The Association between Children’s Numerical Magnitude Processing and Mental Multi-digit Subtraction

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Abstract

Children apply various strategies to mentally solve multi-digit subtraction problems and the efficient use of some of them may depend more or less on numerical magnitude processing. For example, the indirect addition strategy (solving 72-67 as ‘‘how much do I have to add up to 67 to get 72?”) , which is particularly efficient when the two given numbers are close to each other, requires to determine the proximity of these two numbers, a process that may depend on numerical magnitude processing. In the present study, children completed a numerical magnitude comparison task and a number line estimation task, both in a symbolic and nonsymbolic format, to measure their numerical magnitude processing. We administered a multi-digit subtraction task, in which half of the items were specifically designed to elicit indirect addition. Partial correlational analyses, controlling for intellectual ability and motor speed, revealed significant associations between numerical magnitude processing and mental multi-digit subtraction. Additional analyses indicated that numerical magnitude processing was particularly important for those items for which the use of indirect addition is expected to be most efficient. Although this association was observed for both symbolic and nonsymbolic tasks, the strongest associations were found for the symbolic format, and they seemed to be more prominent on numerical magnitude comparison than on number line estimation.

Keywords: numerical magnitude processing; mental multi-digit subtraction; number sense

The Association between Children's Numerical Magnitude Processing and Mental Multi-digit Subtraction

Numerical magnitude processing, or people's elementary intuitions about quantity, has been shown to play a crucial role in mathematical development (Booth & Siegler, 2006a; Butterworth, Varma, & Laurillard, 2011; Price & Ansari, 2012; see De Smedt, Noël, Gilmore, & Ansari, 2013, for a review). Recent studies in typically developing children have provided converging evidence on the role of numerical magnitude processing in predicting individual differences in mathematics achievement (Bugden & Ansari, 2011; De Smedt, Verschaffel, & Ghesquière, 2009; Halberda, Mazzocco, & Feigenson, 2008; Sasanguie, Van den Bussche, & Reynvoet, 2012; see De Smedt et al., 2013, for a review). One important limitation of these studies is that they mainly examined mathematics achievement with general achievement tests, which only yield a total score of mathematics achievement reflecting children's performance across various mathematical subdomains. However, the focus on more specific mathematical skills may help to pinpoint associations between numerical magnitude processing and mathematics achievement in a more precise way, which may be more beneficial to devise appropriate diagnostic instruments and educational interventions. For example, recent data by Vanbinst, Ghesquière, and De Smedt (2012) showed that children's symbolic - but *not* nonsymbolic - numerical magnitude processing skills were associated with single-digit arithmetic performance, indicating that children with better access to magnitude representations from symbolic digits retrieved more facts from their memory and were faster in executing fact retrieval as well as procedural strategies. In the present study, we extended this finding by focusing on another mathematical competency that may be specifically related to numerical magnitude understanding, namely mental multi-digit subtraction. Numerous studies have shown that children develop various strategies to mentally solve multi-digit subtraction problems (Beishuizen, 1993; Blöte, Klein, & Beishuizen, 2000; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009), and the efficient use of some of them may depend more or less on numerical magnitude processing.

In the remainder of this introduction we describe the tasks that are most often used to assess children's numerical magnitude processing and their associations with mathematics achievement as well as the various strategies children use to mentally solve multi-digit subtraction problems. Afterwards we present the design and hypotheses of the current study.

Numerical magnitude processing and mathematics achievement

Research on associations between numerical magnitude processing and mathematics achievement has typically used two types of tasks to measure numerical magnitude processing, i.e. a numerical magnitude comparison task (Sekuler & Mierkiewicz, 1977) and a number line estimation task (Booth & Siegler, 2006). In a numerical magnitude comparison task, children are asked to indicate the numerically larger of two presented numerical magnitudes, which can be presented in either a symbolic (digits) or a nonsymbolic (dots) format (Holloway & Ansari, 2009). In a number line estimation task, children are typically shown a horizontal number line, for example with 0 on one end and 100 on the other, and they are given a number to be positioned on this number line (Ashcraft & Moore, 2012; Booth & Siegler, 2006, 2008). Similar to the numerical magnitude comparison task, this task can be presented in a symbolic or a nonsymbolic format (Sasanguie, De Smedt, Defever, & Reynvoet, 2012).

Performance on a numerical magnitude comparison task has been related to general mathematics achievement, revealing that children who are faster in indicating which of two numbers is the larger, show higher achievement in mathematics more precise representations of numerical magnitude (Bugden & Ansari, 2011; Halberda, Mazzocco, & Feigenson, 2008; Holloway & Ansari, 2009; Sasanguie et al., 2012). Moreover, De Smedt, Verschaffel, and

Ghesquière (2009) provided longitudinal evidence that the speed of comparing numbers assessed at the start of formal schooling is predictively related to subsequent general mathematics achievement in second grade.

