EDT: AN ELASTODYNAMICS TOOLBOX FOR MATLAB

Mattias Schevenels, Stijn François, Geert Degrande

Department of Civil Engineering, K.U.Leuven
Kasteelpark Arenberg 40, B-3001 Leuven, Belgium
mattias.schevenels@bwk.kuleuven.be

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Abstract. *This paper presents the ElastoDynamics Toolbox (EDT) for MATLAB.[1] EDT provides an extensive set of MATLAB functions to model seismic wave propagation in layered soils. It is based on the direct stiffness method and the thin layer method, presented by Kausel and Roësset. The toolbox is developed at K.U.Leuven, where it is used both for educational purposes and in research projects. It is used to solve a variety of problems governed by wave propagation in the soil, such as (1) site amplification, (2) the computation of dispersive surface waves in layered soils, and (3) the calculation of the forced response of the soil due to harmonic and transient loading. The present paper focuses on recent research applications of EDT in each of these categories.*

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[1]A trial version of the ElastoDynamics Toolbox for MATLAB can be requested on the following web page: http://www.kuleuven.be/bwm/edt.

1 INTRODUCTION

The ElastoDynamics Toolbox (EDT) is a MATLAB toolbox to model seismic wave propagation in layered soils [1]. The toolbox is based on the direct stiffness method, developed by Kausel and Roësset [2, 3].

The direct stiffness method is based on a transformation of the wave equation from the time-space domain to the frequency-wavenumber domain. Stiffness matrices for a homogeneous layer element and a homogeneous halfspace element are formulated in the transformed domain. In order to model a layered soil, an element assembly procedure is followed, in a similar way as in the finite element method.

EDT can be used to model site amplification, to compute dispersive surface waves in layered soils, and to calculate the forced response of the soil due to harmonic and transient loading.

The user can interact with EDT at a low level of abstraction (e.g. to compute the stiffness matrix of a layer or halfspace element) or a high level of abstraction (e.g. to calculate the dispersion curves of a layered halfspace). Due to this multi-level approach, the toolbox is suitable for educational purposes and for use in a research environment: the high level functions allow for an easy and efficient solution of many common problems, while the low level functions facilitate customization and the development of new techniques.

EDT serves as an electronic learning environment for the simulation and processing of seismic wave propagation in layered media. It is actively used at K.U.Leuven in the frame of the master's programs of civil engineering and geotechnical and mining engineering.

The toolbox is also used in the frame of a number of research projects. The aim of this paper is to present three recent research applications of EDT. Section 2 focuses on the modelling of site amplification in order to assess the seismic hazard at sites where the top soil layers are soft. Section 3 addresses the Spectral Analysis of Surface Waves (SASW) method, which is used to determine the properties of shallow soil layers. The SASW method is based on the dispersive character of surface waves in layered media. In this section, the non-uniqueness in the SASW method is considered. Section 4 finally focuses on the modelling of a vibration isolating screen in the soil. The screen is modelled with finite elements and the soil with boundary elements. In order to account for the stratification of the soil, the boundary element model is based on the 2.5D Green's functions of a layered halfspace. These Green's functions are computed with EDT. The results in sections 3 and 4 have already been published elsewhere and are only briefly reviewed here.

2 SITE AMPLIFICATION

Site amplification is an important issue in the assessment of the seismic hazard at sites where the top soil layers are particularly soft. In such cases, the seismic motion at the surface can be much higher than the outcrop motion due to resonance of the soft layers. EDT is currently used at K.U.Leuven to model site amplification in the frame of a research project where the seismic hazard in Flanders is investigated.

Site amplification can be modelled assuming linear, equivalent linear, or nonlinear constitutive behavior of the soil. In a low seismicity region such as Flanders, the importance of the nonlinear behavior of the soil is expected to be limited, allowing for the use of a linear material model. This assumption is verified by comparing two site response analyses performed with EDT, using a linear and an equivalent linear material model, respectively.

