Multi-criteria optimisation design of shapers with piece-wise equally distributed time-delay

Dan Pilbauer [*,**] **Wim Michiels** [**] **Tomáš Vyhlídal** [*]

[*] *CIIRC - Czech Institute of Informatics, Robotics and Cybernetics, and Dept. of Instrumentation and Control Eng., Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, 166 07 Praha 6, Czech Republic,* {Dan.Pilbauer, Tomas.Vyhlidal}@fs.cvut.cz

[**] *Department of Computer Science, Katholieke Universiteit Leuven, Celestijnenlaan 200A, B-3001 Heverlee, Belgium,* Wim.Michiels@cs.kuleuven.be

Abstract: The paper presents an optimisation based method to parametrize input shapers with time delays of piece-wise-equal distribution. The design respects the two key requirements on the shaper performance that are the prompt response time and the robustness represented by residual vibration functions, which are however conflicting criteria as a rule. Thus, the optimisation task is solved as a constrained multi-objective problem where the constraints respect the additional requirements on the shaper performance such as a non-decreasing step response or zero residual function at some frequency points. The result is a Pareto front that shows a trade-off between the two main requirements. Next to the presentation of the results in an example, a user friendly shaper-design tool is presented.

Keywords: time-delay, signal shapers, vibration suppression, multi-objective optimisation, Pareto frontier

1. INTRODUCTION

The idea of shaping reference commands in order to remove oscillatory modes from the signal was published by Smith (1957), where feed-forward input shaping based on the time delays was introduced as so-called PosiCast, known also as the zero-vibration (ZV) shaper. Advantageous robust shapers as zero-vibration-derivative (DZV) or extra-insensitive (EI) were introduced and investigated by Singer and Seering (1990), Singhose et al. (1994). Further modifications followed. Multi-mode shapers were developed to compensate two or more flexible modes, see Singh and Heppler (1993), Singh and Vadali (1995) or Sung and Singhose (2009). Next to the continuous-time domain shapers, discrete versions of the shaper were firstly published by Murphy and Watanabe (1992), followed by discussion about implementation aspects by Magee and Book (1993) and more recent publications by Baumgart and Pao (2007), Cole (2011). A comparison and an comprehensive analysis of robustness of signal shaping techniques were published by Singer and Seering (1990) or more recently by Vaughan et al. (2008).

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As an alternative to signal shaping, various command profiles, such as trapezoidal, S-curve or trigonometric functions can be used to smooth fast changes in the reference input signal of flexible systems, Meckl (1998), Singhose et al. (2010). As shown in Vyhlídal and Hromčík (2015), these finite-length input filters can be represented by distributed time delays. This applies also for the Jerk-limited input shapers proposed by Singh (2004). Compared to the classical shapers, the output of the signal is smoothed by various types of filters to avoid strong discontinuities after shaping the signals. In Vyhlídal et al. (2012), Vyhlídal et al. (2013b), an alternative approach was proposed, by considering an equally distributed delay directly in the shaper design. Subsequently, more complex delay distributions were considered in Vyhlídal et al. (2013c). Next to the smoothening effect at the signal accommodation part, the retarded characteristics of the shaper spectrum can be considered as an implementation benefit, particularly, if the shaper is implemented within a closed loop system Vyhlídal et al. (2013a), Vyhlidal et al. (2015).

This paper is a subsequent work to Vyhlídal and Hromčík (2015), where an optimisation based method was proposed for the direct design of robust shapers with piece-wise equally distributed delays. The routine was based on constrained linear least squares optimisation applied directly to the minimization of the sensitivity function in the frequency domain. The key contribution with respect to Vyhlídal and Hromčík (2015) is in considering not only the

robustness, but also the action time in the shaper design. Next, the design task is reformulated a constrained miltiobjective optimization problem, which can be solved using convex optimization techniques. Besides, a software design tool is presented. The paper is also related to the recent work Pilbauer et al. (2015), where an alternative approach was applied to a shaper with continuous distribution of the delay by optimising smooth kernel functions.

