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Aircraft maintenance operations: state of the art

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This paper provides a survey on aircraft maintenance in operations research and management science. Although it is quite related with other airline operations such as flight or crew scheduling, literature on aircraft maintenance is clearly outnumbered in this research area. The literature is classified according to many fields that are related with the problem characteristics or the decisions that need to made. This paper tries to provide a clear overview of the different types of aircraft maintenance and their applications. The main contribution of this review, however, is to facilitate the tracing of the published work in relevant fields of interest. We also identify some trends in the available literature and indicate the areas which should be of interest for future research.

Key words: Aircraft maintenance, literature review, classification, OR in airport operations.

1. Introduction

Throughout the years, the understanding and especially the planning of maintenance checks for airplanes have changed substantially. In the very beginning, the structure of airplanes was quite basic and simple. Therefore, maintenance was usually straightforward and often planned manually. When necessary, it used to be performed after a short period of flying time. Even more comprehensive activities such as repairs and overhauls, which take place on a frequent basis, used to be performed ad hoc. However, the manual planning of maintenance became more and more impracticable due to a more dynamic environment in which both costs and the complexity of the airplanes kept rising. As a result, systematic planning of maintenance was necessary in order to save costs and achieve a greater efficiency [3, 13].

The airline industry is not comparable to any other transportation industry. Flights consist of more than just the take-off and the landing: all has to be put in place, authority and maintenance requirements have to be met. Maintenance checks have to be performed with care to make sure that every plane leaving the ground is reliable, safe and airworthy, this of course at the lowest possible cost. It is obvious that maintaining fleets properly is of key importance in order to stay one of the safest transportation ways. This paper provides a structured overview of the different aspects of aircraft maintenance and an insight into how airline companies are trying to achieve the right balance between three key values: profit, safety and optimal planning of their activities.

Aircraft maintenance is discussed thoroughly in the literature on engineering and technical aspects of an airplane. In this paper, the technical aspect which comes along with maintenance activities will be of minor importance. The focus lies on descriptive classifications fields, some of which are: types of maintenance, (integrated) airline scheduling, maintenance optimization, facility location, workforce for maintenance, personnel training, etc. All articles were published in the research area of ‘Operations Research/Management Science’. After consulting databases like ‘Web of Knowledge’, ‘ScienceDirect’, ‘Google Scholar’ and ‘Springerlink’, the most important classification fields from these papers were chosen. References which were cited in these papers were examined as well, which eventually led to a set of 102 manuscripts (i.e., 85 journal papers, 14 conference papers, 3 other types). Figure 1 shows that the number of contributions to the field of aircraft maintenance is following a positive trend in the last few decades.

Figure 1: Total number of manuscripts on aircraft maintenance

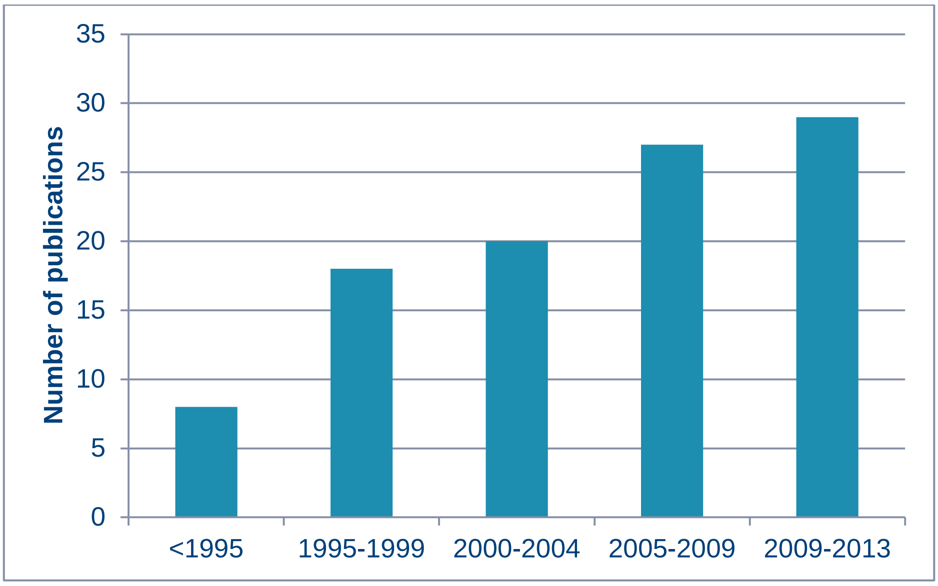


Table 1 gives an overview on the different journals. It is clear that Transportation Science contributes the most to the research area of aircraft maintenance, followed by Interfaces, Computers & Operations Research and the International Journal of Industrial Ergonomics.

Table 1: Overview of journals

Transportation Science	12
Interfaces	8
Computers & Operations Research	7
Annals of Operations Research	6
European Journal of Operational Research	5
Journal of the Operational Research Society	5
Reliability Engineering and System Safety	5
Naval Research Logistics	4
Journal of Air Transport Management	3

2. Organization of the review

This paper presents an overview of the operational aspects of aircraft maintenance, starting from the available literature in the areas of Operations Research and Management Science. In order to give the reader a clear view on the key involvements in aircraft maintenance, this paper is structured in different sections, also called descriptive fields. Each of them involves aircraft maintenance in general, but focuses on one or several of the different subcategories. The different perspectives are:

Type of maintenance (Section 3*):* Indication of differences between A-, B-, C- and D-checks, line maintenance, hangar maintenance, scheduled and unscheduled maintenance, etc.

(Integrated) Airline scheduling (Section 4*):* The evolution of flight scheduling, fleet assignment, aircraft maintenance routing and crew scheduling, all discussed separately as well as interrelated. How are these problems formulated, constrained and solved and how are they related to aircraft maintenance?

Aircraft maintenance optimization (Section 5*):* How can aircraft maintenance be optimized? Papers that specifically study engine scheduling or maintenance planning for other components, task allocation and maintenance scheduling problems. What is the objective of aircraft maintenance scheduling problems and how are they formulated? Extra attention is paid to which constraints are incorporated and the solution methodologies.

Facility location (*Section* 6*):* Analysis of papers that address either aircraft maintenance routing or aircraft maintenance scheduling. Is the choice of the location where the maintenance is performed part of the optimization problem? Is the choice related to a single base or to multiple bases, and does it depend on the flight schedule?

Workforce and training (Section 7*):* Which skills and/or licenses do the technicians and engineers need, and how are the typical workforce aircraft maintenance problems scheduled and solved? Investigation of which types of training are possible or necessary to educate the personnel? When and how should they be trained? A presentation of the different training forms.

Uncertainty and application of research (Section 8*):* How uncertainty is modelled, the availability of data for testing (theoretic or real data based) and a critical view on whether this is put into practice or not.

Per section, a concise analysis of the specific field is presented, based on a selection of appropriate manuscripts. Specific terminology and abbreviations will be explained, detailed tables will be included to give the reader a clear overview of all the relevant papers. A combination of these tables will enable the reader to reconstruct the content of specific papers. The listing summarizes the subset of articles which correspond to a certain subcategory and one can search for papers of interest, corresponding to certain characteristics. Section 9 will give the conclusion of this thesis.