A fictitious soil profile is considered, with soil properties that are realistic for a site in Flanders. The profile is described in table 1. For each layer, the thickness h, the small-strain shear

modulus μ_0, the density ρ, the shear wave velocity $C_\mathrm{s} = \sqrt{\mu_0/\rho}$, and the small-strain material damping ratio β_0 are given.

Layer	h [m]	μ_0 [MPa]	ρ [kg/m^3]	C_s [m/s]	β_0 [$-$]
1	6	39.2	2000	140	0.005
2	9	64.8	2000	180	0.005
3	15	156.8	2000	280	0.005
4	18	320.0	2000	400	0.005
5	∞	1620.0	2000	900	0.005

Table 1: Dynamic soil properties at small deformation ratios.

Soil typically exhibits a softening nonlinearity, or a decrease in modulus as strain increases. Increasing strains also cause progressively larger hysteresis in the stress-strain relation, leading to strain-dependent wave attenuation. These phenomena are taken into account through the use of an equivalent linear material, with an equivalent shear modulus μ and an equivalent damping ratio β. The values of the soil properties μ and β depend on the actual strain level in each soil layer, and can be expressed by means of degradation curves. These curves differ for different types of soil. In this case, the degradation curves for sandy soils proposed by Seed et al. [4] are used (figure 1).
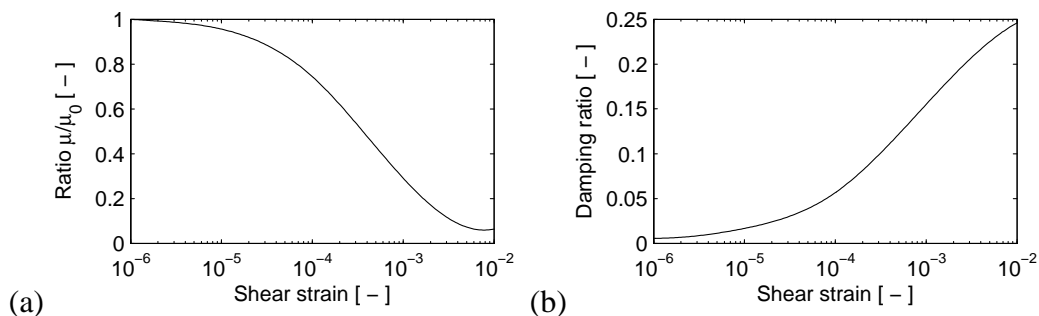

Figure 1: Degradation curves for (a) the shear modulus μ and (b) the material damping ratio β.

Due to the fact that the equivalent material damping ratio β becomes relatively large, the equivalent linear method tends to underestimate the response of the soil in the higher frequency range. A frequency dependent equivalent linear method has therefore been developed by Kausel and Assimaki [5]. In this method, the frequency spectrum of the strain in each layer is considered to determine the equivalent dynamic soil properties. This implies that the degradation curves are evaluated at a different strain level for each frequency, resulting in frequency dependent equivalent dynamic soil properties μ and β. Kausel and Assimaki [5] demonstrate that this approach leads to more realistic results by means of a comparison with a fully nonlinear calculation.

A site response analysis starts from a seismogram that represents the outcrop motion. For the present analysis, a seismogram from the Kocaeli is selected. This seismogram is scaled in order to make it compatible with the design response spectrum from Eurocode 8 [6]. The seismogram and the corresponding response spectrum are shown in figure 2.

EDT is used to perform both a linear and an equivalent linear site response analysis. The equivalent analysis is performed following the frequency dependent approach proposed by
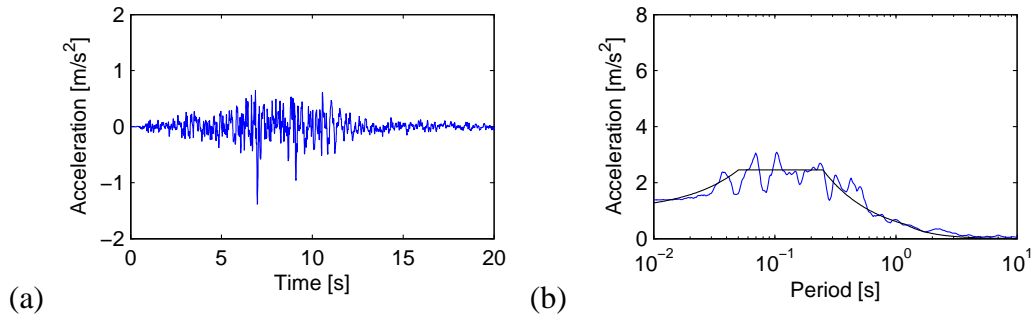
Figure 2: (a) Outcrop motion and (b) corresponding response spectrum (blue line) compared with the design response spectrum from Eurocode 8 (black line).

