Comparison of Lumped and Coupled mass matrix for flexible multibody simulation

In multibody (MB) models based on finite-element (FE) models, the way in which the mass-properties for the FE models are computed can highly impact the accuracy of the resultant MB model. This work compares the effect of using a lumped mass-matrix versus using a coupled or consistent mass matrix. In commercial software, a lumped mass approach is typically adopted and this work shows that this approach might lead to considerable errors in the mass properties alone. Forces and frequencies computed from this kind of simulation can be expected to deliver associated errors.
This work only explores the effect on the rigid mass properties and the effects on the flexible mass properties and mass-coupling should be further investigated.

Computation of mass invariants

In multibody system dynamics based on a floating-frame-of-reference, the mass-properties are typically determined from a lumped mass FE model. In this case, only diagonal terms are present in the FE mass-matrix, such that storage requirements are minimal. The rigid mass-properties (mass m and inertia's I) can be easily determined form this approach. Assuming n nodes this becomes:

$$m = \sum_{i=1}^{n} m_i \ ,$$

$$I_{xx} = \sum_{i=1}^{n} (y^2 + z^2) \cdot m_i + I_{xx,i}$$

$$I_{xy} = \ldots$$

$$\ldots$$

This approach allows an easy computation but neglects the coupling between different degrees of freedom.

For the coupled mass approach, all the couplings have to be taken into account properly. Therefore the FE mass-matrix has to be multiplied with a coordinate transformation matrix. For a 3D case this is a (6*n x 6) matrix to determine the rigid mass properties. This projection matrix looks like:

$$S = \begin{bmatrix} \begin{bmatrix} I_{3\times3} & -\tilde{x}_1 \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \\ \ldots \\ \begin{bmatrix} I_{3\times3} & -\tilde{x}_n \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \end{bmatrix},$$

And the rigid mass-matrix can be determined as:

$$M_{rigid} = S^T \cdot M_{FE} \cdot S\,.$$

Furthermore, for multibody models, also the different coupling terms between the rigid and flexible motion are important. These are typically captured in the *mass-invariants*. This work however, does not investigate the effect of the FE mass matrix on these terms.

Rigid body properties

A simple model was created to compare the resultant mass and rotational inertia computed for a FE model with lumped or coupled mass matrix.

Nastran input-file

The original model is a beam-like structure modeled with solid elements in Patran/Nastran. The model is shown in Fig.

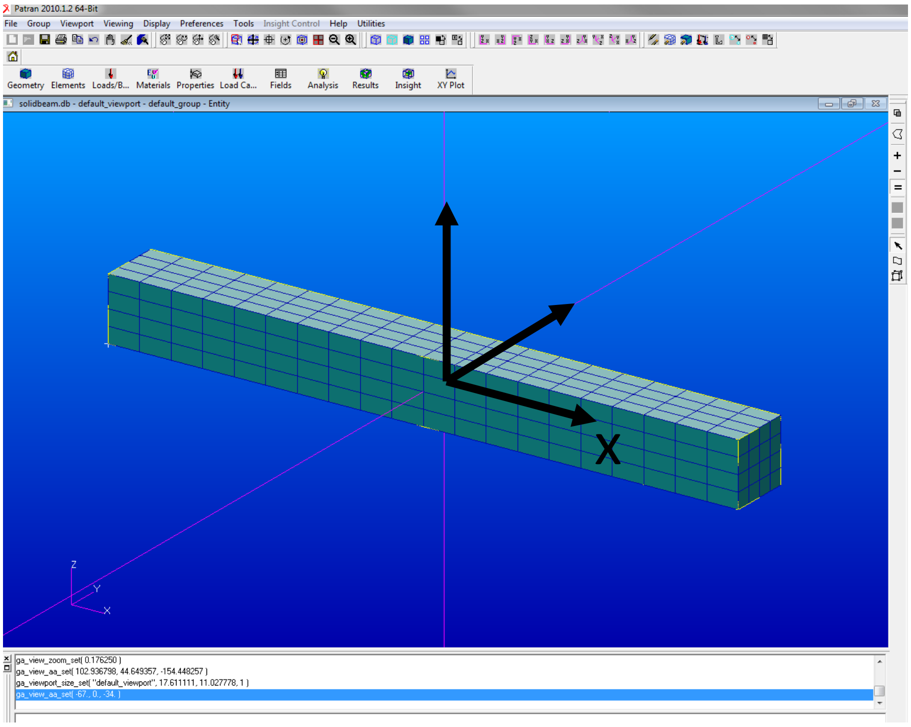

Figure 1: Finite element model in Patran

In Patran, the inertial-properties of the body can be extracted from the geometric data. The mass of the system is 9.75kg and the rotational inertias are: Ixx = 4.063e-3, Iyy=Izz=2.053e-1. These values serve as a reference for the rigid inertial properties of the system.