A similar association with mathematics achievement has been observed in studies with number line estimation as a measure for numerical magnitude processing. Siegler and Booth (2004), for example, demonstrated that individual differences in number line estimation for kindergarteners, first and second graders were strongly correlated with their general mathematics achievement test scores. In line with these results, Sasanguie et al. (2012) observed that more linear estimation patterns on the symbolic number line estimation task were associated with higher general mathematics achievement for kindergarteners, first, second and sixth graders.

It has been assumed that the performance in the numerical magnitude comparison task and number line estimation task relies on the same underlying magnitude representation (Deheane, 1997; Laski & Siegler, 2007). However, this idea has recently been questioned (Paladino & Barth, 2011; Sasanguie and Reynvoet, 2013). For instance, Sasanguie and Reynvoet (2013) compared the performance in the numerical magnitude comparison task and the number line estimation task directly and observed no significant relations between performance in both tasks. Based on these results, the authors suggested that different mechanisms might underlie these numerical magnitude processing tasks. They speculated that the performance in a numerical magnitude comparison task is reflecting decisional mechanisms upon activated magnitude representations (see Verguts, Fias & Stevens, 2005; Van Opstal, Gevers, De Moor & Verguts, 2008), whereas the number line estimation task taps more into number-space associations such as, reliance on anchor points to position a number on the number line. Although different processes may lie at the basis of performance in both tasks, previous studies have extensively shown that both tasks are related to mathematics achievement. Therefore, we included a numerical magnitude comparison task and a number line estimation task to verify which processes plays a more prominent role in explaining performance on subtraction items, or whether both tasks are equally important.

The distinction between the symbolic and nonsymbolic formats relates to the question whether the representation of numerical magnitudes per se, or its access via symbolic digits, is important for mathematical achievement in general. To address this issue, previous studies have compared performance on numerical magnitude tasks with symbolic processing requirement, i.e. using Arabic digits as stimuli, and without symbolic processing requirement, i.e. using dots as stimuli (De Smedt & Gilmore, 2011; Holloway & Ansari, 2009; Landerl & Kölle, 2009; Lonnemann, Linkersdörfer, Hasselhorn, & Lindberg, 2011; Mussolin, Mejias, & Noël, 2010; Rousselle & Noël, 2007; Sasanguie et al., 2012; Vanbinst et al., 2012). If numerical magnitude processing per se is crucial for mathematical achievement, then both symbolic and nonsymbolic tasks should predict individual differences in mathematical achievement. If only symbolic tasks predict general mathematical skill, the hypothesis of the access to numerical meaning from symbolic digits is favored. Several attempts have been made to disentangle both hypotheses and evidence favoring both has been reported (De Smedt & Gilmore, 2011; Halberda et al., 2008; Holloway & Ansari, 2009; Landerl & Kölle, 2009; Libertus, Feigenson, & Halberda, 2011; Lonneman et al., 2011; Mussolin et al., 2010; Rouselle & Noël, 2007; Sasanguie et al., 2012, Vanbinst et al., 2012; see De Smedt et al., 2013, for a review). In the present study we addressed this issue by using both symbolic and nonsymbolic formats of the numerical magnitude processing tasks. This allowed us to examine whether the representation of numerical magnitudes per se, or their access via symbolic digits, is important for mental multi-digit subtraction.

Although the studies reviewed above have shown that the ability to represent numerical magnitudes is related to general mathematics achievement, little is known about how numerical magnitude processing is related to more specific mathematical skills. Booth and Siegler (2008), for example, explored this relation for mental addition in the number domain from 0 to 100, showing a positive correlation with numerical magnitude processing. Other research examined numerical magnitude processing in relation to both single- and multi-digit addition strategy use in children with different levels of mathematical achievement (Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Hoard, Geary, Byrd-Craven, & Nugent, 2008), showing that children with difficulties in mathematics achievement used more immature counting strategies and made more computational errors, coupled with a poorer performance on number line estimation tasks. However, to the best of our knowledge, no study has yet explored the association between numerical magnitude processing and mental multi-digit subtraction. We focused on mental calculation rather than on written computation, because the former is considered to require operating insightfully and flexibly on numbers whereas the latter comes down to routine-based operating on digits (Fuson et al., 1997; Verschaffel, Greer, & De Corte, 2007).

