c &= \pi_1 (S - \frac{1}{2}), \\ v_2^c &= \frac{1}{3} (S - \frac{1}{2}) - (\frac{2}{3} - \pi_1) d_{2.13}^2, \\ v_3^c &= -(\frac{1}{3} + \pi_1) d_{3.12}^2 + (\frac{2}{3} - \pi_1) (S - 2). \end{aligned}$$

Again, party 2's expected utility is higher than party 3's expected utility. The expected utilities of parties 2 and 3 are lower than in the previous case, since they are

more distant from each other $(d_{2,3} \ge 2)$ and both seek party 1's support. Party 1's expected utility is higher than in the previous case, since both the other parties seek its support.

Minimal Winning Coalitions and Minority Governments

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Suppose party 1 initially formed a minimal winning coalition or a minority government, as in the right branch of Figure 2. Also assume that the party whose support it obtained withdraws its support in the third stage. Since party 1 is the only remaining coalition party, it cannot reshuffle the government and the government collapses. Therefore, party i withdraws its support in the third stage if its utility v_i^g under the government is lower than its expected utility v_i^c after collapse.

In the second stage party *i* supports a government if its utility v_i^p when the government takes office is higher than its utility v_i^f when the government fails. Suppose the government fails in the second stage. Parties then get a second chance to form a government. The parties' selection probabilities are assumed to be the average of their selection probabilities in the first and seventh stages. Party 1's selection probability is thus $\frac{1+\pi_1}{2}$, since it is certain to be selected in the first stage and its selection probability at the end of the process is equal to its share of the seats in the legislature. Party 2's selection probability is $\frac{\pi_2}{2} = \frac{1}{6}$, and party 3's selection probability is $\frac{\pi_3}{2} = \frac{1}{3} - \frac{\pi_1}{2}$. The parties' proposal strategies are the same as in the seventh stage, but their expected utilities are different since their selection probabilities are different. If parties 2 and 3 are closer to each other than to party 1, the parties' expected utilities v_i^f after the failure of party 1's initial formation attempt are:

$$\begin{aligned} v_1^f &= \left(\frac{1+\pi_1}{2}\right) \left(S - \frac{1}{2}\right) - \left(\frac{1}{3} - \frac{\pi_1}{2}\right) d_{1.23}^2, \\ v_2^f &= \frac{1}{6} \left(S - \frac{1}{2}\right), \\ v_3^f &= -\left(\frac{2}{3} + \frac{\pi_1}{2}\right) d_{3.12}^2 + \left(\frac{1}{3} - \frac{\pi_1}{2}\right) \left(S - 2d_{2.23}^2\right) \end{aligned}$$

If parties 2 and 3 are closer to party 1 than to each other, the parties' expected utilities v_i^f are:

$$v_1^f = \left(\frac{1+\pi_1}{2}\right)(S - \frac{1}{2}),$$

$$v_2^f = \frac{1}{6}(S - \frac{1}{2}) - \left(\frac{1}{3} - \frac{\pi_1}{2}\right)d_{2.13}^2,$$

$$v_3^f = -\left(\frac{2}{3} + \frac{\pi_1}{2}\right)d_{3.12}^2 + \left(\frac{1}{3} - \frac{\pi_1}{2}\right)(S - 2)$$

. .

If party 1 seeks party *i*'s support in the first stage, party *i*'s utility v_i^p when the government takes office is at least its expected utility v_i^c after collapse, since it can obtain v_i^c by withdrawing its support in the third stage. To attract party *i*'s support for a lasting government in the first stage, party 1 thus needs to offer party *i* its expected utility v_i^c after collapse. By voting against the government in the second stage, party *i* only receives utility v_i^f , which is lower than v_i^c because party *i*'s selection probability after the failure of party 1's initial formation attempt is lower than after the collapse of the government.²³ Party 1's maximization problem in the first stage then is:

 $\max_{s_{1},p} \quad U_{1} = s_{1} - d_{1,g}^{2}$ s.t. $S - s_{1} - d_{i,g}^{2} \ge v_{i}^{c}$