3. Type of maintenance

This section gives an outline of the different types of maintenance for aircraft: scheduled and unscheduled maintenance, line and hangar maintenance, routine and non-routine checks, etc. Various terms are used to refer to the same type of maintenance. For example, there are four major types of maintenance checks (A, B, C and D) which each airplane has to undergo after a certain amount of flying hours, as regulated by the FAA. These four types can also be considered as part of scheduled or preventive maintenance types. Instead of considering them as separate subfields, overlaps are rather common, even if the authors use other terms in their papers. Table 2 indicates which type(s) of maintenance are studied by which papers.

Table 2: Type of maintenance

A-check	4, 11, 12, 13, 18, 24, 29, 31, 34, 50, 51, 82, 87, 89, 93, 97, 100
B-check	13, 18, 29, 31, 34, 87, 89, 100
C-check	13, 18, 29, 31, 34, 89, 100
D-check	13, 29, 31, 34, 89, 100
Scheduled maintenance	1, 4, 6, 8, 13, 15, 18, 24, 25, 28, 31, 33, 35, 43, 44, 47, 48, 49, 56, 57, 59, 60, 64, 67, 69, 76, 77, 79, 86, 88, 90, 92, 100, 102
Unscheduled/emergency maintenance	4, 28, 35, 43, 52, 57, 58, 59, 60, 68, 75, 77, 86
Routine/non-routine maintenance	2, 7, 10, 17, 21, 26, 28, 35, 36, 38, 43, 51, 61, 66, 68
Short/mid/long maintenance	2, 5, 36, 38, 97
Light or heavy/base/hangar maintenance	11, 12, 31, 65, 66, 77, 93
Line maintenance	4, 10, 11, 12, 36, 65, 68, 77, 93
Preventive maintenance	1, 3, 5, 6, 11, 12, 14, 22, 27, 28, 48, 49, 64, 70, 72, 76, 77, 84, 85, 91, 93
Corrective maintenance	1, 8, 14, 16, 22, 28, 48, 49, 72, 75, 76, 84, 85
Predictive/on-condition maintenance	5, 14, 41, 43, 48, 49, 53, 101
Layover maintenance	
Short-term layover	
Pre/(post) flight inspection	4, 11, 12, 59, 60, 68, 77, 93, 97, 98, 99
Transit check	4, 11, 12, 34, 89, 97, 98, 99
Daily check (night stop check/service check)	4, 11, 12, 54, 93, 97, 98, 99
Regular checks	98, 99
OR, IN, DE level maintenance	47, 57, 59, 60
Turnaround inspection	59, 60
Other	11, 12, 34, 59, 60, 86, 93

As mentioned in the previous paragraph, there are four significant types of maintenance checks: A-, B-, C-, and D-checks. Each of them varies in scope, duration and frequency [34]. The first one, the A-check, occurs most frequently, and has to be performed about every sixty-five flight-hours, or approximately once a week. This check comprises the inspection of landing gear, engines and control surfaces [87]. The second substantial check, also called B-check, is performed slightly less frequently, about every 300 to 600 hours. This involves a more extensive visual inspection and also lubrication of all moving parts, for example of the horizontal stabilizers of the plane [87]. The two largest checks, types C and D, are called the heavy maintenance checks. A C-check is an inspection that takes about one to two weeks, once every year. A D-check, which includes, among others stripping, painting and cabin refurbishment, varies from a three-week to two-month inspection and is done once every four years [11]. Within these types of checks, sometimes an extra division is made. Clarke et al. [18] divided the A-check in an M- and Av-check. The M-check is an inspection, carried out every two to three days on every aircraft, the Av-check consists of the M-check in addition to an avionics inspection and is carried out every four to five days. Some airline companies break the C-check into four quarter C-checks, which Talluri [89] calls balance-checks.

Certain papers distinguish between scheduled and unscheduled maintenance. The former is often stated as a preventive form of maintenance which ensures the aircraft is airworthy, conducted at pre-set intervals, and consisting of a range of checks such as A-, B-, C- and D-checks, 48-hour checks and transit checks. The transit checks involve a visual inspection to check if the aircraft carries what is called a Minimum Equipment List or MEL [89], which is a mandatory list of items which allows a plane to fly, even if a number of these items are inoperative but the safety and operation of the flight are not affected. Transit checks are performed at every transit stop of the airplane. The 48-hour checks are performed every two days and are more detailed than the transit checks. The unscheduled maintenance is necessary when components fail and have to be repaired before the plane's next flight.

In the literature, further classifications are used: routine and or non-routine maintenance in for example [2, 10, 38, 61, 68], short, mid and long range maintenance checks in [2, 18, 38, 97] and light or heavy/base/hangar maintenance in [11]. Routine maintenance can be used as a synonym for scheduled maintenance, whereas non-routine maintenance groups the unscheduled maintenance. This definition differs somewhat from that of Haouari et al. [38], who state that routine checks are those checks that need to be executed frequently. Hence, it is a synonym for short-term maintenance at ‘TunisAir’.

As shown in Table 2, other papers also use the terms line and preventive maintenance instead of the terms scheduled, line or routine maintenance. For example, Beliën et al. [11] state that line maintenance includes pre-flight inspections, transit checks, daily checks (visual inspection, fluid levels, general security and cleanliness of the flight deck, emergency equipment), weekly checks and on-call assistance. This line maintenance is preventive in this paper and thus opposed to unplanned or emergency maintenance.

Kumar, Crocker and Knezevic [48] look at the concept for evolutionary maintenance for aircraft engines, a special type of maintenance, but distinguish between three big groups of maintenance in their paper: preventive, corrective and predictive or on-condition maintenance. Preventive or scheduled maintenance are synonyms according to Kumar et al. and done at predefined ages of the system with the purpose of reducing the probability of failure of the system. Corrective maintenance, on the other hand, is performed to restore a system to functioning after it has already failed. Predictive or on-condition maintenance is effectively unscheduled preventive maintenance in which the monitored “condition” of the system triggers the maintenance action [48].

Another term that is used in the literature of aircraft maintenance is layover maintenance [97-99]. Generally, it is performed on-line at the gate or at a connecting airport. Consequently, it has to be planned perfectly to fit in the overall schedule of aircraft that come and go at the gates. Otherwise, delays might occur which incur extra operating costs. Layover maintenance consists of two parts: short-term layover maintenance and regular checks. Since both parts have other features and concerns, their planning is separated. Short-term layover maintenance includes, in addition to a pre-flight check, also a transit and a daily check. On average, they take one or two hours. It usually requires one or more days to finish the whole regular check. It is imperative for the aircraft to stand at the parking ramp while the tasks are performed [99].

Other papers, mainly situated in the military application field, mention a partition such as maintenance at organizational (OR), intermediate (IN) or depot (DE)-level [47, 57, 59, 60]. Kozanidis and Skipis [47] mention a military application for the Hellenic Air Force (HAF) in which the maintenance program is divided into three types of maintenance: first or organizational (OR)-level, second or intermediate (IN)-level and third or depot (DE)-level maintenance. OR-level checks are conducted on site and include inspection, repair and parts replacement. IN-level checks are performed on site as well, yet they consist of a more thorough inspection, repair and parts replacement than OR-level checks. DE-level checks need to be performed by trained workforce in specially designed facilities. This is the most thorough repair and parts replacement.