This model is exported to a bdf-file which is used as an input for Nastran (R3b). A dmap is added to the file in order to output the mass- and stiffness-matrix from the FE-model to two op4-files:

ASSIGN OUTPUT4 = 'Kgg.op4', UNIT = 45, FORM = FORMATTED

ASSIGN OUTPUT4 = 'Mgg.op4', UNIT = 46, FORM = FORMATTED

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$ Direct Text Input for Nastran System Cell Section

$ Direct Text Input for File Management Section

$ Direct Text Input for Executive Control

COMPILE SUBDMAP=SEMODES, SOUIN=MSCSOU, NOLIST, NOREF, NODECK

ALTER 'CALL SUPER3 CASECC'

OUTPUT4 KGG//-1/45/2/FALSE/ $

OUTPUT4 MGG//-1/46/2/FALSE/ $

$ Normal Modes Analysis, Database

CEND

This dmap-alter creates 2 op4-files, one with the stiffness- and one with the mass-matrix. These are formatted, such the resulting files are simple text-files which can be easily read in MATLAB.

In Nastran, the switch between lumped and coupled mass can be done by setting PARAM COUPMASS respectively to -1 or 1.

Matlab processing

The op4-files created by Nastran can be loaded into MATLAB using *readop4_full.m*. This file creates a matrix with all the data and only works for this specific for of the output.

The coordinates of the original mesh are already created in MATLAB but could also be read in from the dat-file by *original_coords_reader.m*.

Rigid mass convergence

The rigid mass and rotational inertias are computed for an increasingly fine mesh. In the Y- and Z-direction, the mesh is constant with 4 elements. In the x-direction, the mesh is refined from 2 to 14 elements. Due to the symmetry of the system Iyy = Izz. The evolution of the mass and inertial properties is shown in Figure 2.

For the total mass it doesn't matter whether a lumped or consistent mass approach is adopted. For the rotational inertia however, important differences can be seen between the 2 approaches. For the consistent-mass approach, the rotational inertia is independent of the mesh refinement and even for a very coarse mesh, the exact rigid body inertias are found.
For the lumped mass approach however, it is clear that a sufficiently fine mesh is necessary to get accurate rigid inertia estimates. In the case of the coarse mesh for example, Iyy shows an error of around 30%. As the number of elements in the x-direction increases, Iyy shows convergence to the exact and coupled properties. A residual error remains because also the number of elements in the y- and z-direction should be increased to get closer to the correct result. Since the number of elements remains constant in the y- and z-direction, Ixx remains constant. As expected the value for the lumped mass approach doesn't show any convergence here and a constant error remains.