Mental multi-digit subtraction

Numerous studies have shown that children develop various strategies to mentally solve multi-digit subtraction problems (Beishuizen, 1993; Blöte et al., 2000; Carpenter et al., 1998; Torbeyns, De Smedt, Stassens, et al., 2009). These strategies can be described along different dimensions (Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012; Torbeyns, De Smedt, Stassens, et al., 2009). One way to categorize these strategies is by determining the operation that underlies the solution process, which can be either subtraction or addition. Using this categorization, two types of strategies can be distinguished: (1) *direct subtraction strategies*, in which the subtrahend is directly subtracted from the minuend and (2) *indirect addition strategies,* in which one determines how much needs to be added to the subtrahend to get to the minuend[1]. A rational task analysis of subtraction problems reveals that indirect addition is particularly efficient on subtractions with a relatively small difference between the minuend and subtrahend (Anghileri, Beishuizen, & van Putten, 2002; Beishuizen, Van Putten, & Van Mulken, 1997; Torbeyns, De Smedt, Stassens, et al., 2009). In these cases, the answer is found quickly and easily by means of indirect addition, whereas the direct subtraction strategy would take more, and more difficult, problem solving steps. For example, if one is asked to solve the problem "81 - 79 = ?", the indirect addition strategy "how much do I have to add up to 79 to get 81? Answer: 2", leads to a faster and less error-prone answer than the direct subtraction strategy "81 - 70 = 11 and 11 - 9 = 2" (De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2012; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009b; Verschaffel et al., 2007; Woods, Resnick, & Groen, 1975). The meaningful and flexible use of this strategy requires a good understanding of the numerical magnitudes in the problem and their mutual relation (Baroody, Torbeyns, & Verschaffel, 2009; Verschaffel, Bryant, & Torbeyns, 2012), and, more particularly, an appropriate estimation of the difference between minuend and subtrahend. Arguably, this determination of the proximity of two given numbers may depend on one's numerical magnitude processing, because children with good numerical magnitude processing skills will be more accurate and faster in estimating the difference between the two given numbers and in comparing this difference with the size of the subtrahend, to determine if this difference is relatively small or not. Against this background, we predicted that the association between children's numerical magnitude processing and their performance on items in which the use of indirect addition is expected, will be more prominent than for the items in which the use of a direct subtraction strategy is expected.

[1] It should be noted that there is also a third class of strategy, namely the *indirect subtraction strategy*, in which one determines how much has to be decreased from the larger given number to get the smaller one (81 – 72 = ? ; 81 – ? = 72) (De Corte & Verschaffel, 1987; Torbeyns, De Smedt, Stassens, et al., 2009).

Most available studies on children's and adults' use of the indirect addition in multi-digit subtraction relied on verbal protocol data. Although these studies showed that adults frequently reported the use the indirect addition strategy, children hardly reported its use (Selter, 2001; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a; Torbeyns, De Smedt, et al., 2009b). Peters, De Smedt, Torbeyns, Ghesquière, and Verschaffel (2010a, 2010b) argued, however, that verbal protocols may be less suited to identify the indirect addition strategy in multi-digit subtraction, because this strategy is often executed in a very fast and quasi-automatic way and, second, because particularly children might have difficulties in articulating precisely how they found the answer when using indirect addition (see also Kirk & Ashcraft, 2001). Using a combination of several non-verbal research methods, Peters et al. (2012) recently showed that primary school children are indeed using the indirect addition strategy, particularly on subtraction items in which the difference between the minuend and the subtrahend is relatively small, as in "81 – 79 = .". Therefore, in order to create items that maximally elicit this indirect addition strategy, we selected items with a relatively small difference.

The present study

Against the background of the aforementioned research, the present study set out to explore the role of numerical magnitude processing in children's mental multi-digit subtraction, and, more particularly, in their use of the indirect addition strategy in mental multi-digit subtraction in the number domain 20-100. We administered a numerical magnitude comparison task and a number line estimation task, both presented in symbolic (digits) and nonsymbolic (dots) formats, to children who were in the middle of third grade. First, we expected an association between children's performance on the multi-digit subtraction problems and both numerical magnitude processing tasks. Second, these children solved a task consisting of multi-digit subtraction items that were constructed in a way that they aimed to elicit the use of an indirect addition strategy or a direct subtraction strategy; and, we hypothesized that the association between children's numerical magnitude processing and their performance on mental multi-digit subtraction would be particularly prominent for those items in which the use of indirect addition is considered to be the most efficient. Third, we directly compared the numerical magnitude comparison task and the number line estimation task to explore the contribution of each task in explaining the performance on mental multi-digit subtraction. Fourth, by comparing symbolic and nonsymbolic tasks, we additionally tried to verify whether the numerical representations of magnitude per se or the access to numerical magnitude from symbolic digits, is related to mental multi-digit subtraction. Finally, to evaluate alternative explanations for associations between numerical magnitude processing and mental multi-digit subtraction, children's general intellectual ability and general motor speed were assessed as control measures.

Method

Participants

The sample consisted of 56 third-graders selected from two schools located in the Flemish part of Belgium. Both schools used the same general educational approach and mathematics curriculum. Participants were all typically developing children (27 boys, 29 girls), with a mean age of 8 years 9 months (*SD* = 5 months) and none of them had repeated a grade. All children were recruited from schools with children from middle-class socioeconomic backgrounds.

Measures

The experimental tasks were administered on a laptop with a 15-inch screen. Stimulus presentation and the recording of behavioral data were controlled by E-prime 2 (Schneider, Eschman, & Zuccolotto, 2002) and a voice-key that was connected to the serial response box (Psychology Software Tools, http://www.pstnet.com)

Numerical magnitude processing

Numerical magnitude comparison

Participants had to select the numerically larger of two presented numerical magnitudes, one on the left and one on the right of the screen, by pressing a key at the side of the numerically largest magnitude. The left response key was d, the right response key was k. Response keys were indicated by a color on the keyboard, green or yellow. Two practice trials were included for each task, to make children familiar with the task requirements. A trial started with a fixation cross for 200 ms, followed by a pause of 800 ms, after which the two stimuli appeared that had to be compared. The experimenter initiated each trial by means of a control key. Children were instructed to perform both accurately and quickly.