2. PRELIMINARIES

The objective of applying input shapers in a serial interconnection with a system is to fully or partially compensate its oscillatory mode, which is determined by a couple of poles $\hat{s}_z = -\beta \pm j\Omega, \beta = \omega\zeta, \Omega = \omega\sqrt{1-\zeta^2}$, where ζ, ω are the damping ratio and natural frequency of the mode to be compensated. As mentioned in the previous section, this task can be performed by a number of shaper classes. A common property of the above mentioned shapers is the neutral spectrum of shaper zeros, as demonstrated in Vyhlídal et al. (2013a), Vyhlidal et al. (2015). Thus, next to the *active* shaper zeros that compensate the effect of the oscillatory pole, there are infinitely many higher frequency zeros distributed close to the imaginary axis or even in the right half plane. Once, the shaper is included in the feedback loop as an inverse shaper, these zeros can be projected to the closed loop poles and can be responsible for the stability loss. For example, this will happen if the scheme in Fig. 1 is applied to handle the task of oscillatory mode suppression os System 2 when induced not only by the setpoint w, but also by disturbance $d_{1,2}$, see Vyhlídal et al. (2013a), Vyhlidal et al. (2015). The only option for the effective loop scheme in Fig. 1 is to involve a shaper with a distributed time delay, as its spectrum of zeros is retarded - i.e. the high frequency zeroes lie safely far from the stability boundary Vyhlidal et al. (2015); Vyhlídal et al. (2013b); Vyhlídal and Hromčík (2015).

2.1 Input shapers with distributed delay

A general form of a shaper with distributed delay can be described as

$$u(t) = Aw(t) + (1-A)\int_0^T w(t-\eta)dh(\eta). \qquad (1)$$

with the parameter $A \in [0,1]$ and the delay distribution $h(\eta)$ over the finite length segment $\eta \in [0,T]$, satisfying $h(\eta) = 1, \eta \geq T$. A common requirement on the delay distribution is the non-decreasing step response shape over $\eta \in [0,T]$. The transfer function of the distributed delay (1) is given by

$$G(s) = A + (1-A)F(s,T), \qquad (2)$$

where $F(s,T) = L\left\{\int_0^T w(t-\eta)dh(\eta)\right\}$ is the transfer function of the delay. The spectral synthesis of the shaper (1) consists of placing the dominant zero of (2) to the position of the oscillatory pole $\hat{s}_z$. As already mentioned, the rest of the infinitely many zeros of (2) follows the exponential asymptotic curves departing of the stability boundary with increasing moduli of the roots.

In Vyhlídal et al. (2012), Vyhlídal et al. (2013b), a zero vibration shaper with equally distributed delay (DZV) was proposed, considering $F(s) = \frac{1-e^{-sT}}{sT}$. Next to the retarded spectrum, a smoother accommodation and filtering effect are the benefits compared to the classical ZV shaper with the distributed delay. A negative aspect compared to the classical ZV shaper was the increased response time length. This inefficiency can partly be overcome by combining the lumped and equally distributed delay $F(s,T,\alpha) = \frac{e^{-\alpha Ts} - e^{-Ts}}{(1-\alpha)s}$, see Vyhlídal et al. (2013c), where parameter α determines the ratio between the lengths of the lumped part and the overall delay length T. An extension towards more general delay distributions, e.g. with S-shape $F(s) = \frac{4(1-2e^{-s\frac{T}{2}}+e^{-sT})}{T^2s^2}$ or trigonometric delay $F(s,T) = \frac{4\pi^2\left(1-e^{-Ts}\right)}{Ts(T^2s^2+4\pi^2)}$ was proposed in Vyhlídal and Hromčík (2015), along with a fully analytical parameterization procedure.