Kausel and Assimaki [5]. The resulting acceleration at the surface and the corresponding response spectrum are shown in figures 3 and 4. If the response spectra are compared with the spectra for the outcrop motion, it can be concluded that the presence of the soft layers leads to a much higher structural response for relatively small structures (fundamental eigenperiod smaller than $1\,\mathrm{s}$). For larger structures (fundamental eigenperiod smaller than $1\,\mathrm{s}$), the impact of site amplification is smaller. Obviously, these conclusions only hold for the present case, and should not be interpreted as generally valid.
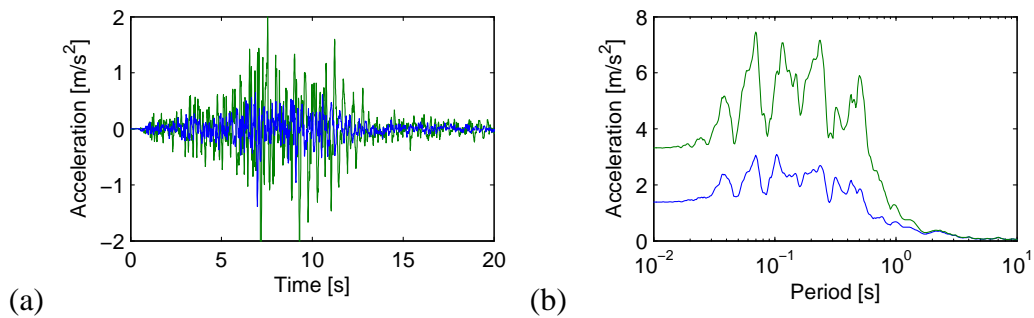

Figure 3: (a) Acceleration at the surface and (b) corresponding response spectrum using a linear material model (green line) compared with the outcrop motion (blue line).
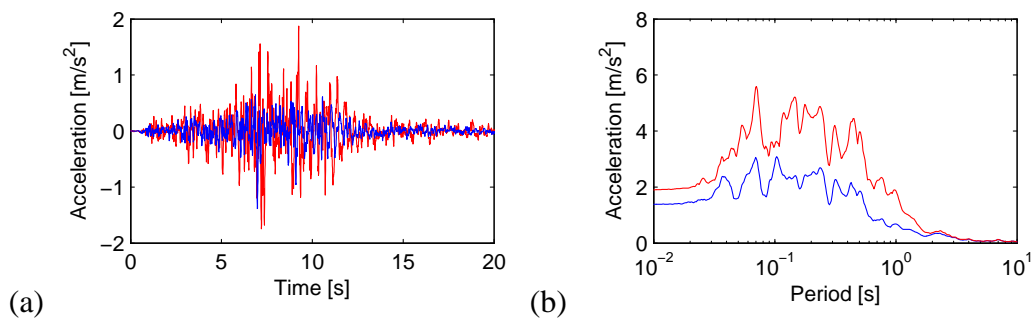

Figure 4: (a) Acceleration at the surface and (b) corresponding response spectrum using an equivalent linear material model (red line) compared with the outcrop motion (blue line).

If the response spectra resulting from the linear analysis and from the equivalent analysis are compared, a large difference is observed for very small structures (fundamental eigenperiod smaller than $0.1\,\mathrm{s}$). For larger structures, the difference is relatively small, apparently justifying

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the use of a linear material model. However, this conclusion cannot be generalized without performing a more elaborate study based on a variety of soil profiles and outcrop motion accelerograms.

