Symmetry Propagation

Improved Dynamic Symmetry Breaking in SAT

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SymCon'12

Outline

SAT

- SAT theory: conjunction of clauses
  - e.g. $T = (a \vee b \vee c) \wedge (d \vee e) \wedge \dots$
  - (or $T = \{a, b, c\} \wedge \{d, e\} \wedge \dots$)
- Assignment α: set of literals currently true
  - e.g. $\alpha = \{a, \neg b, \neg e, d, g\}$
- Decision literals $\delta \subseteq \alpha$: choices made by search
  - e.g. $\delta = \{a, \neg e\}$
- Explanations: why is $\ell \in \alpha$?
  - $expl(\ell) = clause$
  - e.g. $expl(d) = \{d, e\}$
  - Only for propagated literals (those in $\alpha \setminus \delta$)

Symmetries

- A **symmetry** of T is a permutation on the literals of $T \ldots$
  - e.g. $\sigma = (ab\neg c)(de)$
- ... that satisfies these conditions:
  - $\sigma(\alpha)$ is a model of $T \Leftrightarrow \alpha$ is a model of T
  - $\sigma(\neg \ell) = \neg \sigma(\ell)$
- Symmetries lift naturally to clauses and theories.

Symmetries

- If S is the symmetry group of theory T, and T entails the clause $\alpha \rightarrow \ell$, then T also entails the clause $\sigma(\alpha) \rightarrow \sigma(\ell)$.
- There are many such $\sigma(\alpha) \rightarrow \sigma(\ell)$ clauses (too many!)
- We use a **weak activity** heuristic to detect useful $\sigma(\alpha) \rightarrow \sigma(\ell)$ clauses: ones that propagate.

Activity

- Symmetry σ is **active** under α if:
  - $\sigma(\alpha) = \alpha$
- During search:
  - α is the set of true literals,
  - $\delta \subseteq \alpha$ is the set of search decisions.
- Symmetry σ is **weakly active** for δ under α if:
  - $\sigma(\delta) \subseteq \alpha$
- **Asymmetric** literal: literal $\ell \in \alpha$ where $\sigma(\ell) \notin \alpha$
  - (σ is not active, but might be weakly active)

Overview

repeat
  while there is a unit clause **do**
    run unit propagation
  end while
  for each weakly active symmetry σ in S **do**
    if there is an asymmetric literal ℓ for σ **then**
      add $\sigma(\ell)$ to α
      define $expl(\sigma(\ell)) = \sigma(expl(\ell))$
      add $expl(\sigma(\ell))$ to T
      break and go back to unit propagation
    end if
  end for
until conflict, or no new literals propagated

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \emptyset$
$\delta = \emptyset$

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \{\mathbf{a}\}$
$\delta = \{\mathbf{a}\}$

- Search chooses *a*

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \{a, \mathbf{d}\}$
$\delta = \{a\}$

- Unit propagation infers d
- σ is not weakly active (because $\sigma(\delta) \not\subseteq \alpha$)
- a is first asymmetric literal for σ (because $\sigma(a) \notin \alpha$)

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \{a, d, \mathbf{f}\}$
$\delta = \{a, \mathbf{f}\}$

- Search chooses f

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \{a, d, f, \mathbf{b}\}$
$\delta = \{a, f\}$

- Unit propagation infers b

Example

$$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$$
$$S = \{\sigma\} = \{(ab)(de)\}$$
$$\alpha = \{a, d, f, b\}$$
$$\delta = \{a, f\}$$

- σ is now weakly active (but not active)
- First asymmetric literal for σ is now d

Example

$$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$$
$$S = \{\sigma\} = \{(ab)(de)\}$$
$$\alpha = \{a, d, f, b, \mathbf{e}\}$$
$$\delta = \{a, f\}$$

- σ is now weakly active (but not active)
- First asymmetric literal for σ is now d
- Symmetric propagation infers $\sigma(d)$, which is e
- $expl(e) = \sigma(expl(d)) = \{\neg b, e\}$

Example

$T = \{\neg f, a\} \wedge \{\neg f, b\} \wedge \{\neg a, d\} \wedge \{\neg b, e, c\} \wedge \{\neg c, \neg g\}, \{\neg c, g\}$
$S = \{\sigma\} = \{(ab)(de)\}$
$\alpha = \{a, d, f, b, e\}$
$\delta = \{a, f\}$

- Unit propagation resumes (but nothing to do)
- σ is still weakly active
- No more asymmetric literals in α (σ is active)
- Search continues

Tracking weak activity

- Symmetry σ is weakly active if $\sigma(\delta) \subseteq \alpha$.
- Each symmetry has a counter, initialised to zero.
- When a literal ℓ is added to α, for each symmetry σ:
  - if $\ell \in \delta$ and $\sigma(\ell) \notin \alpha$, increment counter
  - if $\sigma^{-1}(\ell) \in \delta$, decrement counter
- Symmetry is weakly active if counter is zero.
- Constant time per symmetry that involves ℓ.