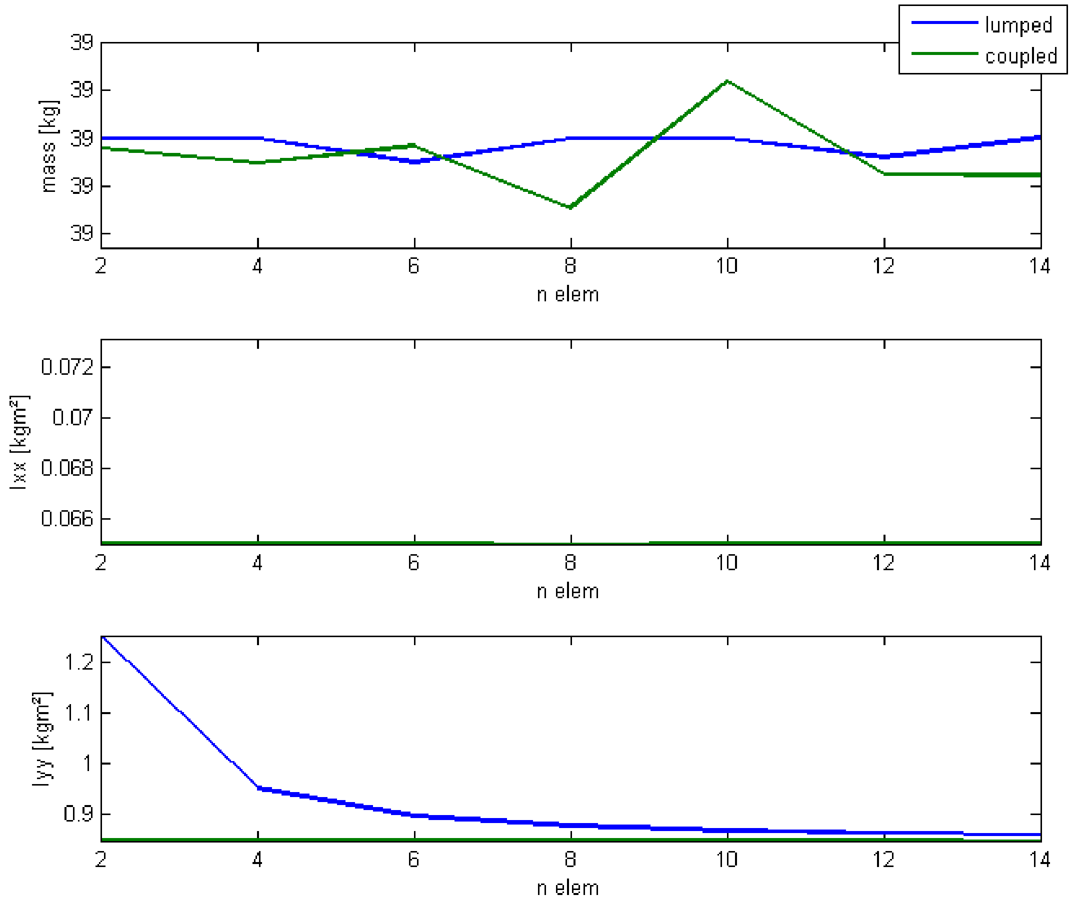

Figure 2: convergence of rigid inertial properties

Practical validation: Jet model

The comparison between the mass-properties from the lumped and coupled mass-matrix are also performed for a small jet-model with a realistic mesh, shown in Figure 3.

The total mass of the system is 6570kg, and the principal lumped rotational inertias are: 0.3759e5kgm², 1.3735kgm² and 1.6505kgm², whereas the coupled inertias are: 0.3743e5kgm², 1.3704e5kgm² and 1.6469kgm². The differences between the different inertias are respectively: 4.3%, 2.3% and 2.2%.

The principal axes for this system are shown in Figure 4 . In this figure the principal axes for both methods are indistinguishable and the error is below 1%, so the error is mainly concentrated in the magnitude of the inertias rather than their axes.

