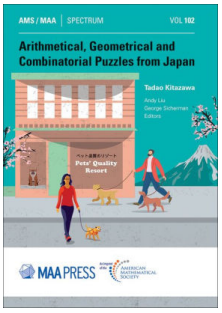
Arithmetical, Geometrical and Combinatorial Puzzles from Japan by *Tadao Kitazawa*, MAA/AMS, 2021 (xii+184 p.), isbn: 978-1-4704-6736-4. Edited by A. Liu and G. Sicherman



Tadao Kitazawa, member of the Japanese Academy of Recreational Mathematics, is a puzzle designer who has published many puzzles in Konwakai News. The editors of this book have collected many of his puzzles and organized them in this collection while Kitazawa has added several new formats.

As announced in the title, there are three parts collecting problems of arithmetical, geometrical or combinatorial nature. Each part consists of several chapters that contain puzzles of a certain format or concept, followed by answers and sometimes also solutions, if the answers need more explanation.

The book starts with an interview of Kitazawa and when asked what his favorite puzzle is, he refers to his 'tower squares' puzzles. Its idea is a bit like a sudoku where you have to fill an $n \times n$ square grid where each row and column must contain the numbers 1 to n exactly once. In the tower squares the numbers that have to appear in every row and every columns are one times 1, two times 2, three times 3, etc. so that n can only be 1,3,6,10,... $n = 1$ is trivial and for $n = 3$ there are only 12 different solutions. The sizes 6 and 10 are most common. No numbers are given, and the player has to fill the grid with n^2 numbers. The n^2 little squares that from the grid are formed by $(n+1)n$ vertical borders of length 1 and as many horizontal ones. The outer borders of the big square are drawn in black, thus leaving $(n-1)n$ vertical and as many horizontal boundaries drawn in light grey. Some of these are drawn in black too and the constraining rule is that on both sides of such a black border the same numbers must appear. A trivial example of a 3×3 square with a unique solution has 4 black borders for its NE and SW squares and no other.

Kiwazawa has a special preference for puzzles involving grids. There are for example grids where some of the borders are matchsticks or toothpicks generating a graph representing some geometrical shape and the problem is to move a small number of these matchsticks and obtain a figure that has certain symmetry properties, or that consists of two identical polygons. Even the arithmetical puzzles are sometimes formulated in grid form. Take for example an $n \times 5$ grid where for example the first row has the 5 symbols "$1 + 2 = 3$" the remaining rows are filled up with similar simple expressions such as $8 - 5 = 3$ or $2 \times 4 = 8$ etc. Next the grid is cut up in L-shaped pieces of 3 grid squares. The player then is given all these pieces and has to put this puzzle back together such that all the sums are correct.

Other puzzles are like tangrams with specific rules about allowed rotations or reflections. Origami type puzzles require to fold paper such that coloured parts fit together to form a certain shape after folding. In the combinatorial part one has to move chess-like pieces on a grid to move them from a starting configuration to a target one.

A type of puzzles that are simple to state, but that can be rather challenging to solve goes as follows. For example if the solution of the puzzle is that 136 is a multiple of 68, then all you get is the row if numbers 16386. The solution is to be found by moving 1, 3 and 6 to a second row, leaving the 6 and 8 on the top row. Thus, given a sequence of digits, move some of them to the second row, so that the number that appears on the second row becomes a multiple of the number that remains on the first row. Variants are possible like the answer $16 \times 3 = 48$ which has to be extracted from the sequence 16438. Move 4 and 8 to row 2 leaving 16-blank(s)-3 on the first.

This collection of puzzles is refreshingly original, and some have indeed some Japanese flavour. The degree of difficulty is varying, but no specific education is needed. This will guarantee many hours of puzzling pleasure. A single authored mathematical puzzle book with so many different formats and variations is exceptional in Japan and even outside Japan. Some exceptions are the *Puzzles 101: A puzzlemaster's challenge* by Nobuyuki "Nob" Yoshigahara (A K Peters, 2004) and *The puzzles of Nobuyuki Yoshigahara* (Springer, 2020). The latter are also edited by the editors of the present book helped by Nob's son.

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