3 SURFACE WAVES

Surface waves are the natural modes of vibration of a layered medium. These waves travel along the free surface, or along interfaces between layers, while they decay exponentially with depth. Surface waves are dispersive: their phase velocity is frequency dependent due to the variation of the dynamic soil properties with depth. This is the basis of the Spectral Analysis of Surface Waves (SASW) method, where the dispersion curve of one or more surface waves is used to identify the dynamic soil properties as a function of depth.

The SASW method consists of two parts. First, an in situ experiment is performed where surface waves are generated by means of a falling weight, an impact hammer, or a hydraulic shaker. The free field response is measured with geophones or accelerometers and used to determine the experimental dispersion and attenuation curves, i.e. the phase velocity and the attenuation coefficient of the surface waves as a function of frequency. Next, an inverse problem is solved to determine the dynamic soil properties. The theoretical dispersion and attenuation curves of a soil with a given profile are calculated (in this case with EDT). The soil profile is iteratively adjusted in order to minimize a misfit function that measures the distance between the theoretical and the experimental dispersion and attenuation curves.

The dispersion and attenuation curves are insensitive to variations of the soil properties on a small spatial scale or at a large depth. The information on the soil properties provided by these curves is therefore limited. As a result, the solution of the inverse problem is non-unique: the soil profile obtained with a classical deterministic optimization scheme is only one of the profiles that fit the experimental data.
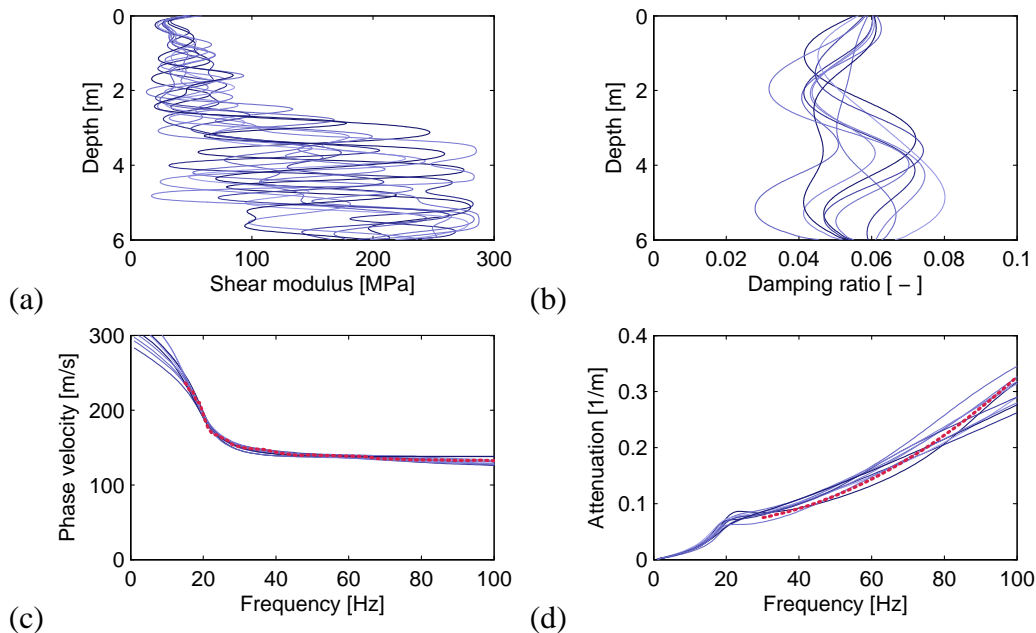

Figure 5: (a) Shear modulus and (b) damping ratio as a function of depth for ten different soils and corresponding theoretical (c) dispersion and (d) attenuation curves (blue lines) compared with the experimental curves (red dots).

The non-uniqueness in the SASW method has been studied in references [7] and [8]. A

Monte Carlo inversion scheme has been used to solve the inverse problem, resulting in an ensemble of soil profiles that fit the experimental data. Figure 5 shows ten different soil profiles from the ensemble: while the shear modulus and the damping ratio vary considerably, the corresponding dispersion and attenuation curves are very similar. This clearly illustrates the non-uniqueness in the SASW method.

