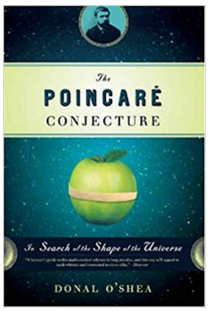
Problem solved. What's next?

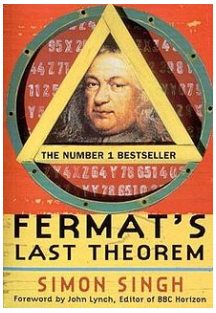


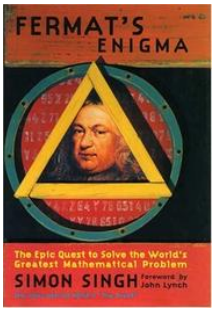


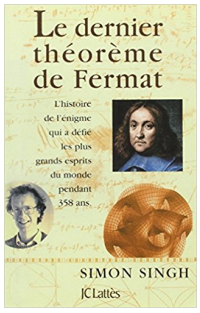
If you have read my previous reviews here, you will know that I am addicted to book shops, and I cannot pass a second-hand book market without stopping. There I recently found two older books popularizing the solution of two major mathematical problems. First I found on a Sunday market Donal O'Shea's *The Poincaré Conjecture: In Search of the Shape of the Universe* (2008) and two days later in a nearby second-hand shop, I found *Het laatste raadsel van Fermat*, a Dutch translation of Simon Singh's *Fermat's Last Theorem* (1997) also distributed as *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*.

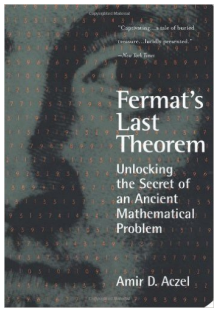
Of course a million dollar prize for solving a mathematical problem like the Poincaré Conjecture (PC) is something worth mentioning in the newspaper. The PC has been an open problem for a long time and many great mathematicians have tried to solve it. This arouses interest, even of a public otherwise not interested in mathematics. And Fermat's Last Theorem (FLT) has the advantage of being a problem easy to explain, yet difficult to solve. So it easily gained some popularity with amateur mathematicians. As Singh writes, hundreds of false proofs were received in the early 1900's when a Wolfskehl Prize was awarded for its solution.

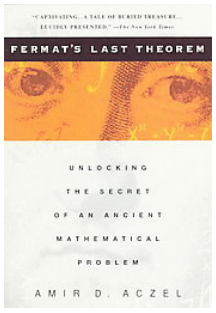
There are of course many more open mathematical problems that remain unsolved, and as soon as one has been solved, and the media have calmed down, another holy grail is shining brightly above the horizon for next generations to earn eternal fame. If we have to choose among the Clay Mathematical Institute's Millennium Prize Problems (CMI-MPP), the Riemann Hypothesis (RH) is by now the most documented and most popularized one.

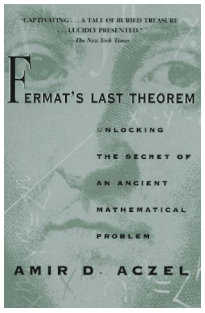












When Andrew Wiles finally proved FLT in 1994 this caused a big media storm after his first announcement in 1993. Simon Singh has made his career by writing his first book on the proof of FLT. He is now (among other things) a well known writer of popular science books (many with a strong mathematical bias). Mind you, his book is not just an explanation of how Wiles did it. No it is a thick book that sketches the whole history of mathematics listing all the big names from the Greek till Gödel and Turing and obviously Wiles and his contemporaries. It is written almost like a novel and accessible to any lay person. There was also an award winning BBC film accompanying the book, which made the book even more accessible and popular. The book has been translated in many languages.

A bit less merchandized is the also nice book by Amir Aczel *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem* (2007). Aczel is an Israeli born American mathematician, who also wrote many popular science and mathematics books mostly related to history.

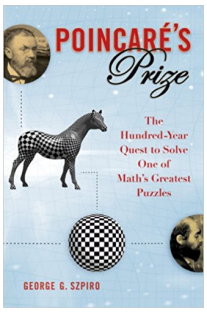
Perhaps you need somewhat more mathematics and more formulas. Then of course there is the more technical literature. You might have a look at Paulo Ribenboim's *13 Lectures on Fermat's Last Theorem* (1979, i.e. pre-Wiles) and *Fermat's Last Theorem for Amateurs* (1999, post-Wiles) which, despite its title, is much more technical.