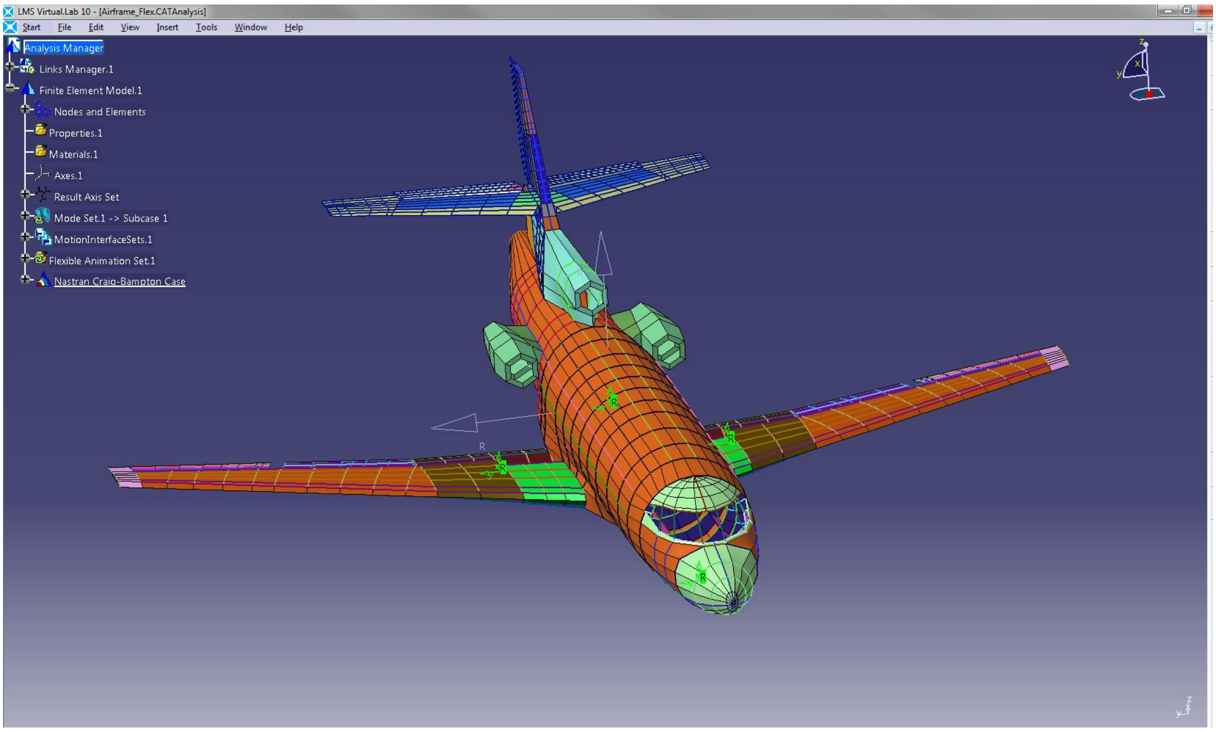

Figure 3: finite element jet model

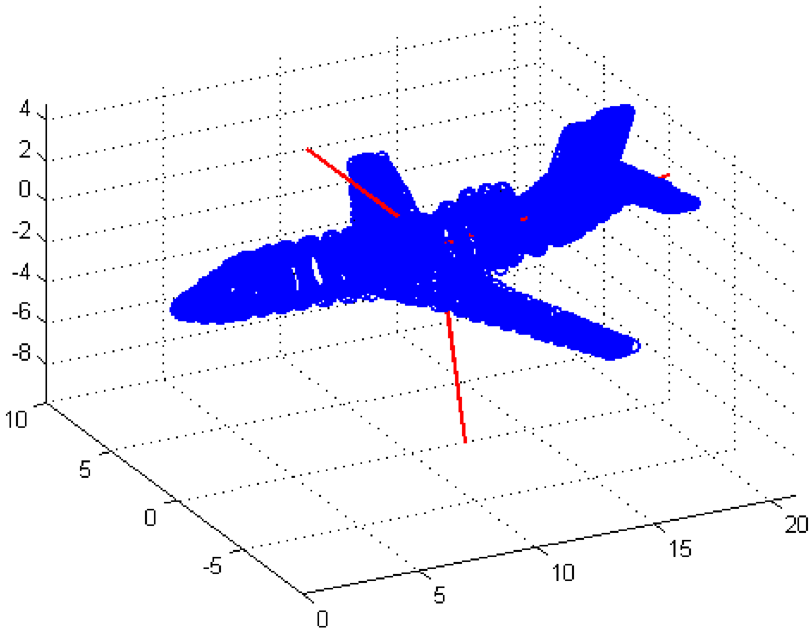

Figure 4: principal rotational axes for jet (lumped = green, coupled = red)

Flexible motion properties

Besides the effect on the rigid body properties, also the flexible dynamic behavior of the system is important. These properties are discussed in this section.

Structural behavior: eigenfrequencies, modal mass and stiffness

Eigenfrequency convergence in Figure 5. Difficult to draw a general conclusion on which approach is best from a structural dynamics point of view. In literature it has been shown that a combination of lumped and coupled mass give the best approximation for the eigenfrequencies (ref handbook). However, from a physical point of view, the coupled mass results make most sense because a coarser model is expected to be stiffer and should thus yield higher eigenfrequencies.