Symbolic numerical magnitude comparison

Symbolic stimuli involved Arabic digits ranging from 1 to 100 presented in white on a black background in Arial font (size 72). The stimuli were presented in two separate blocks, one including only single-digit numbers and one including only two-digit numbers. Stimuli in the single-digit condition comprised all possible pairs of numbers 1 to 9, resulting in 72 trials. Stimuli in the multi-digit condition comprised a selection of number pairs with numbers ranging from 10 to 99. These number pairs involved eight different types of pairs, systematically balanced on the decade distance, the unit distance and the compatibility of the digits. The distance between the decades and units of two numbers was either small (1-4) or large (5-9). Half of the trials were compatible , i.e. when both comparisons between decades and units led to the same decision (e.g. 53 and 68; both $5 < 6$ and $3 < 8$) whereas half of the trials were incompatible, i.e. when the two comparisons for decades and units lead to different decisions (e.g. 59 and 74; 5 < 7 but 9 > 4) (Nuerk, Weger, & Willmes, 2001). For each type of pair, four items were presented, resulting in 32 trials. Pairs were presented in a semi random order, with no two pairs of the same type presented after one another. The position of the larger number was counterbalanced and the stimuli remained on the screen until response.

Nonsymbolic numerical magnitude comparison

Nonsymbolic stimuli comprised two sets of white dots simultaneously presented on a black background. Dot patterns were generated with a Matlab script (Gebuis & Reynvoet, 2011) that controlled for information concerning the visual properties of stimuli that co-vary with number (area extended, item size, total surface, density, and circumference). This was done to ensure that children could not reliably use these non-numerical cues or perceptual features to make their decision. To avoid subitizing, stimuli in the nonsymbolic numerical magnitude comparison task involved number pairs with numbers ranging from 8 to 32. Pairs were constructed using ratios 2.0, 1.5, 1.3, 1.2, 1.12, around the standard number 16, resulting in 10 different pairs (i.e. numbers 8, 11, 12, 13, 14, 18, 19, 21, 24 and 32; combined with number 16). The order of the quantity in a pair was balanced and each type of pair was presented six times, yielding 60 trials. The stimulus disappeared after 840 ms to avoid counting the number of dots.

Number line estimation

Children were presented with 25 cm long blank lines in the center of white A4 sheets, representing the interval 0-100. Children were instructed to mark on the line where they thought that the quantity, which was presented on a computer screen, had to be positioned. To ensure that children were aware of the interval size, an example was provided by the experimenter solving the first item of the task while saying: “This line goes from 0 to 100. If here is 0 and here is 100, where would you position this number?”, in the symbolic number line estimation task. In the nonsymbolic format, the experimenter provided an example by the first item saying: “This line goes from 0 dots to 100 dots. If here is 0 dots and here is 100 dots, where would you position this quantity?”. A time limit of 5 seconds was administered, after which the page was turned and children had to start with the next trial.

Following Siegler and Booth (2004) we calculated the percentage absolute error (PAE), as a measure of children’s estimation accuracy, by dividing the absolute value of difference between the estimate and the estimated quantity with the scale of estimates. For example, if a child was asked to estimate 26 on a 0-100 number line and placed the mark at the point on the line corresponding to 41, the PAE would be (41 - 26) / 100 or 15%.

Symbolic number line estimation

Symbolic stimuli involved Arabic digits. The numbers to be positioned were 17, 52, 90, 48, 61, 39, 4, 33, 42, 79, 14, 3, 24, 29, 64, 81, 8, 96, 12, 84, 21, 18, 6, 57, 72, 25, which were the same as in Ashcraft and Moore (2012). Numbers were presented in a fixed random order. The end points of the number line in the symbolic task were labeled with 0 on the left and 100 on the right.

Nonsymbolic number line estimation

Nonsymbolic stimuli comprised white dot patterns on a black background. The quantities to be positioned concerned the same set as in the symbolic number line task, but in a different order: 81, 17, 39, 61, 57, 29, 84, 42, 52, 12, 90, 64, 18, 8, 21, 72, 24, 33, 25, 14, 79, 96, 48. Similar to the nonsymbolic comparison task, we excluded the numbers 3, 4, and 6, to avoid subitizing. Dot patterns were generated with a MATLAB script[2] provided by Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004) and were controlled for non-numerical

[2] It was not possible to use the MATLAB script by Gebuis and Reynvoet (2011). This script only controls for non-numerical parameters when a set of pairs is created. For the nonsymbolic number line estimation task there are no sets of pairs created, only single dot patterns. Therefore, we opted to use the script by Piazza et al. (2004) which also controls for non-numerical parameters.

parameters, i.e. individual dot size, total occupied area, and density. This prevented that decisions were dependent on non-numerical cues or perceptual features. Quantities were presented in a fixed random order. The end points were labeled on the left with an empty circle and on the right with a circle with 100 dots.