²³Party 2's expected utility v_2^f after failure is half its expected utility v_2^c after collapse. If party 3 is closer to party 2 than to party 1, party 2's expected utility v_2^c after collapse is $\frac{1}{3}(S-\frac{1}{2})$, which is positive since it was assumed that $S \ge \frac{14}{3}$ to enable a caretaker government to give all parties a zero-utility. If party 3 is closer to party 1 than to party 2, party 2's expected utility v_2^c after collapse decreases as party 1 becomes smaller (π_1 decreases) and more central ($d_{2.12}$ increases). The minimum value of v_2^c is $\frac{1}{3}S - \frac{3}{2}$, which is also positive. Thus v_2^c is higher than v_2^f . After the failure of party 1's initial formation attempt, party 3 receives a positive utility of $S - 2d_{2.23}^2$ or $S - \frac{1}{2}$, if it is selected to form the government. If it is not selected, it obtains a negative utility of $-d_{3.12}^2$, since neither party 1 nor party 2 seek its support. After the failure of party 1's initial formation attempt party 3's selection probability is lower than after the collapse of the government. Thus v_3^c is higher than v_3^f .

$$s_1 \ge 0$$
$$S - s_1 \ge 0.$$

. .

Suppose party 1 seeks party 3's support. Then there are two different situations depending on party 3's expected utility v_3^c .

(1) $d_{3.13}^2 = 1 \ge -v_3^c$. For a government policy equal to the midpoint between the policy positions of parties 1 and 3, party 3's utility if it gets no government portfolios is lower than its expected utility v_3^c . In this case party 1 proposes the midpoint between its policy position and the policy position of party 3 as government policy, and it offers party 3 government portfolios such that party 3 obtains its expected utility v_3^c , i.e. $s_3 = 1 + v_3^c$, which is non-negative in this case. If party 3 is closer to party 2 than to party 1, party 1's utility U_1^{mw3} under a minimal winning coalition with party 3 is $U_1^{mw3} = S - 2 - v_3^c = S - 2 + (\frac{1}{3} + \pi_1)d_{3.12}^2 - (\frac{2}{3} - \pi_1)(S - 2d_{2.23}^2)$. If party 3 is closer to party 1 than to party 2, party 1's utility is $U_1^{mw3} = S - 2 - v_3^c = (\frac{1}{3} + \pi_1)(S - 2 + d_{3.12}^2)$. In either case party 1 prefers this minimal winning coalition with party 3 to the failure of its formation attempt and to the collapse of the government, since its utility U_1^{mw3} is higher than its expected utilities v_1^f and v_1^c .²⁴

(2) $-v_3^c > 1$. With a government policy equal to the midpoint between the policy positions of parties 1 and 3, party 3 obtains more than its expected utility v_3^c even

²⁴To see this, if party 3 is closer to party 2 than to party 1, party 1 prefers the minimal winning coalition with party 3 to the failure of its formation attempt, because $U_1^{mw3} - v_1^f \ge 0$. In particular, $U_1^{mw3} - v_1^f = S - 2 + (\frac{1}{3} + \pi_1)d_{3.12}^2 - (\frac{2}{3} - \pi_1)(S - 2d_{2.23}^2) - (\frac{1+\pi_1}{2})(S - \frac{1}{2}) + (\frac{1}{3} - \frac{\pi_1}{2})d_{1.23}^2$. This difference decreases as party 1 becomes smaller (π_1 decreases) and less central ($d_{1.23}$ increases, $d_{3.12}$ and $d_{2.23}$ decrease). Its minimum value is $S - 2 + \frac{2}{3}(\frac{9}{4}) - (S - \frac{1}{2}) + \frac{1}{6}(\frac{9}{4}) = \frac{3}{8}$. If party 3 is closer to party 1 than to party 2, party 1 also prefers the minimal winning coalition with party 3 to the failure of its formation attempt, because $U_1 - v_1^f \ge 0$. In particular, $U_1^{mw3} - v_1^f = (\frac{1}{3} + \pi_1)(S - 2 + d_{3.12}^2) - (\frac{1+\pi_1}{2})(S - \frac{1}{2})$. This difference decreases as party 1 becomes smaller (π_1 decreases) and less central ($d_{3.12}$ decreases). Its minimum value is $\frac{2}{3}(S - 2 + \frac{15}{4})\frac{2}{3}(S - \frac{1}{2}) = \frac{3}{2}$. In either case party 1 prefers the minimal winning coalition to the collapse of the government since $U_1^{mw3} \ge v_1^f \ge v_1^c$.