Properties

- Symmetry propagation preserves completeness (never loops infinitely).
- Symmetry propagation preserves soundness, and does not choose solutions a priori.
- Choosing a choice literal can decrease or increase the set of weakly active symmetries.
- Propagating a literal only increases the set of weakly active symmetries.
- After propagation, all weakly active symmetries are active again
  - i.e. for all weakly active symmetries σ we have $\sigma(\alpha) = \alpha$

Optimisation 1: Inverting Symmetries

- An symmetry σ is *inverting* if $\sigma(\ell) = \neg\ell$ for some ℓ.
  - Such a literal ℓ is *inverting* for σ.
- When an inverting literal ℓ is propagated, symmetry propagation will cause $\sigma(\ell) = \neg\ell$ to be propagated, causing a conflict.
- When an inverting literal ℓ becomes a decision literal, then σ will become weakly inactive *permanently* (until backtracking undoes the choice of ℓ).
- Optimisation: make the search avoid choosing inverting literals as its decisions.

Optimisation 2: Inactive propagation

- Symmetry propagation is about finding implied unit clauses.
  - We find an explanation clause c such that weak activity guarantees $\sigma(c)$ is unit.
- In addition: generate clauses from existing explanations and weakly *inactive* symmetries and propagate with them if they are unit.

for each literal $\ell \in \alpha \setminus \delta$ **do**
  for each weakly inactive symmetry σ **do**
    if $\sigma(expl(\ell))$ is unit **then**
      Propagate with it and resume unit propagation
    end if
  end for
end for

Results Overview

- Compared Minisat, Minisat+Shatter, Minisat+SP, Minisat+SP+optimisations.
- Symmetric problems from SAT2011 competition and standard symmetric benchmarks.
- On satisfiable problems:
  - All methods work well.
  - Minisat+Shatter best.
- On unsatisfiable problems:
  - Symmetry breaking much more important.
  - Minisat+SP+optimisations best.

Results Examples

- Satisfiable problem

Problem name	Minisat	$+\mathrm{SP}^{\mathrm{reg}}$	$+\mathrm{SP}^{\mathrm{opt}}$	+Shatter
	Solve Time (s)			
battleship-07-13-sat	**0.0**	0.4	0.4	0.4
battleship-08-15-sat	**0.0**	**0.0**	**0.0**	**0.0**
battleship-09-17-sat	**0.0**	0.1	0.1	0.1
battleship-10-17-sat	3.9	**1.4**	2.2	5.4
battleship-10-18-sat	**0.0**	**0.0**	0.1	0.1
battleship-10-19-sat	**0.0**	0.1	0.1	0.1
battleship-12-23-sat	**0.0**	0.1	0.1	0.1
battleship-14-26-sat	718.2	1060.2	546.1	**14.3**
battleship-15-29-sat	386.0	**16.5**	296.6	88.1
battleship-24-57-sat	16.5	**2.8**	21.9	34.3

Results Examples

- Unsatisfiable problem

	Solve Time (s)			
Problem name	Minisat	+SP^{reg}	+SP^{opt}	+Shatter
battleship-05-08-uns	**0.0**	**0.0**	**0.0**	**0.0**
battleship-06-09-uns	0.1	**0.0**	**0.0**	**0.0**
battleship-07-12-uns	485.1	17.3	2.0	**1.4**
battleship-10-10-uns	1.6	1.1	0.2	**0.0**
battleship-12-12-uns	402.3	45.6	**0.7**	1.3
battleship-14-14-uns	-	-	1372.2	**736.6**
battleship-15-15-uns	-	-	**149.0**	-
battleship-16-16-uns	-	-	**32.9**	-

Results Examples

- Inverting Literals

Problem name	Minisat	Solve Time (s) +SP$^{\mathrm{reg}}$	+SP$^{\mathrm{opt}}$	+Shatter
Urq3_5-uns	139.7	**0.0**	**0.0**	0.1
Urq4_5-uns	-	**0.0**	**0.0**	39.0
Urq5_5-uns	-	7.0	**0.2**	3810.3
Urq6_5-uns	-	-	**0.6**	-
Urq7_5-uns	-	-	**1.3**	-
Urq8_5-uns	-	-	**3.3**	-

Conclusions and Future Directions

- New approach to dynamic symmetry breaking
  - Better than existing methods on unsatisfiable instances
- Application to general constraint programming
- Search heuristics to maximise weakly active constraints

Thank you!

Questions?