Mental multi-digit subtraction

Children had to mentally solve a series of 32 horizontally presented multi-digit subtractions. All problems required carrying. The items in the subtraction task were constructed in such a way that they triggered the use of either direct subtraction or indirect addition. Against the background of previous studies on mental multi-digit subtraction in the number domain 20-100 (e.g. Peters et al., 2012, Torbeyns et al., 2009b), items eliciting direct subtraction (DS-items) were those with (a) a subtrahend smaller than the difference and (b) a relatively large distance ($M_{distance}$ = 54.38) between the minuend and the subtrahend, e.g. “83 - 27 = .”, and items eliciting indirect addition (IA-items) were characterized by (a) a subtrahend larger than the difference and (b) a relatively small distance ($M_{distance}$ = 13.81) between the minuend and the subtrahend, e.g. “72 - 67 = .”. Children were only allowed to solve the items using these two different types of strategies, either the *direct subtraction* strategy or the *indirect addition* strategy. Both strategies were explained and demonstrated at the start of the experiment.

Children were shown a subtraction problem on the computer screen after which they were expected to mentally solve the problem as fast and accurately as possible. They were not allowed to use paper and pencil or any other materials. Children had to say their answer out loud into a microphone that recorded their response time, using a voice key, after which the experimenter typed the child’s answer into the computer. The experimenter initiated each trial by means of a control key.

Control tasks

Motor reaction time

This task was included to control for children's response speed on the keyboard. Two figures appeared on the screen. One of them was colored white and one of them was colored black. The child was asked to press as soon as possible on the side of the white figure. Twenty experimental trials were presented.

Intellectual ability

Children's general cognitive ability was assessed with Raven's standard progressive matrices (Raven, Court, & Raven, 1992). For each child, a standardized score (*M* = 100; *SD* = 15) was calculated.

Procedure

All tasks were administered at the children's own school. The measure of intellectual ability was group-based. All other tasks were completed individually. These tasks were administered in two different sessions, starting with the motor reaction time task and the numerical magnitude comparison tasks in session 1, followed by the number line estimation tasks and the subtraction task in session 2. All children completed the tasks in the same order.

Results

On a trial level, we removed trials in the numerical magnitude comparison tasks for which children had a response time higher than 5 seconds (0.1% of the trials). We also removed trials in the subtraction task for which children had a response time higher than 35 seconds (3.6% of the trials). Response times were calculated on the basis of the correct trials only. All analyses were carried out by means of SPSS Version 18.

Descriptive statistics

Numerical magnitude processing

Table 1 shows the descriptive statistics for the numerical magnitude processing tasks. The accuracy on symbolic numerical magnitude comparison was very high and therefore we only considered mean reaction times on this task in subsequent correlational analyses. On the other hand, the accuracy on the nonsymbolic numerical magnitude comparison task was substantially lower, and therefore we included accuracy rather than speed on this task in subsequent analyses. This echoes previous work with similar symbolic and nonsymbolic comparison tasks where respectively speed rather than accuracy (De Smedt et al., 2009; Landerl & Kölle, 2009) and accuracy rather than speed is analyzed (Sasanguie et al., 2012).

Mental multi-digit subtraction

Table 2 shows the descriptive statistics for the multi-digit subtraction task. The accuracy was high for both types of subtraction items. A repeated measures ANOVA on the accuracies with type of item as a within-subject factor showed that the accuracy for the DS-items was significantly higher than for the IA-items ($F(1,55) = 5.47$, $p < .05$). Furthermore, the reaction time for the DS-items was significantly faster than for the IA-items ($F(1,55) = 4.21$, $p < .05$).

Correlational analyses

To examine the associations between the numerical magnitude processing tasks and the mental multi-digit subtraction task, we performed partial Pearson correlational analyses (Table 3), controlling for intellectual ability. For the symbolic numerical magnitude comparison task, there was a strong correlation between the blocks with the numbers 1-9 and 10-100 ($r = .77$, $p < .01$). Analyses with the two tasks separately showed that the results for both tasks were very similar. Therefore, performance on both blocks was averaged and this average score was used in all subsequent analyses.

Significant positive correlations were found between the response times on the symbolic numerical magnitude comparison task and the response times on the DS-items and IA-items of the subtraction task. This means that children who were faster in comparing two numbers, were also faster in solving the DS-items and the IA-items. In a second step we additionally controlled for motor reaction time when the correlation between two reaction times measures was calculated. Results showed that all previously found associations remained significant (*r*s > .39; *p*s < .01). Symbolic numerical magnitude comparison was also significantly negatively correlated with accuracy on the IA-items, indicating that children who were faster on symbolic comparison performed more accurately on the IA-items. The nonsymbolic numerical magnitude comparison task was only positively correlated with accuracy on the IA-items, showing that children who performed more accurately on the nonsymbolic numerical magnitude comparison task were also more accurate in solving the IA-items. Turning to number line estimation, we only observed significant positive correlations between the symbolic number line estimation task and the response times for both types of subtraction items, indicating that children who made more accurate number line estimates, were also faster in solving DS-items and IA-items. There were no other significant associations between the number line estimation tasks and mental multi-digit subtraction (Table 3).

