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THE FIRST RESOURCE FOR MATHEMATICS

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Sharpening mathematical analysis skills. (English) Zbl 1482.00001
Problem Books in Mathematics. Cham: Springer (ISBN 978-3-030-77138-6/hbk; 978-3-030-77141-6/pbk; 978-3-030-77139-3/ebook). xx, 536 p. (2021).

Over many years of practice in teaching and research, the authors have collected a set of mostly challenging and non-standard exercises in mathematical analysis. The intended audience are smart undergraduate to young graduate students. Think of competition level problems like those of the Putnam competition or the Mathematical Olympiads like SEEMOUS. This book can be used to prepare for such contests both by potential participants as well as by those that have to set up the questions.

The topics covered are classic but the authors have strived to include problems requiring an unusual approach or using a long forgotten technique. For example it is proved that there are infinitely many prime numbers as a consequence of the Euler sum identity $\sum_{n=1}^{\infty} n^{-2} = \pi^2/6$. Thus what one should not expect is the classical analysis course where a particular technique is proposed (for example integration by parts) followed by a set of drilling exercises that are just applying this particular technique. To guide the student, some exercises are subdivided in steps that lead to the eventual problem and its solution. This results in problems with a wide range of difficulty, sometimes ending with an open problem.

The book has two parts. The first one has six chapters with exercises that cover the usual topics of an analysis course (limits, series, derivatives and integrals and their applications). Chapter seven contains some particularly challenging problems. Some involve series that are evaluated in terms of values of the zeta function, logarithms and polylogarithm. Others are so called 'gems' of mathematical beauty and some are open problems. These problems are all original, while in previous chapters some are recycled from contests or problem sections in journals, in which case the origin is mentioned. A short chapter eight contains two new proofs of the quadratic Euler sum (called here the Sandham-Yeung series) $\sum_{n=1}^{\infty}(\frac{H_n}{n})^2 = \frac{17}{4}\zeta(4)$ where H_n is the nth harmonic number.

Part two gives almost all the answers and in most cases also the elaborate solutions for the problems of part one. The solutions vary from computations that are fully worked out to a minimal remark like: 'this is a special case of...', with a forward reference to a more general exercise, or 'this is analogous to...' or in the extreme cases, just giving the numerical answer. Only few exercises are skipped without an answer.

There is a minimal interlacing of definitions, identities, or properties, but no systematic development of theory like in a classical course. The essence of the book are obviously the exercises that in most cases have the form 'prove that...' or 'calculate ...' or 'find (all) the solution(s) of...'. Some problems, properties, and even solutions are framed in grey boxes to put some emphasis on items that are particularly important, challenging, interesting, or beautiful.

This is a challenging, surprising, and interesting way to explore the classical topics in analysis clearly intended for the those who want to excel in mathematics. It is not a replacement of, but should come in excess of a classical textbook with traditional exercises. Weaker students should probably not be exposed to most of these exercises since the level of difficulty could easily depress them, hence doing more harm than good.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

00A07 Problem books
97U40 Problem books, competitions, examinations (aspects of mathematics education)

Cited in **2** Documents

Keywords:

mathematical analysis; sequences; series; limits; derivatives; integrals; power series; quadratic series; convergence; partial derivatives; implicit functions; Sandham-Yeung series; Putnam; SEEMOUS; coursework

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