if it gets no portfolios. Party 1 then proposes a government policy between its own policy position and the midpoint between the policy positions of parties 1 and 3. It does not offer party 3 any portfolios and forms a minority government. Since party 1 keeps all government portfolios for itself and the government policy is not farther from its ideal policy than the midpoint between the policy positions of parties 1 and 3, party 1's utility U_1^{mg3} under a minority government supported by party 3 is at least $S - d_{1.13}^2 = S - 1$. Party 1 prefers such a minority government to the failure of its initial formation attempt and the collapse of the government.²⁵

1.1

Party 3's expected utility v_3^c in the sixth stage decreases as party 1 becomes larger $(\pi_1 \text{ increases})$ and more central $(d_{3,12} \text{ and } d_{3,23} \text{ increase})$. The minimum values of v_3^c are $-\frac{5}{6}(\frac{15}{4}) + \frac{1}{6}(S-2) = \frac{1}{6}S - \frac{83}{24}$, when party 3 is closer to party 2 than to party 1, and $-\frac{5}{6}(\frac{25}{4}) + \frac{1}{6}(S-2) = \frac{1}{6}S - \frac{133}{24}$, otherwise. The former is smaller than -1 if $S \leq \frac{59}{4}$; the latter is smaller than -1 if $S \leq \frac{109}{4}$. For a low enough total value S of the government portfolios, the government thus changes from a minimal winning coalition to a minority government as party 1 becomes larger and more central.²⁶

Suppose party 1 seeks party 2's support in the first stage. Party 1 then proposes 2^{5} Its minimal utility under a minority government $U_{1}^{mg3} = S - 1$ is larger than the maximum value of $v_{1}^{f} = \frac{3}{4}S - \frac{3}{8} > v_{1}^{c}$.

²⁶Let u denote the utility the parties receive under a caretaker government. Then this conclusion holds for any utility $u \in [-\frac{1}{4}, S-2]$ such that the party that is selected in the sixth stage forms a minimal winning coalition. If $u < -\frac{1}{4}$, parties 1 and 2 propose a minority government in the seventh stage. With u > S-2 and maximum centrality of party 1, party 3 prefers the caretaker government to the optimal proposal it can make and thus makes a proposal that will not be accepted. As $u \in [-\frac{1}{4}, S-2]$ increases, the party that is selected to form a new government in the sixth stage has to offer the other parties a higher utility to attract their support. Its utility is thus lower, whereas the utility of the party whose support it seeks is higher. Since neither party 1 nor party 2 seek party 3's support, party 3's expected utility v_3^c in the sixth stage declines as u increases. Thus, as $u \in [-\frac{1}{4}, S-2]$ increases, the government changes from a minimal winning coalition to a minority government. the midpoint between its policy position and the policy position of party 2, and it offers party 2 government portfolios such that party 2 obtains its expected utility v_2^c , i.e., $s_2 = \frac{1}{4} + v_2^c$, which is positive as shown above. If party 3 is closer to party 2 than to party 1, party 1's utility $U_1^{mw^2}$ under a minimal winning coalition with party 2 is $U_1^{mw^2} = S - \frac{1}{2} - v_2^c = \frac{2}{3}(S - \frac{1}{2})$. If party 3 is closer to party 1 than to party 2, party 1's utility $U_1^{mw^2} = S - \frac{1}{2} - v_2^c = \frac{2}{3}(S - \frac{1}{2}) + (\frac{2}{3} - \pi_1)d_{2.13}^2$. In either case party 1 prefers to seek the support of party 3.²⁷

Surplus Majorities

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Suppose party 1 initially forms a surplus majority, as in the left branch of Figure 2 shown in Figure 3, but party 2 withdraws its support in the third stage. In the fourth stage party 1 can then reshuffle the government by proposing a minority government or a minimal winning coalition with party 3. To obtain party 3's confidence in the