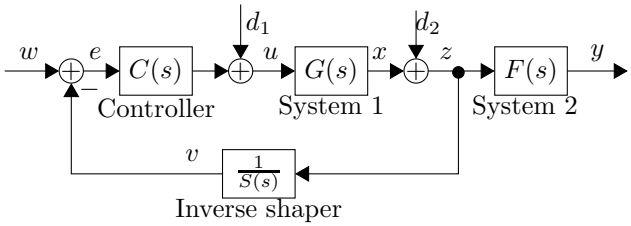

Fig. 1. Inverse shaper in the feedback

In order to increase robustness, i.e. provide distributed delay alternatives to the classical EI, DZV shapers, a least squares approach was proposed in Vyhlídal and Hromčík (2015), considering

$$F(s) = \frac{\sum_{k=0}^{N} \bar{a}_k e^{-s\tau_k}}{s}. \qquad (3)$$

In the procedure, the delays $\tau_k, k = 0..N$ covering equidistantly the interval the delay length interval $[0,T]$ are fixed and the parameters A and $\bar{a}_k, k = 0..N$ are free parameters to optimise a residual vibration characteristic.

A technique using smooth kernel function was proposed by Pilbauer et al. (2015) where transfer function of the shaper is given by

$$G(s) = A + \int_0^T g(\theta)e^{-s\theta}d\theta. \qquad (4)$$

with the kernel function g chosen as the polynomial $g(\theta) = \sum_{i=0}^{N_p} a_i\theta^i$. This approach has more degrees of freedom in the design but leads to more complicated multiobjective optimisation problem. Compared to (2)-(3), the resulting shaper is also of an enhanced complexity from the implementation point of view.

2.2 Residual vibration function

The idea of measuring robustness of the shaper was introduced by Singhose et al. (1994), where term *residual vibrations* were firstly used. The residual vibration function takes into account changes of the parameters of the System 2 (Fig. 1). The robustness in this paper is expressed in terms of residual vibrations and more precisely in terms of the transfer function

$$V(\zeta,\omega) = \left|G\left(-\omega\zeta - j\omega\sqrt{1-\zeta^2}\right)\right| e^{\zeta\omega T}, \qquad (5)$$

which expresses the amplitude of the residual vibrations at the time $t = T$. The transfer function has uncertainties

in both, natural frequency ω and damping ratio ζ, on a given region of the parameters. Notice that if $s = -\omega\zeta - j\omega\sqrt{1-\zeta^2}$, i.e. if the shaper zero has the position of the oscillatory pole to be compensated, the residual vibration function is zero. Thus, there are no residual vibration in the time domain responses. However, if the frequency or the damping of the oscillatory mode change, the residual vibrations of the magnitude $\left|G\left(-\bar{\omega}\bar{\zeta} - j\bar{\omega}\sqrt{1-\bar{\zeta}^2}\right)\right|$.

3. MAIN RESULT - DESIGN OF DISTRIBUTED DELAY SHAPER

3.1 Problem formulation

We consider the robust shaper form proposed in Vyhlídal and Hromčík (2015), i.e. given by (2)-(3), which can be reformulated to

$$G(s,T) = A + \frac{\sum_{k=0}^{N_p} a_k \mathrm{e}^{-s\tau_k}}{s}. \tag{6}$$

The parameters of (6) are the gain of the non-delayed shaper part A, the gains a_k and delays τ_k $(k = 0, 1, ..., N)$ determining the overall distribution of the delay. In the optimisation procedure which follows, we consider that the delays are equidistantly distributed over the interval $[0, T]$, i.e. $\tau_m = \tau_{m-1} + \Delta\tau, m = 1, ..., N_p$ with $\tau_0 = 0$ and $\Delta\tau = \frac{T}{N_p}$. The two criteria to optimise over the free parameters $A, a_k, k = 0, ..., N$ are the residual function (5) and the action time T. Additionally, a set of (partly optional) constraints is to be taken into the consideration as addressed below.

3.2 The primary objective functions

In minimizing the residual vibration function over the interval covering the vicinity corresponding the oscillatory mode $\hat{s}_z = -\omega\zeta - j\omega\sqrt{1-\zeta^2}$ to be compensated, the uncertainties in both, natural frequency ω and damping ratio ζ are taken into account.