To compare the unique contribution of each numerical magnitude processing task in explaining performance on the DS-items and IA-items, we calculated partial correlations in which the correlation between performance on a numerical magnitude processing task and one type of item was controlled for the performance on the other type of item, and for intellectual ability (Table 4). Results showed that there was no significant correlation between the performance on the symbolic numerical magnitude comparison task and the response times on the DS-items, when controlled for the performance on the IA-items. On the other hand, a significant positive correlation was found between response times on the symbolic numerical magnitude comparison tasks and response times on the IA-items, when controlling for the response times on the DS-items. The comparison of these two partial correlations, using the Williams-Steiger test, showed that they were significantly different from each other (t = -2.89; p < .01), suggesting that symbolic numerical magnitude comparison yields a more unique contribution in explaining the response times on IA items. No significant correlation was found between performance on the symbolic numerical magnitude comparison task and accuracy on the DS-items, when we controlled for the accuracy on the IA-items. However, there was a significant negative correlation between the performance on symbolic numerical magnitude comparison task and the accuracy on the IA-items, when controlling for the accuracy on the DS-items. These partial correlations did not significantly differ from each other (t = 1.50; p = .14).

A similar pattern of results was found for the nonsymbolic numerical magnitude comparison task. No significant correlation was found with the accuracy on the DS-items when controlling for the accuracy on the IA, but there was a significant positive correlation with the accuracy on the IA-items when controlling for the accuracy on the DS-items. A comparison of these partial correlations showed that they were significantly different from each other (t = 2.04; p < .05).

To assess the unique contribution of symbolic number line estimation in explaining the performance on the DS-items and IA-items, we again calculated partial correlations in which the correlation between the PAE on the symbolic number line estimation task and the performance on one type of item was controlled for the performance on the other item type. Results showed that there was no significant correlation between PAE and performance on the DS-items, when controlled for performance on the IA-items, and no significant correlation between PAE and performance on the IA-items, controlling for performance on the IA-items.

All associations remained significant after additionally controlling for motor reaction time (*r*s > .28; *p*s < .05).

Discussion

Studies in typically developing children have shown that the ability to represent numerical magnitudes, as measured with the numerical magnitude comparison tasks and number line estimation tasks, is related to general mathematics achievement (Booth & Siegler, 2006; Holloway & Ansari, 2009). From these tests it is unclear how children's numerical magnitude processing contributes to specific mathematical skills. In the current study, we therefore investigated the association between numerical magnitude processing and individual differences in mental multi-digit subtraction in the number domain 20-100. First, we expected an association between children's numerical magnitude processing and their performance on the multi-digit subtraction problems. Second, we anticipated this association to be more prominent for those items in which the use of indirect addition is considered most efficient. Third, comparing the performance on two numerical magnitude processing tasks could give us insight on which of these tasks is most important for mental multi-digit subtraction. Finally, by comparing symbolic and nonsymbolic tasks, we tried to verify whether the numerical representations of magnitude per se or rather the access to numerical magnitude from symbolic digits, is related to mental multi-digit subtraction.

First, extending the existing body of data, our results indicated that there was a significant association between numerical magnitude processing and mental multi-digit subtraction, even after we additionally controlled for intellectual ability and motor speed. Children with a better numerical magnitude processing ability were faster and more accurate in mentally solving multi-digit subtractions. The present findings replicated those of previous studies on the role of numerical magnitude processing in general mathematics achievement (Booth & Siegler, 2006; Bugden & Ansari, 2011; Butterworth, Varma, & Laurillard, 2011; De

Smedt et al., 2009; Halberda & Feigenson, 2008; Price & Ansari, 2012), and go beyond the previous ones, by showing that this association also applies to a more specific mathematical skill, i.e. mental multi-digit subtraction.

Second, while our findings showed that children with better numerical magnitude processing ability performed better on both types of subtraction items, the role of numerical magnitude processing ability seemed to be more important in the IA-items, as indicated by the partial correlation analyses in which we controlled for the performance on the DS-items and vice versa, and by the direct comparison of these correlations. This could be explained by the fact that the flexible use of the indirect addition strategy requires a good understanding of the numerical magnitudes in the problem and their mutual relation (Baroody et al., 2009; Verschaffel et al., 2012). For example, the flexible use of the indirect addition strategy requires an appropriate estimation of the difference between minuend and subtrahend, on which children with better numerical magnitude ability may perform better. Our findings are in line with this idea and suggest that indeed the determination of the proximity of two given numbers, which is required for applying the indirect addition strategy efficiently, i.e. fast and accurate, depends on one's numerical magnitude processing.