If the soil profile resulting from an SASW test is used for the numerical prediction of ground vibrations, these predictions are uncertain due to the non-unique character of the soil profile. This uncertainty has been investigated in references [7] and [8]. It has been shown that the variability of the vibration predictions is relatively low in a limited frequency range. Below this range, the wavelength of the waves in the soil is large and the waves reach deeper soil layers. The properties of these layers can not be determined from an SASW test on account of its limited resolution in terms of depth. The variability of the vibration predictions is therefore high. Above this frequency range, the waves in the soil are affected by the small scale variations of the soil properties. These variations are poorly resolved in the SASW test, also resulting in a high variability of the vibration predictions.

4 FORCED VIBRATIONS

The calculation of the forced response of the soil due to a unit load (i.e. the Green's functions of the soil) is the basis of the boundary element method, which can be used to calculate foundation impedance curves or to model dynamic soil-structure interaction.

EDT has been used to solve a variety of dynamic soil-structure interaction problems. The most recent application is the modelling of a vibration isolating screen in the soil [9]. The screen is assumed to be infinitely long, allowing for the use of a 2.5D coupled finite element - boundary element model. The screen is modelled with finite elements and the soil with boundary elements. In order to account for the stratification of the soil, the boundary element model is based on the 2.5D Green's functions of a layered halfspace (as opposed to a homogeneous fullspace). These functions represent the response of the soil due to a line load with an amplitude that varies harmonically in time and space. The 2.5D Green's functions are computed with EDT.
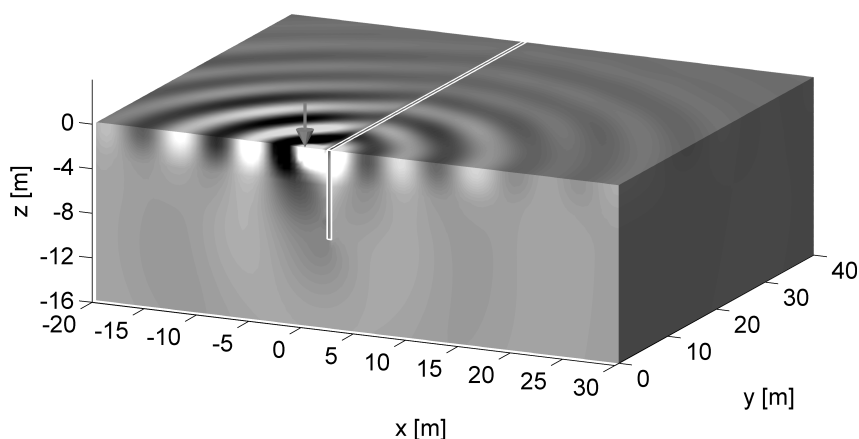

Figure 6: The response of a soil due to a harmonic point load in the presence of a vibration isolating screen.

The results of the coupled finite element - boundary element analysis are shown in figure 6. This figure shows the response of the soil due to a harmonic point load at the surface, next to the vibration isolating screen. It is clear that the vibration isolating screen reflects the energy in

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the direction of the source, resulting in a decrease of the vibration levels at the other side of the screen. In reference [9], the results of the numerical analysis are verified using experimental data, obtained from a measurement campaign on a site in Brussels (Belgium).

5 CONCLUSION

This paper introduces the ElastoDynamics Toolbox (EDT) for MATLAB, developed at K.U.Leuven. The focus is on three recent research projects where EDT is used to model seismic wave propagation. The first example involves the modelling of site amplification using an equivalent linear material model with frequency dependent moduli and damping ratios. The second example focuses on the non-uniqueness in the Spectral Analysis of Surface Waves method. The third example concerns a dynamic soil-structure interaction problem where the efficiency of a vibration isolating screen in the soil is studied. In all three examples, EDT has proven to be a useful and efficient tool to solve relatively complex problems involving seismic wave propagation in layered soils.

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