²⁷If party 3 is closer to party 2 than to party 1, party 1 prefers the minimal winning coalition with party 3 to the minimal winning coalition with party 2, because $U_1^{mw3} - U_1^{mw2} \ge 0$. In particular, $U_1^{mw3} - U_1^{mw2} = S - 2 + (\frac{1}{3} + \pi_1)d_{3.12}^2 - (\frac{2}{3} - \pi_1)(S - 2d_{2.23}^2) - \frac{2}{3}(S - \frac{1}{2})$. This difference decreases as party 1 becomes smaller (π_1 decreases) and less central ($d_{3,12}$ and $d_{2,23}$ decrease). Its minimum value is $S - 2 + \frac{2}{3}(\frac{9}{4}) - (S - \frac{1}{2}) = 0$. If party 3 is closer to party 2 than to party 1, party 1 prefers the minority government with party 3's support to the minimal winning coalition with party 2, because $U_1^{mg3} - U_1^{mw2} \ge 0$. In particular, $U_1^{mg3} - U_1^{mw2} \ge S - 1 - \frac{2}{3}(S - \frac{1}{2}) = \frac{1}{3}S - \frac{2}{3} > 0$. If party 3 is closer to party 1 than to party 2, party 1 also prefers the minimal winning coalition with party 3 to the minimal winning coalition with party 2, since $U_1^{mw3} - U_1^{mw2} \ge 0$. In particular, $U_1^{mw3} - U_1^{mw2} = (\frac{1}{3} + \pi_1)(S - 2 + d_{3,12}^2) - \frac{2}{3}(S - \frac{1}{2}) + (\frac{2}{3} - \pi_1)d_{2,13}^2$. This difference decreases as party 1 becomes smaller (π_1 decreases) and less central ($d_{3,12}$ and $d_{2,13}$ decrease). Its minimum value is $\frac{2}{3}(S-2+\frac{15}{4})-\frac{2}{3}(S-\frac{1}{2})+\frac{1}{3}(\frac{3}{2})=2$. If party 3 is closer to party 1 than to party 2, party 1 prefers the minority government with party 3's support to the minimal winning coalition with party 2, because $U_1^{mg3} - U_1^{mw2} \ge 0$. In particular, $U_1^{mg3} - U_1^{mw2} \ge S - 1 - \frac{2}{3}(S - \frac{1}{2}) + (\frac{2}{3} - \pi_1)d_{2.13}^2$. This difference decreases as party 1 becomes larger (π_1 increases) and less central ($d_{2.13}$ decreases). Its minimum value is $S - 1 - \frac{2}{3}(S - \frac{1}{2}) + \frac{1}{6}(\frac{3}{2}) = \frac{1}{3}S - \frac{5}{12} = \frac{14}{9} - \frac{5}{12} > 0.$

fifth stage, party 1 has to offer party 3 its expected utility v_3^c . Party 1 then solves the maximization problem described above. For party 2 the withdrawal of support leads to a government reshuffle and a non-positive utility v_2^r after reshuffle, since party 2 does not receive any portfolios in the reshuffled government. Thus, party 2 withdraws its support if its utility v_2^g under the government is lower than its utility v_2^r under a reshuffled government.

In the second stage party *i* supports a government if its utility v_i^p when the government takes office is higher than its utility v_i^f when the government fails. If one party rejects the proposal, the government fails and the parties have continuation values v_i^f , i = 1, ..., 3.

In the first stage, to attract the support of parties 2 and 3 for a lasting surplus majority, party 1 needs to offer them their respective expected utilities, v_2^f and v_3^f . Although party 2's expected utility v_2^c after collapse is higher than v_2^f , party 1 does not have to offer v_2^c to party 2, since party 1 would reshuffle the government with party 3 if party 2 withdrew its support. Party 2 would then obtain a non-positive utility v_2^r . Party 1's coalition partners are thus in a prisoner's dilemma in a surplus majority, and neither has an incentive to withdraw. Party 2's non-positive utility v_2^r under a reshuffled government is smaller than its expected utility v_2^f if the initial formation attempt fails. Thus, party 2 does not withdraw its support in the third stage if its utility v_2^g under the government is greater than or equal to v_2^f . Party 1's maximization problem then is:

$$\begin{aligned} \max_{s_1, s_2, p} & U_1 = s_1 - d_{1,g}^2 \\ \text{s.t.} & s_2 - d_{2,g}^2 - v_2^f \ge 0 \\ & S - s_1 - s_2 - d_{3,g}^2 - v_3^f \ge 0 \\ & s_1 \ge 0 \\ & s_2 \ge 0 \end{aligned}$$