A third goal was to unravel which of the numerical magnitude processing tasks provided the best predictor of mental multi-digit subtraction. Sasanguie et al. (2012) found that the symbolic numerical magnitude comparison task and the symbolic number line estimation task were both related to children's mathematics achievement. This is consistent with our results, suggesting that that both tasks indeed are measures of a common mechanism, namely numerical magnitude processing. When performing the partial correlational analyses, however, to compare the unique contribution of each numerical magnitude processing task, the association between the number line estimation task and mental multi-digit subtraction disappeared, while the association with numerical magnitude comparison was still significant.

This suggests that numerical magnitude comparison has a more unique contribution in explaining performance on IA items. It is possible that different mechanisms underlie these numerical magnitude processing tasks, which has also been shown in a recent study by Sasanguie and Reynvoet (2013). The numerical magnitude comparison task is reflecting decisional mechanisms on activated magnitude representations (see Verguts, Fias & Stevens, 2005), whereas the number line estimation task also taps into visuo-spatial processes. Our results suggests that the fluency by which number representations are available, i.e. how fast decisions can be made based on the activated magnitude representations, is an important factor, as shown by the role numerical magnitude comparison plays in explaining mental multi-digit subtraction. However, more work needs to be done to provide a fine grained characterization of the specific mechanisms involved in the numerical magnitude comparison task and the number line estimation task.

Finally, in contrast to the findings of Vanbinst et al. (2012), who observed similar associations between numerical magnitude processing and mental subtraction in the number domain up to 20 and only revealed a significant association with the symbolic numerical magnitude comparison task, we also found a significant association between mental calculation and nonsymbolic numerical magnitude comparison. A possible explanation is that nonsymbolic processing does not play a role in the processing of small numbers, and only comes into play when larger, multi-digit numbers are used. This further relates to the question whether the representation of numerical magnitudes per se or its access via symbolic digits is important for mathematics achievement (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007) and consequently for mental multi-digit subtraction. To address this issue, we compared the performance on numerical magnitude tasks with symbolic processing requirement, i.e. using Arabic digits as stimuli, and without symbolic processing requirement, i.e. using dots as stimuli. Some studies have shown that symbolic but not nonsymbolic magnitude processing skills correlate with individual differences in general mathematics achievement (Holloway & Ansari, 2009; Rousselle & Noël, 2007; Sasanguie, De Smedt, et al., 2012), whereas others observed significant associations between nonsymbolic magnitude processing and mathematics performance (Halberda et al., 2008; Libertus et al., 2011; Mazzocco, Feigenson, & Halberda, 2011). Our results revealed significant correlations between mental multi-digit subtraction and both symbolic and nonsymbolic tasks. This is in line with previous studies on general mathematics achievement (Mundy & Gilmore, 2009; Mussolin, Mejias, & Noël, 2010). These associations, however, were different for speed and accuracy on the multi-digit subtraction task. Accuracy on the subtraction task was associated with the symbolic numerical magnitude comparison task, the nonsymbolic numerical magnitude comparison task, and the symbolic number line estimation tasks, indicating that both symbolic and nonsymbolic representations seem to play a their role in mental multi-digit subtraction. By contrast, the speed by which children solved the subtraction items was only associated with the two symbolic tasks and not with the nonsymbolic tasks, indicating the association to be more prominent for the symbolic tasks. This suggests that particularly the access to numerical magnitudes from symbolic digits is crucial for mental multi-digit subtraction.

The current data were collected at only one measurement point and therefore they do not allow us to make causal interpretations. It would be interesting to conduct, as a next step, a longitudinal study that examines the development of children's numerical magnitude processing and its association with mental multi-digit subtraction. Even more compelling would be to carry out carefully controlled intervention research that examines the effect of training children's numerical magnitude processing on their mental subtraction performance. It has been shown that interventions that use games based on magnitude processing, e.g. number lines, enhance children's performance in numerical magnitude processing tasks

(Kucian et al., 2011; Ramani & Siegler, 2011) as well as learning of multi-digit addition (Booth & Siegler, 2008). Against the background of these findings, it would be interesting to examine whether interventions that use such games affect mental multi-digit subtraction.

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Table 1

Descriptive statistics for numerical magnitude comparison and number line estimation tasks

Measure	*M*	*SD*
Numerical magnitude comparison		
Symbolic		
Reaction time (ms)	965.11	246.03
Accuracy (%)	94.67	2.98
Nonsymbolic		
Reaction time (ms)	1009.49	238.65
Accuracy (%)	71.35	7.26
Number line estimation		
Symbolic		
Percentage absolute error	6.30	2.25
Nonsymbolic		
Percentage absolute error	17.48	4.65

Table 2

Descriptive statistics for mental multi-digit subtraction

Variable	*M*	*SD*
DS-items		
Reaction time (ms)	10842.18	3319.78
Accuracy (%)	96.53	4.72
IA-items		
Reaction time (ms)	11532.08	4242.00
Accuracy (%)	94.50	6.23

Note. DS = direct subtraction. IA = indirect addition.