Since there is a great variety of terminology for the different types of maintenance in Operations Research and Management Science, it is not always forthright to know what exactly the authors mean in their papers. Certainly, an explanation is provided to give the reader more information but even then, general terminology would be convenient. As can be seen in Table 2, the authors use the A-D-checks quite often. Nevertheless, a few of them state that A- and B-checks are short-term checks and C- and D-checks are long term checks. Others state that an A-check is short or light and a B-check is long or heavy. There is no real consistency, which makes it difficult for the readers to completely understand the different types of maintenance. The terms scheduled, preventive, line and corrective maintenance, are the other most commonly used types. Scheduled and preventive maintenance both prove that proper planning is of key importance in aircraft maintenance. Multiple examples across the papers show that cost savings can be made via proper planning while still delivering safe and airworthy aircraft. Again, the downside here is that the subdivisions are not clearly separated from each other. Only preventive and corrective maintenance are noticeably different. Preventive and scheduled maintenance are often used as synonyms. Nonetheless, a clear overall definition would be convenient. Therefore, we provide a framework in Table 3, by using the most common definitions. With respect to uncertainty, aircraft maintenance can be divided into scheduled (preventive or routine) maintenance and unscheduled or non-routine maintenance, indicated by the rows in the table. The columns refer to the intensity of the workload, starting from short-term (i.e., frequent and light) until long-term (heavy) maintenance. The research community seems to agree that line maintenance consists of both scheduled and unscheduled maintenance. Confusion arises, however, when the types of tasks need to be specified. Some see line maintenance as only the short-term maintenance, others also add the A- (and even B-) checks. Line maintenance got its definition from “on line” maintenance, referring to all the maintenance that can be done at the gate or at the apron. All other maintenance is categorized as “hangar” maintenance. In our table, we categorize both classification fields as mid-term maintenance (i.e., A- and B-checks), whereas the heavy maintenance (i.e., C- and D-checks) is categorized as hangar maintenance only. Note that one paper suggests that line maintenance is the maintenance carried out by the airline company, opposite to the base maintenance which is assigned to a third party.

Table 3: Taxonomy for aircraft maintenance

	Lay-over maintenance or light maintenance			Heavy maintenance	
	Line maintenance	Line or hangar maintenance		Hangar maintenance	
SCHEDULED or PREVENTIVE or ROUTINE	**Short-term**	**Mid-term or regular checks**		**Long-term**	
	Pre-flight, transit, daily checks	**A-check**	**B-check**	**C-check**	**D-check**
		Av- check **M-check**		**Balance checks**	
UNSCHEDULED or NON-ROUTINE	*Predictive or on-condition maintenance*	*Predictive or on-condition maintenance*		*Predictive or on-condition maintenance*	
	Corrective or emergency maintenance	*Corrective or emergency maintenance*		*Corrective or emergency maintenance*	

4. (Integrated) Airline scheduling

The airline industry is a very competitive environment. Besides revenue management, profit maximization can be achieved by the minimization of operational costs. These costs are related to four substantial subproblems of airline scheduling: flight scheduling, fleet assignment, (maintenance) routing and crew scheduling. Table 4 gives an overview of which papers address which subpart(s).

Table 4: (Integrated) Airline scheduling

A.	Flight scheduling	29, 33, 46, 47
B.	Fleet assignment	8, 18, 64, 74, 83, 88
C.	Routing	2, 6, 17, 23, 34, 54, 79, 89
D.	Flight scheduling + Fleet assignment	82
E.	Flight scheduling + Routing	45, 50
F.	Fleet assignment + Routing	9, 30, 37, 38, 63
G.	Fleet assignment + Crew scheduling	31
H.	Routing + Crew scheduling	20, 21, 25, 61, 96
I.	Flight scheduling + Fleet assignment + Routing	24
J.	Fleet assignment + Crew scheduling + Routing	67
K.	Flight scheduling + Routing + Crew scheduling	56, 62
L.	Flight scheduling + Fleet assignment + Routing + Crew scheduling	51

Mostly, aircraft maintenance in (integrated) airline scheduling can be found in all the combinations with routing. Although the majority of the literature on the other subproblems of (integrated) airline scheduling tend to leave out all the maintenance considerations, there are some exceptions. These will be elaborated in the following paragraphs, while discussing the characteristics of each subproblem.

The first subproblem is flight scheduling. Here is determined where and when flights will depart and arrive. It is mostly handled together with some of the other three parts of airline scheduling. In one paper [29], it is studied as a standalone problem while considering aircraft maintenance. The purpose of the paper is to present a model for planners to both locate maintenance stations and to develop flight schedules that better meet the cyclical demand for maintenance. The second part, usually solved subsequent to flight scheduling, is the fleet assignment problem (FAP), where an aircraft type is assigned to each flight that is scheduled. It is an essential step in the whole airline scheduling process because it highly impacts the companies' revenues [37]. It takes into account equipment availability and capability, operational costs and, consequently, also potential revenues. Similar types of an aircraft can be categorized in a group. Clarke et al. [18], for instance, incorporate maintenance and crew constraints in their formulation of the fleet assignment problem, without solving the (maintenance) routing or crew scheduling problem. They believe that by incorporating some of the characteristics of the latter two problems, these will be easier to solve in a later phase. The foremost framework to formulate this assignment problem and airline problems in general is the time-space network (TSN), in which nodes represent all the departure and arrival cities and arcs represent all the flights [18]. Some examples of papers where this time-space network is used, are [38, 82]. This time-space network representation has some inconveniences too, for example the lack of ability to determine a specific airplane on the ground in the representation. This constrains its application in the following step of aircraft routing. As a result, it is modelled as a mixed-integer multi-commodity flow problem in for example [18, 30, 37, 74]. Here, the commodities represent the fleets and the constraints make sure that the coverage of flights as well as the operational requirements are satisfied [74]. An overview of the use of problem formulations or representations is given in Table 5.

Table 5: Problem representation

Time-space network (TSN)	9, 21, 31, 38, 54, 61, 62, 82, 88
Integer multi-commodity flow network	18, 25, 29, 30, 31, 37, 74, 76, 87
Set-partitioning based formulation	5, 6, 15, 20, 25, 29, 31, 38, 79, 96
Job shop problem	3, 19, 32, 43
Time-line graph/flight-leg network	9, 17, 18, 31, 34, 88, 89
Connection graph/connection network	2, 6, 50, 79

The aircraft groups retrieved in the second step will form the input for our third subproblem, the aircraft (maintenance) routing problem. Routing is done according to the tail number, the identification number of a plane. Every plane needs to be parked in a maintenance station after a certain number of days of flying without maintenance. It spends then at least one night at the maintenance base station [34]. Aircraft maintenance checks are necessary after a certain number of flying hours, for safety reasons and since authorities compel it. The problem of aircraft routing is often modelled as connection graphs or networks [2, 6, 79] or as timeline graphs or flight-leg networks [17, 34, 89]. A connection network is a structure in which nodes represent flight legs and arcs represent feasible connections among the flight legs. The connection network includes dummy source nodes that are connected to appropriate starting legs and dummy sink nodes that are connected to appropriate ending flight legs. Flight-leg networks are structures in which nodes represent cities and arcs between nodes represent flight legs, connecting these cities. A complication with this flight-leg or time-line representation involves keeping track of the departure and arrival times of each arc in the network [79]. Different solution algorithms are suggested to solve this routing problem. Sarac, Batta and Rump [79] develop an operational routing problem formulation which includes maintenance resource availability constraints and propose a branch-and-price algorithm to solve this problem.