Assume that $\omega \in I_1$, $\zeta \in I_2$, where

$$I_1 = [\omega_{\min}, \omega_{\max}], I_2 = [\zeta_{\min}, \zeta_{\max}], \tag{7}$$

with nominal frequency ω_{nom} and nominal damping ζ_{nom} assumed to be the midpoints of these intervals. To handle the uncertainty in ω and ζ we define a grid of N_ω and N_ζ Chebyshev points, which are more efficient for polynomial approximation, see Stewart (c1996), on I_1 and I_2

$$\begin{aligned}\omega_k &= \left(\frac{\omega_{\max}+\omega_{\min}}{2}\right) - \left(\frac{\omega_{\max}-\omega_{\min}}{2}\right)\cos\left(\frac{(k-1)\pi}{N_\omega - 1}\right),\\ \zeta_l &= \left(\frac{\zeta_{\max}+\zeta_{\min}}{2}\right) - \left(\frac{\zeta_{\max}-\zeta_{\min}}{2}\right)\cos\left(\frac{(k-1)\pi}{N_\zeta - 1}\right).\end{aligned} \tag{8}$$

with $k = 1, ..., N_\omega$ and $l = 1, ..., N_\zeta$. Define the vector of gains to be assessed as

$$\mathbf{x} = \begin{bmatrix} A & a_0 & a_1 & \cdots & a_{N_p}\end{bmatrix}^{\intercal}, \tag{9}$$

then the robustness criterion can be expressed by

$$f(\mathbf{x}, T) = \frac{1}{N_\omega N_\zeta}\sum_{k=1}^{N_\omega}\sum_{l=1}^{N_\zeta} V(\zeta_l, \omega_k)^2.$$

The shaper transfer function (6) can be expressed as

$$G\left(-\omega\zeta - j\omega\sqrt{1-\zeta^2}\right) = L(\zeta, \omega, T)\mathbf{x}$$

with

$$L(\zeta, \omega, T) = \begin{bmatrix}1 & g_0(\bar{s}, T) \cdots & g_{N_p}(\bar{s}, T)\end{bmatrix},$$

where $g_k = \frac{\mathrm{e}^{-s\tau_k}}{s}, k = 0..N_p$ and $\bar{s} = -\omega\zeta - j\omega\sqrt{1-\zeta^2}$. Consequently we have

$$\begin{aligned}V(\zeta,\omega)^2 &= \mathbf{x}^{\intercal}\, L(\zeta,\omega,T)L(\zeta,\omega,T)\,\mathbf{x}e^{2\zeta\omega T}\\ &= \mathbf{x}^{\intercal}\,\Re(L(\zeta,\omega,T)L(\zeta,\omega,T))\,\mathbf{x}e^{2\zeta\omega T}\end{aligned}$$

and the robustness measure takes the form

$$f(\mathbf{x}, T) = \mathbf{x}^{\intercal}H(T)\mathbf{x}, \tag{10}$$

where

$$H(T) = \frac{1}{N_\omega N_\zeta}\sum_{k=0}^{N_\omega}\sum_{l=0}^{N_\zeta} e^{2\zeta_l\omega_k T}\Re\left\{L(\zeta_l,\omega_k,T)L(\zeta_l,\omega_k,T)\right\}. \tag{11}$$

3.3 Linear constraints

The shaper optimisation problem is constrained by requirements on its structure and properties, which can be expressed as linear equality or inequality constraints.