Table 3

Pearson partial correlations between the numerical magnitude processing tasks and subtraction, controlling for intellectual ability

	Direct subtraction		Indirect addition	
Measure	Accuracy	RT	Accuracy	RT
Numerical magnitude comparison				
Symbolic (RT)	-.16	.43**	-.32*	.50**
Nonsymbolic (Accuracy)	.10	-.26	.33*	-.24
Number line estimation				
Symbolic (PAE)	-.12	.37**	.15	.35*
Nonsymbolic (PAE)	-.16	.08	-.22	.06

Note. * $p < .05$; ** $p < .01$
Note. PAE = percentage absolute error.

Table 4

Pearson partial correlations between the numerical magnitude processing tasks and one type of subtraction item, controlling for the other type of subtraction item and intellectual ability.

	DS (controlling for IA)		IA (controlling for DS)	
Measure	Accuracy	RT	Accuracy	RT
Numerical magnitude comparison				
Symbolic (RT)	-.07	.06	-.29*	.30*
Nonsymbolic (Accuracy)	-.01	-.11	.32*	-.05
Number line estimation				
Symbolic (PAE)	-.16	.17	.20	.08

Note. * $p < .05$; ** $p < .01$

Note. DS = Direct subtraction. IA = Indirect addition

Appendix 1

Studies that have examined the association between numerical magnitude processing and mathematics achievement by means of magnitude comparison tasks have typically included various indicators to index performance on the comparison task, such as distance effects or Weber fractions (see De Smedt et al., 2013, for a recent review). In this appendix, we explore this issue for the current study. To address this issue for the association between numerical magnitude processing and multi-digit subtraction, we performed additional analyses using the distance effect and Weber fraction as other indicators of numerical magnitude processing.

We first re-analyzed our symbolic numerical magnitude processing data using the slope of a regression of numerical distance on RT as a measure for the distance effect (see De Smedt et al., 2009, for a similar analysis). This was done separately for the numbers 1 to 9, where all distances were included in the analysis, and the numbers 10 to 99, where distances between decades were included to calculate the distance effect (see Table 1 below). We found a significant negative correlation between the response times on the IA items and the distance effect on both the numbers 1 to 9 and the numbers 10 to 99 in the symbolic numerical magnitude comparison task. Moreover, a significant negative correlation between the response times on the DS items and the distance effect on the small numbers, and a trend for the large numbers (p = .06), in the symbolic numerical magnitude comparison task was found.

Table 1

Pearson partial correlations between the distance effect on symbolic numerical magnitude comparison and subtraction, controlling for intellectual ability

	Direct subtraction		Indirect addition	
Measure	Accuracy	RT	Accuracy	RT
Symbolic numerical magnitude comparison				
Numbers 1-9	-.01	-.38**	.05	-.36*
Numbers 10-99	.08	-.25°	.15	-.30*

Note. ° p = .06; * p < .05; * p <.01

In a next step, we again performed partial correlation analyses in which we accounted for the performance on DS items when considering the association between IA items and numerical magnitude comparison and vice versa. Results (See table 2 below) showed that the distance effect did not have a unique role in explaining the performance on the subtraction items. These data seem to be less strong and robust compared to data where general reaction time on the comparison task is used as outcome measure. This is in line with the existing body of studies that examined the association between symbolic magnitude comparison and mathematics achievement (De Smedt et al., 2013). In their review, De Smedt et al. show that while there are consistent and robust associations between reaction on the symbolic comparison task and mathematics achievement, the associations between the distance effect and mathematics performance are less consistent (see also Sasanguie et al., 2012, 2013).

Table 2

Pearson partial correlations between the distance effect on symbolic numerical magnitude comparison tasks and one type of subtraction item, controlling for the other type of subtraction item

	DS (controlling for IA)		IA (controlling for DS)	
Measure	Accuracy	RT	Accuracy	RT
Symbolic numerical magnitude comparison				
Slope: numbers 1-9	-.01	-.24	.09	.00
Slope: numbers 10-99	.04	-.01	.13	-.17

Note. * $p < .05$; ** $p < .01$

Secondly, we re-analyzed our nonsymbolic data by using the weber fraction as a measure for nonsymbolic numerical magnitude processing. It was not possible to find a good fit for two children, so this analysis was based on the data of 54 children instead of the complete dataset of 56 children. Table 3 below shows the results of the correlational analysis, which indicates that no significant correlations were found between the Weber fraction and children’s performance on mental multi-digit subtraction items. A recent study by Sasanguie et al. (2013) has used both the weber fraction and accuracy as a measure for non-symbolic comparison and showed that both measures indicated the same results and were strongly correlated. This suggests that both measures index similar processes. On the other hand, the fit for the weber fraction is not always very good in children, making it a less suitable measure for our study, which might explain why not all studies using weber fractions have observed an association between non-symbolic comparison performance and math achievement (see also De Smedt et al., 2013, for a discussion).

Table 3

Pearson partial correlations between the weber fraction on nonsymbolic numerical magnitude comparison and subtraction, controlling for intellectual ability

	Direct subtraction		Indirect addition	
Measure	Accuracy	RT	Accuracy	RT
Nonsymbolic numerical magnitude comparison				
Weber Fraction	-.09	.08	-.05	.16

Note. * $p < .05$; ** $p < .01$

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