The fourth and final subproblem is crew scheduling, a combined problem where the objective is to find and assign a qualified cabin crew per flight. The costs for crews can increase very quickly. In fact, they are ranked as the second largest airline expense after fuel costs [20]. Undoubtedly, considerable savings can be made if the scheduling of these crews is managed better. The ordinary way of modelling the crew-scheduling problem is as an integer program with finding a minimum cost assignment of flights to itineraries. A crew schedule is actually a set of pairings that partitions the legs to be flown by an aircraft and thus it lends itself often to a set partitioning formulation. Columns and rows represent respectively possible crew pairings and scheduled flights. Since crew scheduling is the final subproblem that is solved after the aircraft maintenance routing has been scheduled, there are no papers dedicated only to crew scheduling linked with maintenance.

Through integration of these different subproblems, costs can be saved and greater efficiency can be achieved. In the real world, these four steps are often solved via a sequential approach. Hereby, the complexity is reduced substantially. However, the downside is that it does not provide the best possible solution [20, 61]. Recently, more research has been devoted to new solution methods such as the integration of two or three subproblems into one considerable operating problem [67]. We will focus on some integrated problems and elaborate the most common solution methods for these problems. An overview of these solution techniques is given in Table 6. This table leads us to two conclusions. First, it is clear that the (integrated) airline scheduling problems are very large problems, which are found intractable to solve as is. Therefore, many authors rely on decomposition methods, such as Benders' decomposition, dynamic programming or Dantzig-Wolfe decomposition. The latter is mainly solved by column generation or branch-and-price, two techniques which enable the user to not consider the numerous variables explicitly. Secondly, no meta-heuristics were used solving these problems. To create feasible solutions, constructive heuristics are used in many papers, mainly to turn the linear into an integer solution.

The integration of the flight scheduling and the fleet assignment problem is studied in [82]. Sherali, Bae and Haouari [82] discuss along with the consideration of choosing optimal flight legs, the assignment of the aircraft type which has to execute this flight leg. Additionally, optional legs and multiple fare classes are taken into account. The improvement in these flight connection opportunities results in the increase of revenues. Sherali et al. integrate the passenger-mix model in the main model itself. A special analysis of the resulting MIP is used for this problem and is called ‘a polyhedral analysis’. Hence, it is possible to deduct various classes of valid inequalities for tightening its formulation. The solution approach, applied on the resulting model, is Benders’ decomposition. The use of a sequential fixing heuristic for the largest sized problems could also have great benefits: the solution quality is just slightly worse while the computational effort is considerably reduced.

Table 6: Solution methods for (integrated) airline scheduling

Mathematical programming	
Mixed/Linear/Integer programming	6, 8, 9 ,18, 20, 21, 23, 24, 29, 31, 33, 38, 45, 46, 47, 50, 54, 56, 61, 62, 74, 82, 83, 88, 96
Column generation	21, 25, 30, 38, 61, 62, 67, 83, 96
Branch-and-price	9, 20, 24, 38, 50, 79
Lagrangian relaxation	17, 23, 37
Dynamic programming	64
Branch-and-bound	18, 20, 21, 25, 31, 38, 61, 62, 74, 83, 96
Benders' decomposition	8, 21, 38, 61, 62, 67, 82
Constraint programming	8, 30
Constructive heuristic	2, 6, 8, 17, 23, 29, 30, 34, 37, 46, 61, 62, 64, 67, 74, 79, 82, 83, 89, 96
Discrete-event simulation	56
Polyhedral analysis	82
Euler Tour	17, 34, 89
Other	24, 45, 51, 62, 63

The integration of the flight scheduling and the maintenance routing is addressed in [33, 45-47]. Both parts are essential for a carrier’s profitability, level of service and competitiveness in the market. Models are developed to simultaneously help airline companies improve their fly routes as well as create feasible timetables.

Next is the integration of the fleet assignment with maintenance routing. When these two parts are integrated, the schedule of flights is already determined. The objectives of these problems is then to find a route for every airplane with minimum cost assigning exactly one airplane to every flight while exploiting maintenance constraints. Solution methods differ in the relevant literature: Benders' decomposition [38], column generation [30], (mixed) integer programming [9] and branch-and-price [9, 38] are some of the most encountered solution methods. Haouari, Mansour and Aissaoui and Sherali [38] and Haouari, Mansour and Aissaoui [37] contribute with the proposition of exact as well as heuristic approaches to tackle this integrated aircraft fleeting and routing problem (AFRP). In [38], Haouari et al. propose both an assignment-based as a set partitioning formulation, both solved with Benders' decomposition and a branch-and-price method. Although Benders' decomposition does not provide a solution of the same quality as the branch-and-price method, it still gives a high-quality outcome in much less computation time, which makes the technique preferable.

In some papers, the fleet assignment stage is integrated with the crew scheduling stage. Gao, Johnson and Smith [31] worked on a method to provide fleet assignment solutions that efficiently plan crews and handle real-time operations with robustness. The model ensures that the number of fleet types serving a given station does not exceed a specified limit. A branch-and-bound technique is applied to the resulting MIP model.

The last two integrated subparts are aircraft maintenance routing and crew scheduling. The objective of the integrated model is to determine a set of aircraft routes and crew pairings at minimum cost on condition that exactly one airplane and crew cover each flight leg. Concurrently, side constraints such as maximum flight time, maintenance requirements and minimum connection times for crews need to be taken into account. All of these models are solved with column generation and/or branch-and-bound [20, 21, 25, 61, 96]. To control the linking constraints which impose minimum connection times for crews, depending on aircraft connections, a Benders' decomposition solution approach is used by Mercier, Cordeau and Soumis [61] and Cordeau, Stojkovic, Soumis and Desrosiers [21]. Column generation is applied for the solution process iteration between master and subproblem of respectively the aircraft maintenance routing and crew scheduling. To find integer outcomes, a heuristic branch-and-bound is applied.

In this last paragraph, examples of integration models between three of the subparts of the airline process will be presented. Martin, Jones and Keskinocak [56] and Mercier and Soumis [62] integrate flight scheduling, aircraft routing and crew scheduling. Mercier and Soumis [62] propose a Benders' decomposition method for the overall problem including a dynamic constraint generation procedure and column generation. Martin et al. [56] consider the problem of scheduling commercial aircraft while addressing maintenance routing (demand) and downtime (crew rest) constraints. Maintenance constraints include a maximum number of flight hours, take-offs and landings or engine starts between two maintenance checks. A decision support system based on a mixed integer model computes cost-minimizing solutions.

Clarke, Hane, Johnson and Nemhauser [18] and Papadakos [67] integrate the fleet assignment, aircraft maintenance routing and crew scheduling steps. Clarke et al. [18] provide modelling instruments for the fleet assignment with the objective of maximizing revenues minus costs while taking into account maintenance routing and crew scheduling. The FAP is modelled as a multi-commodity network. Maintenance constraints are considered through incorporating both short and long maintenance constraints. Also crew constraints are part of the problem because lonely overnights are avoided as much as possible. Through branch-and-bound in the mixed integer program, solutions can be derived. Papadakos [67] solves this integrated problem through an enhanced Benders' decomposition method and accelerated column generation.

As a final example, Lapp and Cohn [51] present the objective to improve the robustness of the airline planning process and to reduce the impact of daily disruptions, while considering the four separate parts of the airline process as a whole. Especially maintenance robustness is aimed for by making limited changes to the initial set of planned lines-of-flight (LOF). Solving the mathematical model with a sequential optimization approach shows significant improvements.