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$$S-s_1-s_2\geq 0.$$

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Party 1 proposes the centroid of the three parties' policy positions as government policy, and it offers the two other parties just enough portfolios to obtain their utilities v_i^f . Party 1's utility U_1^{sm} under a surplus majority is $S - d_{1.c}^2 - d_{2.c}^2 - d_{3.c}^2 - v_2^f - v_3^f$, where $d_{i.c}$ is the distance between party *i*'s policy position and the centroid of the three parties' policy positions. There are two cases.

(1) If party 3 is closer to party 2 than to party 1, party 1's utility U_1^{sm} is $U_1^{sm} = S - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - \frac{1}{6}(S - \frac{1}{2}) + (\frac{2}{3} + \frac{\pi_1}{2})d_{3,12}^2 - (\frac{1}{3} - \frac{\pi_1}{2})(S - 2d_{2,23}^2)$. As party 1 becomes smaller and less central its utility under a surplus majority increases relative to its utility under a minimal winning coalition.²⁸ ²⁹ For minimal size and centrality of party 1, its utility under a surplus majority is $\frac{2}{3}S + \frac{1}{24}$, which is larger than its utility $\frac{2}{3}S - \frac{1}{3}$ under a minimal winning coalition. Therefore, the government changes from a minimal winning coalition to a surplus majority as party 1 becomes smaller

²⁸Party 1 prefers a surplus majority to a minimal winning coalition if $U_1^{sm} - U_1^{mw3} \ge 0$, where $U_1^{sm} - U_1^{mw3} = (\frac{1}{3} - \frac{\pi_1}{2})(S - 2d_{2.23}^2 + d_{3.12}^2) + 2 - d_{1.c}^2 - d_{2.c}^2 - d_{3.c}^2 - \frac{1}{6}(S - \frac{1}{2})$. This difference increases as party 1 becomes smaller (π_1 decreases) and less central ($d_{2.23}^2$, $d_{3.12}$ and $\sum_i d_{i.c}^2$ decrease). Party 1's utility under a minimal winning coalition as well as under a surplus majority increase as it becomes larger and more central, but it increases faster under a minimal winning coalition.

²⁹A surplus majority is also preferred to a minority government with party 3's support if it is preferred to a minimal winning coalition with party 3. Under a minority government the constraint that the value of party 3's government portfolios be non-negative is binding. If this constraint were removed, party 1 would prefer to propose the midpoint between the ideal policies of parties 1 and 3 as government policy and give party 3 negative portfolios rather than propose a minority government. A minimal winning coalition can then be thought of as the solution to party 1's maximization problem without the constraint that the value of party 3's government portfolios be non-negative. Thus a surplus majority is preferred to a minority government if it is preferred to a minimal winning coalition. and less central.^{30 31}

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(2) If party 3 is closer to party 1 than to party 2, party 1's utility U_1^{sm} under a 3^{0} This conclusion holds for any utility $u \in [\frac{1}{4}, S-2]$ under a caretaker government. Party 2's expected utility after the failure of party 1's initial formation attempt increases as the parties' expected utilities under a caretaker government increase, because both the other parties seek party 2's support after the failure of the initial formation attempt. In particular, party 2's expected utility increases by two thirds the increase in its utility under a caretaker government. Party 3's expected utility decreases because neither party 1 nor party 2 seek its support, but its utility decreases by no more than one sixth the increase in its utility under a caretaker government. Therefore, the total utility party 1 has to offer the other parties to attract both the other parties' support, increases as the parties' utilities under a caretaker government increase. Party 1's utility under a surplus majority then decreases. The government thus changes from a surplus majority to a minimal winning coalition as $u \in [\frac{1}{4}, S-2]$ increases.