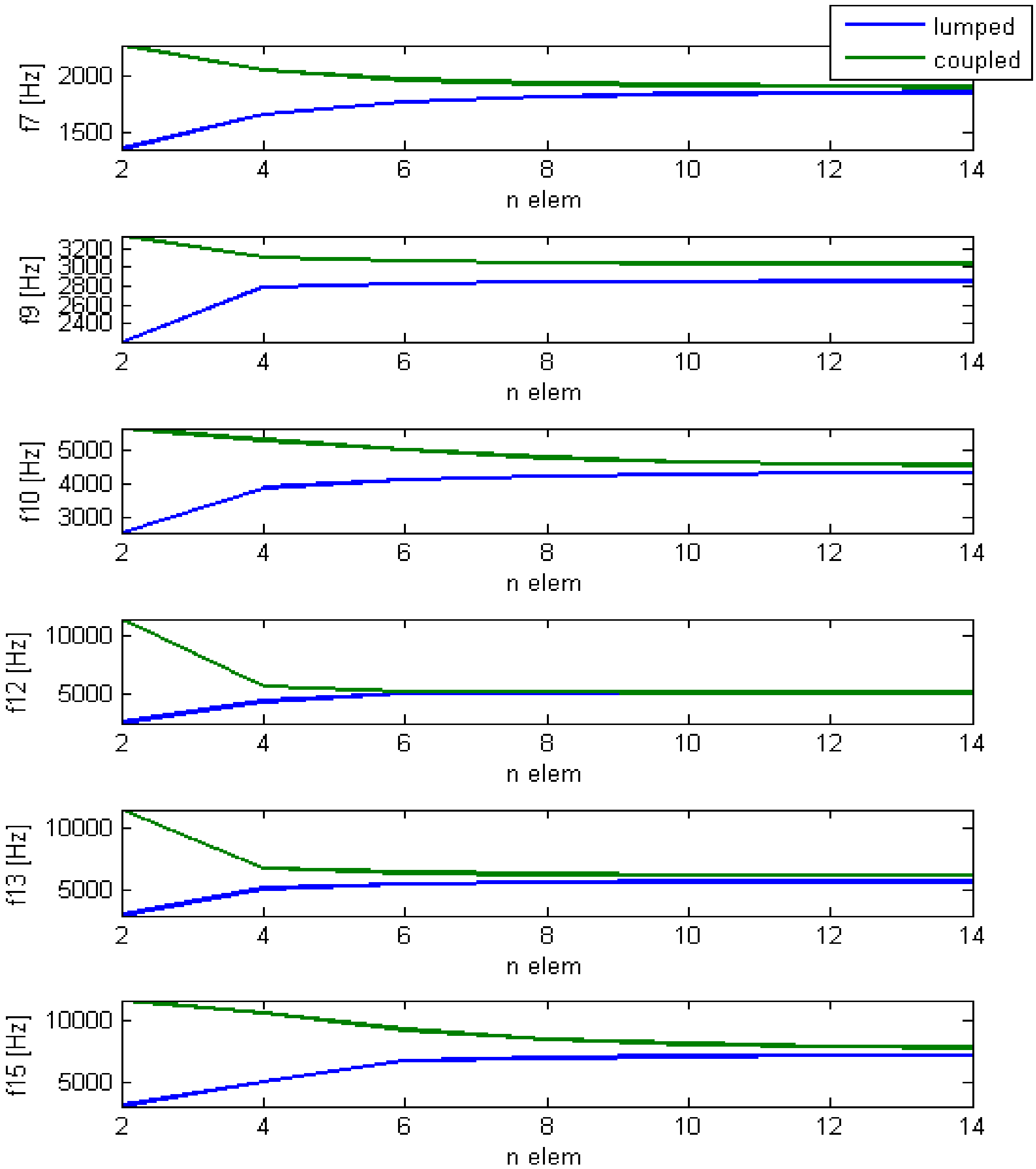

Figure 5: eigenfrequency convergence

Conclusion

This comparison shows that, even though no clear ‘winner’ can be determined for the frequency behavior for FE models employing lumped or coupled matrices, the rigid body mass-properties clearly show that a coupled approach is preferable in the context of flexible multibody dynamics. A next topic to investigate is the convergence of the other mass-properties such as coupling between the rigid motion and the flexible inertia.