Equality constraint I. In order co compensate a specific oscillatory mode entirely, the zeros of the shaper are placed at the position of the oscillatory mode defined as

$$\hat{s}_n = -\zeta_{\mathrm{nom}}\,\omega_{\mathrm{nom}} \pm j\sqrt{1-\zeta_{\mathrm{nom}}^2}\,\omega_{\mathrm{nom}},$$

where ω_{nom} and ζ_{nom} are the nominal frequency and damping ratio. This requirement corresponds to

$$\begin{cases}\Re\left(A + \dfrac{1}{\hat{s}_n}\displaystyle\sum_{k=0}^{N} a_k e^{-\hat{s}_n\tau_k}\right) = 0,\\ \Im\left(A + \dfrac{1}{\hat{s}_n}\displaystyle\sum_{k=0}^{N} a_k e^{-\hat{s}_n\tau_k}\right) = 0.\end{cases} \tag{12}$$

Let's note that this constraint is optional and can be excluded. On the other hand, the design allows to place multiple zeros, which would however reduce the space of feasible solutions and could result in a shaper with considerably higher response time T.

Equality constraint II. The additional equality constraint comes from the basic requirement on input shaping to have the static gain equal to one, i.e. to satisfy

$$\lim_{s\to 0} A + \sum_{i=0}^{N_p} a_i g_i(s) = 1, \tag{13}$$

which turns to

$$\sum_{k=0}^{N} a_k = 0, A - \sum_{k=0}^{N} a_k\tau_k = 1. \tag{14}$$

Inequality constraint I. The first constraint comes from the basic requirement on the delay free part of the shaper $0 \le A \le 1$. One can also fix parameter A, which is convenient in inverse shaper applications, see Vyhlídal et al. (2012), Vyhlídal et al. (2013b).

Inequality constraint II. The second requirement which is common for all shapers is a non-decreasing step response. This requirement can be easily expressed as a set of N inequality constraints formulated as

$$-\sum_{k=0}^{l} a_k \le 0 \text{ for } l = 0, 1, 2, ..., N_p - 1. \tag{15}$$

Compactly, all the above mentioned constraints for the coefficients $(A, a_0, ...a_{Np})$ can be rewritten as

$$\begin{cases} A_1(T)\mathbf{x} \geq b_1, \\ A_2(T)\mathbf{x} = b_2, \end{cases} \tag{16}$$

where we underline the dependence of the selected (in)equality $A_1(T), A_2(T)$ on the response time T.

3.4 A multi-objective problem definition

In almost every practical application the design is a compromise between conflicting requirements. This is also the case for the design of the shaper, where decreasing response time T goes against increasing robustness and vice versa. The final attributes of the shaper then rely on a decision maker. The compromise is selected by a given parameter $\alpha \in [0, 1]$, which brigs the definition of the optimisation problem

$$\begin{aligned} &\min_{T,\mathbf{x}} \ \alpha \left(\frac{T}{T_{\text{nom}}}\right)^2 + (1-\alpha)\frac{\mathbf{x}^\intercal H(T)\mathbf{x}}{R_{\text{nom}}^2}, \\ &\text{subject to} \\ &\begin{cases} A_1(T)\mathbf{x} \geq b_1, \\ A_2(T)\mathbf{x} = b_2. \end{cases} \end{aligned} \tag{17}$$

As can be seen, the quadratic criteria has scaling factors. The scaling of the criterion is important to obtain comparable values of the functions. The scale factor $T_{\text{nom}} = \frac{2\pi}{\omega_{nom}}$ allows us to consider response time in a relative sense, in comparison to the nominal period time of the oscillation to be suppressed. The second scaling factor R_{nom} is the reference value for the residual vibrations. The scale factor is taken as $R_{\text{nom}} = 0.05$ since 5% of the residual vibrations is considered as a standard limit for the robustness, see Singer (1989).

Optimisation problem (17) is in general non-convex. However, for a fixed value of T, the resulting problem is convex due to $H(T) \geq 0$, and it corresponds to a quadratic programming problem. Therefore, (17) can be solved by a parameter sweep over scalar parameter T. Note that, by varying parameter α from zero to one, the full *Pareto front* is generated of the multi-objective optimisation problem

$$\begin{aligned} &\min_{T,\mathbf{x}} \ \left\{ \left(\frac{T}{T_{\text{nom}}}\right)^2, \ \frac{\mathbf{x}^\intercal H(T)\mathbf{x}}{R_{\text{nom}}^2} \right\}, \\ &\text{subject to (16).} \end{aligned} \tag{18}$$

The overall optimisation procedure is then summarised in the following algorithm.