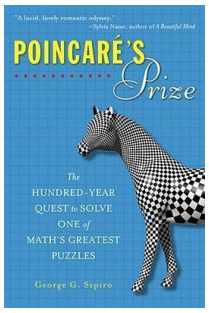
The Poincaré conjecture is mathematically more involved and more difficult to explain to a lay person. So books on that topic are written more by mathematicians specialized in the field. A more accessible one is written by George G. Szpiro (not to be confused with Lucien Szpiro who has his own conjecture related to the abc conjecture). *Poincaré's Prize: The Hundred-Year Quest to Solve One of Math's Greatest*

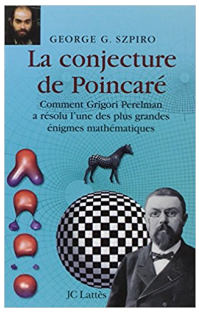
Puzzles (2007). The author is not only a mathematician but also a journalist which perhaps explains why it is more accessible. The book by O'Shea that I mentioned at the beginning is kind of a mixture. It is partly teaching the mathematics and partially telling the (by now well known turbulent) story of Grigori Perelman who solved the problem in 2003 by uploading three papers at arXive. The proof was finally confirmed in 2006 and Perelman was awarded several prizes which he all refused and he has withdrawn from mathematics. O'Shea is a Canadian math professor, now in Florida (not to be confused with the homonymous Donal O'Shea who is an Irish chemist). He has written several technical math books and this one on the Poincaré's Conjecture which is aiming at a more general public avoiding the formulas.

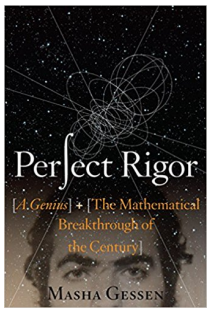
Again both books are not just explaining the problem and the approach by Perelman, they also contain a lot of history, although focussing on more recent history than in Singh's FLT. There exists an excellent review of these two books in the Notices of the AMS, see www.ams.org/notices/200801/tx080100037p.pdf (Jan. 2008). So that saves me the effort of writing it myself.

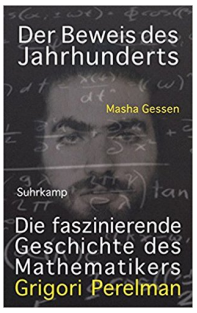






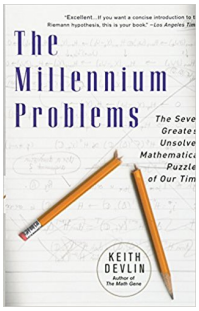


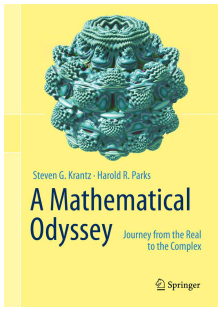


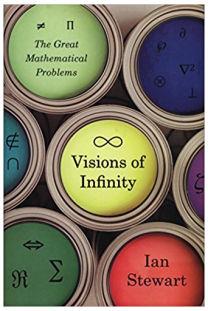


If you need more mathematics, again I refer to the technical literature. Right after Perelman uploaded his papers, mathematicians started to verify his terse papers and completed the holes. *Ricci Flow and the Poincaré Conjecture* (2007) edited by John Morgan and Gang Tian is one of the results of these groups working on Perelman's proofs.

Perelman has a quite peculiar temper and what exactly explains his behaviour is not very clear. Partly of course because he has avoided contact with the media. Masha Gessen is an American journalist who wrote *Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century* (2009). This is a kind of biography of Perelman trying to understand his way of thinking and explaining why he disappeared from the mathematical scenery. Of course there is also much on the PC too.







Will the RH be the next of the great problems to be solved? In any case the math popularizers are prepared and they have introduced the big problems (among them the CMI-MPP) to a wider public already. For example Keith Devlin's *The Millennium Problems. The Seven Greatest Unsolved Mathematical Puzzles Of Our Time* (2003) Stephen Krantz and Harold Parks *A Mathematical Odyssey. Journey from the Real to the Complex* (2014) and Ian Stewart *Visions of Infinity: The Great Mathematical Problems* (2013). Some of their chapters are also discussing Wiles solving FLT and Perelman solving the PC.

Even though some of these books are quite old, they are still interesting to read. You will learn that often the solution of a difficult problem comes, quite unexpectedly, from what is at first sight, a remote corner of mathematics. It is not the originally posed problem that is directly solved, but a remote one that was previously shown to imply a solution to the original one. Perelman used Ricci flow to prove the geometrization conjecture, a strong result that, almost unnoticed, also proves the PC. Simiarly the RH is a statement about the zeros of a complex function which solves the original problem of finding the distribution of primes. Major steps in mathematics are only taken by combining seemingly unrelated areas of mathematics and hence they rely on previous work by many mathematicians who are all contributing and collaborating to expand the building of mathematics.

Adhemar Bultheel