³¹Party 1's coalition parties receive portfolios $s_2 = d_{2,c}^2 + v_2^f$ and $s_3 = d_{3,c}^2 + v_3^f$. Party 2's portfolios are positive, since $v_2^f = \frac{1}{6}(S-\frac{1}{2}) \ge 0$. Party 3's portfolios are $s_3 = d_{3,c}^2 - (\frac{2}{3} + \frac{\pi_1}{2})d_{3,12}^2 + (\frac{1}{3} - \frac{\pi_1}{2})(S-2d_{2,23}^2)$. They decrease as party 1 becomes larger (π_1 increase) and more central ($d_{3,12}$ and $d_{2,23}$ increase). They are negative if $\pi_1 \ge 2(\frac{d_{3,c}^2 - \frac{2}{3}d_{3,12} + \frac{1}{3}(S-2d_{2,23}^2))$. If party 1 prefers a surplus majority to a minimal winning coalition, then $U_1^{Im} - U_1^{mw3} = (\frac{1}{3} - \frac{\pi_1}{2})(S-2d_{2,23}^2 + d_{3,12}^2) + 2 - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - \frac{1}{6}(S-\frac{1}{2}) \ge 0 \Leftrightarrow \pi_1 \le 2(\frac{\frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2)}{(S-2d_{2,23}^2 + d_{3,12}^2)})$. A sufficient condition for s₃ to be positive is thus that $\frac{\frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2)}{(S-2d_{2,23}^2 + d_{3,12}^2)} \le \frac{d_{3,c}^2 - \frac{2}{3}d_{3,12}^2 + \frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2)}{(S-2d_{2,23}^2 + d_{3,12}^2)} \le \frac{d_{3,c}^2 - \frac{2}{3}d_{2,12}^2 + \frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2)}{(S-2d_{2,23}^2 + d_{3,12}^2)} \Rightarrow \frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2) + 2 - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - \frac{1}{6}(S-\frac{1}{2}) \ge d_{3,c}^2 - \frac{2}{3}d_{3,12}^2 + \frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2)} \Rightarrow \frac{1}{3}(S-2d_{2,23}^2 + d_{3,12}^2) + 2 - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - \frac{1}{6}(S-\frac{1}{2}) \ge d_{3,c}^2 - \frac{2}{3}d_{3,12}^2 + \frac{1}{3}(S-2d_{2,23}^2) \Leftrightarrow S \ge 6(d_{3,12}^2 + 2 - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - d_{3,c}^2 - \frac{1}{6}(S-\frac{1}{2}) \ge d_{3,12}^2 + 2 - d_{1,c}^2 - d_{2,c}^2 - d_{3,c}^2 - d_{3,c}^2) + \frac{1}{2}$ increases. For minimal centrality of party 1 it is equal to 8. Thus, party 3's portfolios s_3 are positive if $S \ge 8$. They can be negative for S < 8. Since the values of the parties' portfolios have to be positive in a surplus majority, party 1 cannot propose the surplus majority charaterized above if it is central, large enough and S < 8. The optimal surplus majority proposal then offers party 3 portfolios wo surplus majority is $U_1^{sm} = S - d_{1.c}^2 - d_{2.c}^2 - d_{3.c}^2 + d_{2.13}^2 + (\frac{1}{3} - \frac{\pi_1}{2})d_{2.13}^2 - \frac{1}{6}(S - \frac{1}{2}) + (\frac{2}{3} + \frac{\pi_1}{2})d_{3.12}^2 - (\frac{1}{3} - \frac{\pi_1}{2})(S-2)$. As party 1 becomes smaller and less central, its utility under a surplus majority increases relative to its utility under a minimal winning coalition. For minimal size and centrality of party 1, its utility under a surplus majority is $\frac{2}{3}S + \frac{19}{24}$, which is smaller than its utility $\frac{2}{3}S + \frac{7}{6}$ under a minimal winning coalition. Surplus majorities thus do not form if party 1 is so central that party 3 is closer to party 1 than to party 2.

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