Algorithm 1 - Overall shaper design procedure.
The result of the shaper design is depending on a decision maker, hence the procedure has to follows the further steps:

- *Step 1* - Definition of the shaper structure by the number of free parameters, i.e. assessing $N_p > 1$ and determining whether the parameter A is excluded from the optimisation and fixed on a specific value or is left as a decision variable.
- *Step 2* - Define nominal values of the flexible structure (System 2 in Fig. 1), i.e. nominal frequency ω_{nom} and nominal damping ration ζ_{nom}. For the given nominal parameters, zeros of the shapers can be placed exactly on the nominal pole or not. Then the uncertainty intervals (7) over which the residual function is to be minimized are to be determined.
- *Step 3* - Finally, the *Pareto front* is obtained by solving the problem (18) with respect to the parameter α. The end user can then select a compromise between the two objective functions, the response time T and average residual vibration on the parameters domain $(\omega, \xi) \in I_1 \times I_2$, just by adjusting the parameter α.

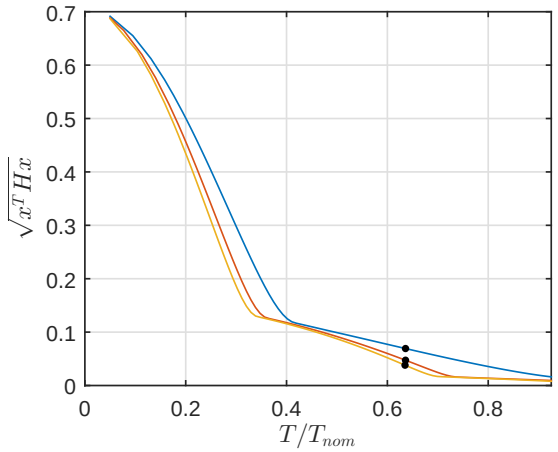

Fig. 2. Pareto frontier generated for different number of coefficients $N = 5$(blue), 10(red), 15(yellow) with marked points for $\alpha = 0.78$

3.5 Demonstration example

Consider the oscillatory mode given by the nominal frequency $\omega_{nom} = 10$ and damping ratio $\zeta_{nom} = 0.01$. The residual vibration function is to be minimized over the the intervals $I_1 = [0.85, 1.15]\omega_{nom}$, $I_2 = [0.85, 0.15]\zeta_{nom}$, covered by $N_\zeta = 10$, $N_\omega = 10$ discretisation points to obtain the mesh (8). Performing the optimisation procedure over the full parameter set consisting of the parameter A and a_k, $k = 0, ..., Np$ and taking into account the Equality constraint II and the Inequality constraints I and II, the whole Pareto front is given in Fig. 2 for $N_p = 5, 10$ and 15. It can nicely be observed that increasing the number of coefficients provides more degrees of freedom forthe optimisation and enhance diversity of the solutions.

Next, for the specific value of $\alpha = 0.78$ the comparison of the residual vibration function (5), the shaper step responses and the spectral properties are given in Figs. 3, 4 and 5 is shown. As can be seen, increasing the number of points from $N_p = 5$ to $N_p = 10$ led to considerable improvement in the residual vibration function over the target region. On the other hand, an increase from $N_p = 10$ to $N_p = 15$ had a negligible effect on the shaper robustness. In the step response in Fig. 4 we can also observe a considerable difference between the case with $N_p = 5$ and the cases obtained for $N_p = 10, 15$ - an increase of N_p provides enhanced variability of speed. Finally, in Fig. 5, we can see that as required all the obtained shapers provide the retarded character of the spectrum.