As a conclusion for this section, the literature suggests that the more the models are integrated, the more constraints and rules need to be taken into account. On the one hand, since the mathematical models become overloaded with variables, the increased required computation time is an inconvenience of problem integration. On the other hand, advantages of handling these issues conjointly can be noticed as well: an increase in profitability and a higher service quality. For future research, the exploration of more concepts and approaches to solve these interrelated operational planning problems as a whole, instead of through the more traditional approach of sequential processes, is highly recommended.

5. Aircraft maintenance optimization

This section gives an overview of the aircraft maintenance optimization problems, specifically for engine maintenance and other components of an airplane. A short section will describe task allocation in aircraft maintenance. Finally, aircraft maintenance scheduling problems will be discussed. These are problems that actually try to schedule the maintenance, rather than just incorporating maintenance constraints. Although there is a small difference in meaning between scheduling and planning, it is not interesting to make this difference in this field. Therefore, scheduling and planning will be utilized as synonyms throughout the rest of this paper. All these options are listed in Table 7.

Table 7: Aircraft maintenance optimization

Engine	1, 5, 14, 22, 32, 39, 40, 41, 43, 48, 49, 81
Other components	1, 14, 19, 32, 39, 42, 53, 78, 80, 84, 85, 101
Task/job allocation	3, 8, 15, 26, 32, 44, 57, 68, 70, 76, 77, 102
Aircraft maintenance scheduling	3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 27, 28, 29, 33, 36, 41, 44, 45, 46, 47, 54, 57, 58, 59, 60, 64, 65, 66, 67, 68, 70, 76, 77, 84, 85, 86, 87, 92, 93, 97, 100

The maintenance of components, especially of the engine of an airplane, is very important and difficult [39]. The engine is complex, subject to wear and failure, safety-critical and expensive. Therefore, the optimization of the policy for maintenance and the maintenance scheduling of engines, which arbitrates the balance cost/safety, is a considerable problem. Most of the airline companies fly with various aircraft families, characterized by a specific engine type for each airplane. The inspection and repairs of the engines are usually done at the airline special facility for engine maintenance [32]. As a result of its complexity, including many parts and repair routings, a job shop environment can be implemented in a component or engine maintenance facility, in for example [19, 32, 43].

Almgren et al. [5] and Kleeman and Lamont [43] consider the maintenance scheduling problem for aircraft engines. Almgren et al. [5] describe an opportunistic replacement problem with single objective of minimizing the total maintenance cost. The components are bound, though to a maximum time interval without replacement. Via MIP and polyhedral analysis, the replacement problem is solved. The results show significant savings compared to simpler maintenance policies. Kleeman and Lamont [43] describe a multi-objective genetic algorithm. They have two goals: the minimization of the time needed to return engines to mission capable status and the minimization of the associated cost by limiting the number of times an engine has to be taken from the active inventory for maintenance [43].

In this paragraph, a couple of examples of task allocation in aircraft maintenance will be presented. This planning problem is usually dealt with through LP/IP/MIP [26, 44, 77]. Dietz and Rosenshine [26] combine the determination of the optimal level of specialisation and optimal task allocation for maintenance workforce. They use a sequential LP algorithm to obtain good results and the model can be easily applied to problems with the objective to maximize the operational efficiency of military aircraft. Safaei, Banjevic and Jardine [77] address the maximization of the availability of a military airplane. Pre and after flight checks must indicate if repair is necessary and the plane thus needs to be stationed at the repair shop. In the shop, the jobs need to be assigned to a limited number of technicians in such a way that next missions can take place as planned. The workforce is therefore the primary resource or constraint in the problem. A network structure is used to simulate the flow of the planes between missions and the repair shop. MIP provides results for this scheduling problem.

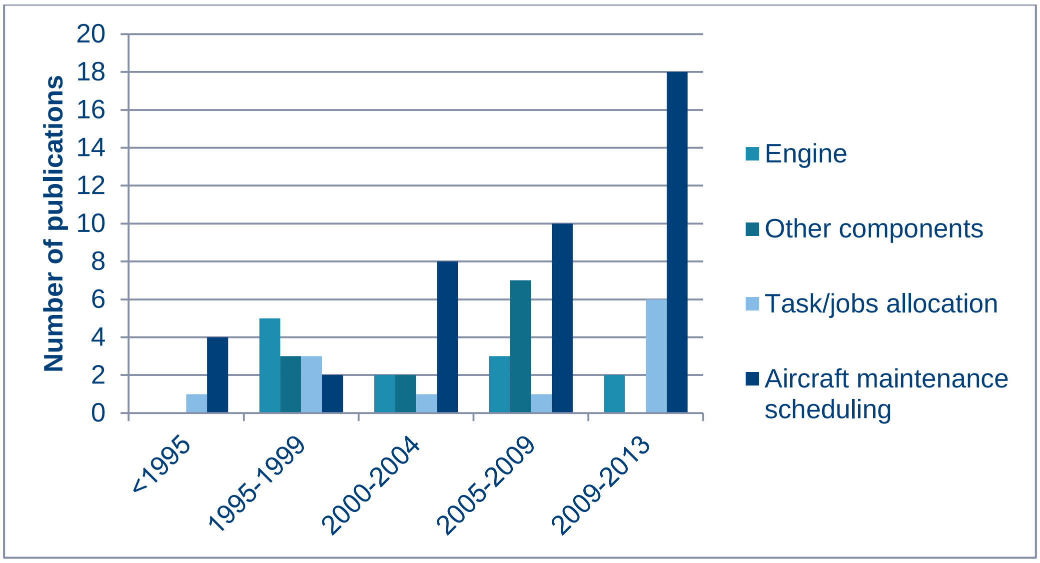
Quan, Greenwood, Liu and Hu [70] study multi-objective preventive maintenance tasks scheduling with evolutionary algorithms aiming for a schedule with a minimum number of technicians to perform these tasks. Quan et al. [70] do this by minimizing worker idle time. However, the tasks need to be completed as fast as possible to make equipment available again. Hence, the completion time needs to be minimized as well, leading to a trade-off between either a smaller workforce to lower the idle time and a larger workforce to lower the completion time. EA is used to find pareto-optimal solutions.

This last paragraph addresses aircraft maintenance scheduling problems. Moudani and Mora-Camino [64], van Buskirk et al. [92] and Bajestani and Beck [8] consider the maintenance scheduling for aircraft in combination with aircraft or fleet assignment. Moudani and Mora-Camino [64] mix two techniques to find solutions: dynamic programming for the fleet assignment and a heuristic approach for the maintenance scheduling. van Buskirk et al. [92] describe a military application and a system which operates in two stages. In the first stage, the qualified planes are assigned to missions. The second stage addresses the usage-based and calendar-based maintenance action scheduling. A greedy search algorithm and constraint programming are used to find better-scheduled preventive maintenance activities. Bajestani and Beck [8] also address a military aircraft maintenance scheduling problem in which the goal is to meet aircraft requirements for a number of missions in the presence of breakdowns. An optimal solution requires both the assignment of airplanes to missions and scheduling the repair jobs while coping with limited capacity. Different approaches are suggested for solving the problem. Experimental results show that a logic-based Benders' decomposition combined with integer and constraint programming outperforms the hybrid heuristic. The latter, though, can compute feasible schedules in a very short time, so advantages and disadvantages need to be considered, depending on the user's interest.