3.6 Graphic user interface for the shaper design

The shaper design procedure proposed and demonstrated above has been implemented as a Matlab tool, including

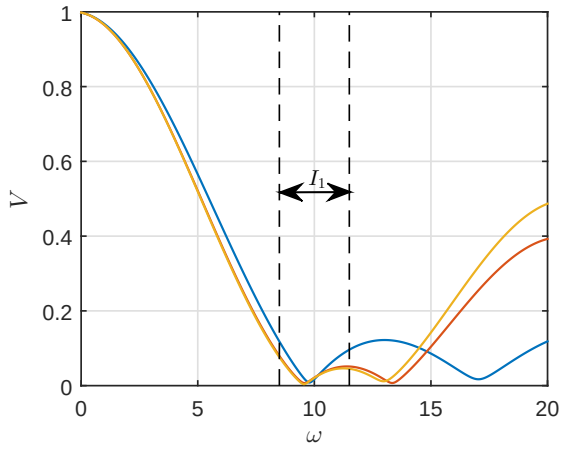

Fig. 3. Residual vibration function of the shapers for given $\alpha = 0.78$ for a different number of coefficients $N = 5$(blue), 10(red), 15(yellow)

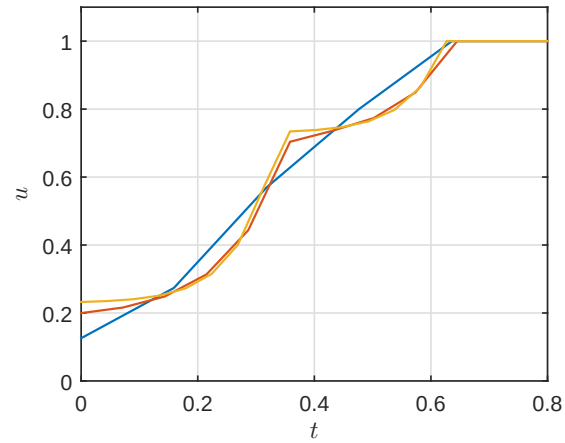

Fig. 4. Step response of the shapers with a different number of coefficients $N = 5$(blue), 10(red), 15(yellow)

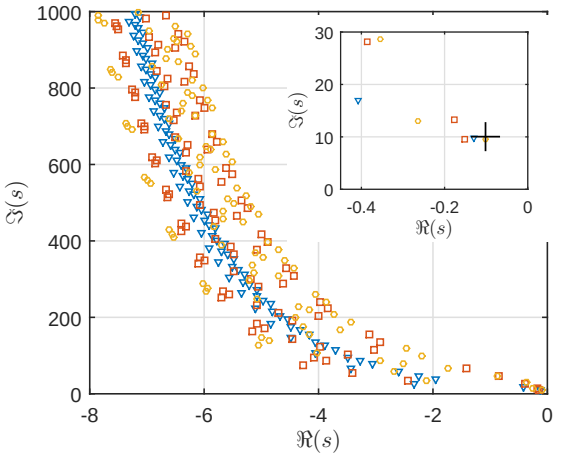

Fig. 5. Zeros of the shapers for a given $\alpha = 0.78$ and number of coefficients $N = 5$(blue), 10(red), 15(yellow) with cropped and zoomed area around target mode $s = -\omega_{nom}(\zeta_{nom} + j\sqrt{1-\zeta_{nom}^2})$ (black cross)

the graphic user interface (GUI) shown in Fig. 6, which consists of four parts:

- The shaper parameterization window to define the structural and required performance properties of the shaper. After defining all the parameters, the optimisation task is initiated by pressing the button *Draw pareto front*.
- The Pareto front window serves for the objective function visualization and for determining the *user-defined shaper performance point* by the slider below the graph.
- The shaper's attributes window corresponding to the selected point on the Pareto front toolbar. The button *Show* is to demonstrates the shaper properties and the button *Export* is to save the results to the Matlab workspace.
- The shaper's characteristics window with the step response, impulse response and residual vibration function for the selected point of the *Pareto front*.