Kozanidis et al. [33, 45-47] and Feo and Bard [29] combine the flight and maintenance planning. After a certain number of flying hours since the last check, a plane is grounded to come in for maintenance. This problem requires planning with the objective of maximum utilization of the planes over time. In these papers, solutions are provided via MIP. Kozanidis and Skipis [47] report a mixed integer bi-objective optimization model. The first objective is to minimize the total residual flight time. The second is to maximize the total number of available aircraft. The residual flight time is the remaining time that an aircraft can fly before it is grounded for a maintenance check. Therefore, minimizing the residual flight time is equal to maximizing airplane utilization.

Figure 2 shows that, whereas the number of manuscripts that focus on engines or other components are decreasing, the task/jobs allocation and the aircraft maintenance scheduling problem are receiving more attention recently.

Figure 2: Evolution of airline/components maintenance scheduling



An efficient maintenance system is of key importance for an airline to meet its objectives. Minimizing costs is the most evident objective. However, also ‘minimal flight cancellations’, ‘minimal delays’, ‘minimal repair turn time’ and ‘efficient utilization of maintenance resources’ can be crucial. Different evaluation techniques are proposed for the aircraft maintenance scheduling/planning applications throughout the literature. Yet many use simulation [13, 28, 36, 57, 59, 60, 65], some use heuristics [57, 87, 100] and others use decision systems [58, 68] or combinations. Table 8 lists the different solution methods applied to all manuscripts that deal with aircraft maintenance, except for those that study the (integrated) airline scheduling problem. Compared to Table 6, less problems are solved with decomposition techniques or branch-and-price. Their share is taken by simulation methods (i.e., to incorporate uncertainty) and meta-heuristics.

Table 8: Solution methods

Mathematical programming	
Mixed/Linear/Integer programming	4, 5, 11, 12, 14, 15, 26, 44, 52, 76, 77, 80, 87, 93, 97, 98, 99
Column generation	69
Lagrangian relaxation	77

Dynamic programming	40, 58, 90
Branch-and-bound	12, 52, 77
Benders' decomposition	69
Constraint programming	92
Constructive heuristic	5, 11, 12, 13, 14, 27, 35, 39, 52, 55, 66, 77, 87, 92, 98
Improvement heuristic	
Simulated annealing	3
Tabu search	93
Genetic algorithm	3, 43, 57, 70, 80, 81, 100, 102
Other	3, 14, 55, 98
Simulation	
Discrete-event	1, 10, 13, 19, 28, 32, 35, 36, 42, 48, 55, 57, 59, 60, 68, 75, 93
Monte-Carlo	22, 35, 65, 66
Polyhedral analysis	5
DEA	73, 93
Observation/questionnaire/interview/survey	4, 94
Other	5, 49, 68, 84, 85, 86, 94, 95, 101

Sriram and Haghani [87] incorporate re-assignment of airplanes to the flight segments. They try to minimize the overall maintenance cost, thus also the extra cost that comes with the eventual re-assignment. They use a constructive heuristic, based on depth-first and random search. Reasonable results are recorded during the experiments in a very short computation time. Papakostas, Papachatzakis, Xanthakis, Mourtzis and Chryssolouris [68] present a decision support system with the criteria of cost, remaining useful life (RUL), operational risk and flight delay. Line maintenance alternatives are evaluated to these criteria to make optimal maintenance planning decisions.

Table 9 shows that aircraft maintenance is studied in various problems, each with their own constraints. It is clear that many of these sets can be linked with the airline (integrated) scheduling problems, such as balance/flow constraints (routing), availability/count constraints (fleet assignment) flight scheduling constraints, crew assignment constraints. Other constraint sets that are connected with the aircraft maintenance problem are, for instance, coverage and resource constraints. Coverage constraints ensure that the jobs are covered by available personnel. Also other resource/capacity constraints arise such as the availability of the hangar. Of course this table gives only a limited view on the constraint sets, since there are many problem specific constraints.

Table 9: Constraints

Balance/flow constraints	9, 10, 11, 12, 14, 18, 20, 24, 26, 27, 29, 31, 34, 37, 50, 54, 61, 63, 74, 76, 82, 83, 87, 89
Demand constraints	24
Budget constraints	26
Availability/count constraints	2, 3, 8, 9, 13, 20, 25, 31, 33, 34, 38, 45, 46, 47, 50, 54, 61, 62, 63, 64, 67, 82, 83, 87, 89, 96
Set partitioning/covering constraints	37, 54
Coverage constraints	3, 4, 6, 9, 11, 12, 20, 21, 24, 28, 31, 34, 44, 50, 56, 61, 62, 63, 64, 65, 66, 68, 69, 74, 79, 80, 82, 83, 86, 88, 89, 93
Legal constraints	11, 12, 27, 52, 68, 69, 79, 80, 82, 93, 97
Flight scheduling constraints	33, 45, 46, 47, 77, 100
Aircraft assignment constraints	23, 30, 37, 63, 69, 77
General maintenance constraints	2, 9, 14, 17, 18, 20, 21, 23, 24, 28, 29, 30, 31, 33, 34, 36, 37, 38, 45, 46, 47, 50, 51, 54, 56, 61, 62, 63, 64, 67, 69, 74, 76, 79, 82, 87, 88, 89, 90, 96, 97, 99
Crew assignment constraints	18, 21, 31, 52, 56, 61, 62, 67, 69, 74, 90
Operational requirements	13, 28, 30, 74, 88, 90, 92, 97
Resource constraints	6, 9, 10, 14, 15, 24, 30, 32, 35, 38, 55, 59, 60, 68, 70, 74, 77, 79, 80, 86, 91, 92, 96
Capacity constraints	15, 24, 37, 44
Specific resources	
Manpower	3, 4, 6, 10, 11, 13, 14, 25, 26, 27, 28, 36, 38, 44, 47, 59, 60, 68, 76, 77, 79, 80, 86, 92, 93, 97, 98
Equipment	10, 13, 14, 28, 59, 60, 77, 80, 86, 92
Capacity maintenance base	2, 6, 8, 13, 14, 15, 28, 33, 38, 45, 46, 47, 54, 57, 58, 59, 60, 68, 69, 77, 79, 80, 86, 87, 100
Capacity plane	24
Other	4, 5, 8, 11, 12, 14, 19, 20, 21, 23, 25, 28, 31, 33, 37, 38, 39, 44, 46, 51, 56, 57, 58, 62, 69, 73, 76, 77, 80, 82, 83, 93, 96, 97, 98, 99, 102

6. Facility location

This section addresses the facility location for aircraft maintenance. It should not be seen as a pure facility location problem, but more as the incorporation of the facility location as a constraint on when and where the maintenance can be performed. More specifically, we explore here if the facility location is part of the optimization problem. If it is part of the problem, does it concern a single base or multiple bases and does it depend on the flight schedule? Maintenance is usually done during the night. Due to limited workforce or resources [54], every maintenance station is only capable of performing a certain number of maintenance checks per night. An overview of the papers that incorporate facility location is given in Table 10.