A user-friendly, MATLAB based, interface for the shaper design is available from

`http://twr.cs.kuleuven.be/research/software/delay-control/shaper-design/`

4. CONCLUSIONS

The key contribution of this paper is an extension of the design of the input shaper with piece-wise equally distributed time delay. In comparison with previous work, the main advantage of the proposed method lies in the definition of the multi-objective constrained problem. Presuming higher number of degrees of freedom, the optimisation allows to design a robust and/or fast shaper, where the final attributes rely on decision maker. Then, for the given requirements, linear (in)equality constraints are defined. Some of the constraints, such as a non-decreasing step response and unity gain are necessary and demands on placing zeros to compensate specific mode of the flexible structure or initial gain are the optional constraints. The presented examples show the applicability of the proposed method and the effect of different number of the coefficients which is mainly beneficial but also brings difficulty for the case where the shaper is applied in the feedback.

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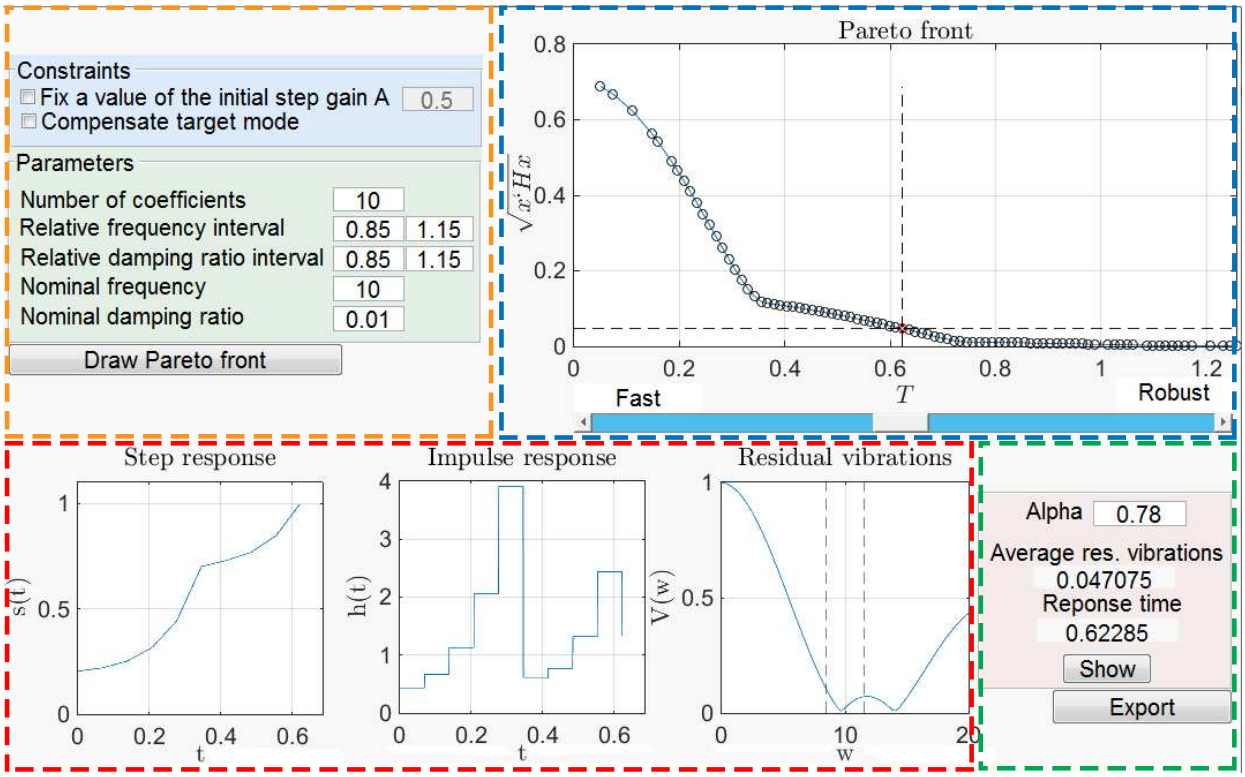

Fig. 6. Graphic user interface for the design of the shaper

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