The incorporation of a single base facility location in the optimisation problem, independent of the flight schedule, was found in only one example [25]. More papers integrate the single base facility location for aircraft maintenance into the optimisation problem when it is linked with the flight schedules. Gavranis and Kozanidis [33], Kozanidis and Skipis [47], Kozanidis et al. [46], Bajestani and Beck [8] and Kozanidis et al. [45] all describe flight and maintenance planning in military applications. In their models, they only consider one maintenance station with limited space capacity. In the problem, the flight schedules are optimized, whereas the maintenance station is a constraint.

Table 10: Facility location

Single base	2, 8, 10, 11, 12, 25, 28, 33, 45, 46, 47, 86, 93
Multiple base	6, 9, 17, 18, 21, 24, 29, 34, 36, 37, 38, 50, 51, 54, 56, 57, 59, 60, 61, 63, 64, 67, 68, 69, 79, 80, 87, 89, 96, 100
Depending on flight schedule	2, 63, 87

The examples of the incorporation of more than one facility location for maintenance in the optimisation problems are quite numerous. Feo and Bard [29] present such a problem, where both the choice for multiple base facility locations and the flight schedule are integrated in the same model. This model, which not only develops flight schedules that meet cyclical demand for maintenance better, also aims to locate fewer maintenance stations in a more strategic way to save costs.

As can be seen from Table 10, authors often incorporate facility location in their aircraft maintenance related problems. Most of them incorporate the decision for multiple bases into the problem instead of just a single base. The papers that only discuss one facility location are usually examples of military application and let the choice of maintenance base repeatedly depend on the flight schedule. Papers that integrate the decision for multiple bases often entail commercial airlines. Table 9 shows that the optimization problems from the papers listed in Table 10 often have resource constraints. Specific resource constraints are, for instance, manpower, equipment and the capacity of the maintenance base(s).

7. Workforce and training

This section addresses the workforce needed to perform the aircraft maintenance. First, the skills and certificates required will be scrutinized. Next, a comprehensive overview of the specific workforce planning and scheduling methods in aircraft maintenance is given, with a focus on the models and algorithms. Finally, the different forms of training are classified. Table 11 gives an overview of all papers that mention the need for skills and licences in aircraft maintenance, the papers that discuss maintenance personnel planning as well as the various types of training.

It is essential that highly qualified personnel performs the maintenance [11]. To get the workforce qualified, they need to be trained. Because the term ‘skills’ is quite general, occasionally a partition or specification is clarified in the relevant literature, such as technical skills, team skills, or skills grouped per different service or aircraft type. Closely linked to skills is the licensing of the workforce. Aircraft maintenance personnel (AMP) or engineers (AME) need to have a valid licence in order to be approved to work. Moreover, after all the maintenance is finished and the airplane is approved to fly again, it can only be released by licenced personnel. Ideally, all engineers would have certificates for all aircraft types. However, depending on the regulations of the state in which the engineers or technicians are working, maintenance crewmembers are only allowed to have licences for a limited number of aircraft types [27, 97]. The safety rules of KLM (the Netherlands), for instance, prohibit engineers to license for more than two aircraft types and one skill [27]. In Taiwan, on the other hand, engineers are allowed to obtain certificates for three or sometimes even four aircraft types [97].

Table 11: Workforce and training

Skills	3, 26, 27, 57, 70, 71, 75, 76, 77, 86, 94
Licenses	26, 27, 28, 44, 97, 98, 99
Maintenance personnel planning	3, 4, 10, 11, 12, 27, 36, 44, 70, 76, 93, 97, 98, 99, 102
General training	59, 91
Aircraft maintenance technician (AMT)	26, 27, 59, 97
Other	26, 59, 71, 88, 94, 95, 97, 98

The problem of specific maintenance personnel planning or scheduling is different from the general crew scheduling that was described in Section 4. Yan, Yang and Chen [98] state that the crew scheduling problems are often modelled as set partition/covering, network and integer program problems. Algorithms to solve these problems can be column generation, heuristics and branch-and-bound procedures. For the specific maintenance personnel scheduling, almost all problems are modelled as (mixed) integer programs [4, 11, 12, 15, 27, 76, 93, 97-99]. The problem is solved by using only heuristics [3, 102], heuristics in combination with IP [12, 27, 98] or by simulation [10, 93].

Brimberg, Hurley and Wright [15] propose an algorithm to schedule two or three technicians that can work at the same time subject to limited physical space but no precedence constraints among tasks. This NP-complete partitioning problem is solved through integer programming. Currently, this problem is solved with the restriction of maximum two or three technicians working at the same time. Future research might focus to relaxing this constraint by considering more than three technicians working at the same time. Dijkstra et al. [27] developed a decision support system (DSS) for the management of KLM airlines, which contains a database module, an analysis module and a graphical user interface. Via integer programming and heuristics, optimal or near-optimal solutions can be retrieved. The management can use this DSS in order to determine the right number of maintenance engineers, their training requirements and the effectiveness of the overall maintenance department.

Training is necessary for all levels of airline personnel and, of course, also for maintenance personnel. A general form of training for the entire personnel of a maintenance department is mentioned in for example [59]. Other papers address training on an individual level, for an aircraft maintenance engineer (AME) or for a technician (AMT), for example [26, 27]. The rest category ‘other’ groups other forms of training such as safety training, on-the-job training, etc.

As a summary of this section, although the licencing of the engineer is absolutely necessary, it is hardly ever part of the maintenance scheduling problem studied in the literature. Certification for personnel is obligatory before the technicians may perform maintenance, but is also very diversified in different parts of the world. That can be a reason why certification may be frequently mentioned in papers of Operations Research and Management Science, but rarely used as a constraint in the workforce scheduling problems for maintenance [97, 99]. The general skill-level of technicians is more incorporated in the planning problems than licensing. In most of the problems, this skill requirement is merely integrated via manpower or other constraints. Some do mention specific skill constraints of the personnel, such as Dijkstra et al. [27] did. Nevertheless, in all relevant papers found in Operations Research and Management Science, various different skills are mentioned. All of those specific skills, however, are not incorporated in workforce scheduling or planning problems. This should be explored more extensively in the future and, preferably, it should be linked with the specific training procedures and costs.

8. Uncertainty and applicability of research

Uncertainty is a substantial problem with respect to the planning or scheduling problems of aircraft maintenance. A listing of the relevant manuscripts based on their uncertainty incorporation is given in Table 12. It is clear that the papers with deterministic setting outnumber those that incorporate stochasticity. Five types of uncertainty are addressed in the relevant literature. The first field deals with the uncertainty of the flight arrivals, i.e., delays. The second group takes into account stochastic failure rates. This field shows some overlap with the aircraft maintenance literature on engines and other components (Table 7). In these manuscripts, one tries to predict the remaining time before a component gets into failure. The repair times for these failures or maintenance processing times in general can also be uncertain, which is the third field. The one but last field addresses the workforce availability: personnel can be absent due to illness, etc. The last type is the most ample one, a kind of residual group called 'Other'. This category lists both the papers that do not mention what type of uncertainty that is incorporated, as well as the other sources that cannot be classified in the previous categories. In [28], for instance, equipment and spare parts availability is uncertain. Of course, not only simulation techniques but also scenario analyses can be used to cope with uncertainty. By varying a number of parameters, one can mimic the stochastic behaviour of real-life scheduling processes. Good planning or scheduling of aircraft maintenance activities could reduce the negative impact of uncertainty.

Mattila and Virtanen [57-60] discuss the scheduling for maintenance of a fighter aircraft under conflict operating conditions. They incorporate uncertainty in the failure rate and repair time for the different components of the fleet.

Table 12: Uncertainty incorporation

Deterministic	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20, 21, 23, 24, 25, 27, 29, 30, 31, 33, 34, 37, 38, 43, 44, 45, 46, 47, 51, 52, 54, 56, 61, 62, 63, 64, 67, 69, 70, 73, 74, 76, 79, 80, 82, 83, 87, 88, 89, 92, 97, 98, 99, 100, 102

Stochastic	
Timing workload/flight arrival	19, 28, 50, 58, 59, 60, 66, 69, 75, 93, 96
Failure rates	22, 26, 32, 35, 40, 42, 48, 49, 53, 55, 57, 58, 59, 60, 68, 77, 81, 84, 85, 101
Repair times	19, 26, 32, 39, 55, 58, 59, 60, 84, 85, 101
Workforce availability	26, 68
Other	26, 28, 36, 47, 59, 65, 66, 90

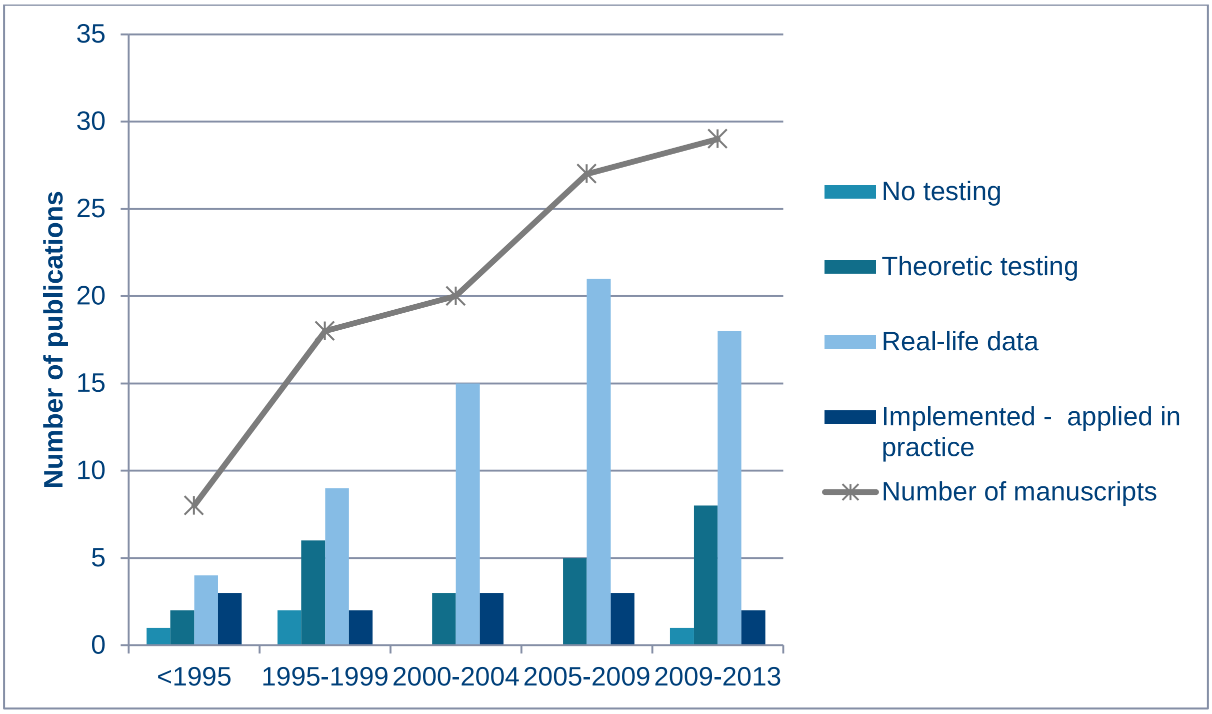
In most of the papers, researchers provide, additionally to the development of a model or formulation, a testing phase. In this testing phase the applicability of their research is illustrated. For this, extensive data is needed. As shown in Table 13, the majority of this data is based on real-life problems. Disappointingly, only a small subset of these real data based examples is actually implemented and put into practice.

Table 13: Applicability of research

No Testing	6, 28, 40, 44
Data for testing	
Theoretic	1, 2, 5, 8, 15, 19, 22, 23, 33, 39, 43, 45, 46, 48, 49, 55, 58, 68, 70, 80, 81, 87, 90, 102
Real data based	3, 4, 5, 9, 10, 11, 12, 13, 14, 17, 18, 20, 21, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 47, 50, 51, 52, 53, 54, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 79, 82, 83, 84, 85, 86, 88, 89, 92, 93, 96, 97, 98, 99, 100, 101
Implemented & applied in practice	4, 11, 12, 13, 25, 27, 47, 56, 60, 64, 74, 86, 88

Figure 3 shows that the number of manuscripts that use real-life data follow more or less the positive trend of the manuscripts that study aircraft maintenance. The solution approaches that are actually implemented, however, do not change the last few decades. Considering the increasing number of manuscripts, the ratio of real-life implementations over publications thus declines.

Figure 3: Evolution of the testing and the application of research



9. Conclusion

In this paper, manuscripts on aircraft maintenance are reviewed which have appeared in the Operations Research and Management Science area. The analysis of these manuscripts resulted in contributions on several levels, which were referred to as descriptive fields. In every one of those fields, the most important trends and concepts were discussed. Every section is accommodated with at least one table. In these tables, the reader can identify the information given in the text, accompanied by more examples. Specific features of papers can be easily derived from the tables and common features among the papers can be identified. By pooling the tables, it is possible for the reader to look for specific contributions over the several fields and reconstruct the papers of interest. In the following paragraphs, a few of the most significant findings are summarized and discussed.

Section 3 serves as an extended introductory. We found the terminology containing many overlapping definitions very confusing. To obtain a better insight into the different types of aircraft maintenance, we provide an overview table that classifies all types. It would be a significant improvement if researchers could work towards an agreement on general accepted terminology and definitions.

In Section 4, the (integrated) airline scheduling concept is elaborated. This consists of flight scheduling, fleet assignment, aircraft maintenance routing and crew assignment. All these different parts are discussed separately as well as interrelated, while focusing on their relation with aircraft maintenance. In Section 5, aircraft maintenance optimization problems are addressed, regarding specific parts such as, for example, the engine. Furthermore, task allocation and aircraft maintenance scheduling problems are discussed. Throughout these two sections, the problems are analysed completely: formulation of the problem, network representation, constraints and possible solution techniques. The challenge often not lies in formulating the problems, but in solving the formulated models. The optimization models are frequently reduced or decomposed in order to develop an efficient solution procedure.

Maintenance optimization models often incorporate facility location in the problem. In such a case, the choice of a single or multiple locations should depend on the flight schedule or flight missions to obtain optimal, cost-efficient results. The workforce plays an important role in aircraft maintenance as well. Workers need to have different skills and licences, and need to follow appropriate training. Workforce scheduling problems are also examined with respect to problem formulation, constraints, and solution techniques.

Finally, the uncertainty and applicability of research in aircraft maintenance is studied. Numerous researches incorporate forms of uncertainty in their problems. Uncertainty occurs with respect to flight arrival, failure rate, repair time, workforce availability, and others. The suggested models are often tested based on real-life data. Henceforth, more models and solution techniques should be implemented and applied in reality.

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