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INFLUENCE OF DESIGN PARAMETER VARIATION ON THE DYNAMIC BEHAVIOUR OF THERMOPLASTIC HONEYCOMB PANELS

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ABSTRACT

Honeycomb sandwich panels are layered structures that consist of at least five layers: two thin face sheets are bonded to a thick honeycomb core. Because of the wide range of panel parameters, numerical modelling is needed to provide insight into the structural characteristics of a certain particular panel.
In this paper the effect of design parameter variations on the dynamic behaviour of honeycomb sandwich panels, in particular of thermoplastic Monopan panels, is studied.
In the first section the specific structure of Monopan honeycomb panels is illustrated. The different design parameters of panels of this type are outlined.
The second section deals with the numerical modal analysis of these honeycomb panels and the design parameter estimation. The procedure that is used to obtain initial design parameter values from various experiments is fully outlined. Several sandwich parameters are found to exhibit a significant amount of scatter. Average parameter values are used to obtain a good initial FE – model. The studied dynamic behaviour of the test panels is outlined.
The third section covers the experimental validation of the dynamic behaviour of a number of test panels.
In the fourth section the model updating procedure is discussed. The estimation of modal parameters from the experiments is outlined and results from sensitivity analysis are discussed. Finally the results from the updated FE model are given.
In the last section conclusions are made, along with an outlook to future research.

1. INTRODUCTION

Honeycomb sandwich panels consist of a thick honeycomb core that is bonded to thin face sheets. The structure of a typical panel is shown in fig. 1. The coordinate system is used throughout this text, although the axes are often indicated with numbers 1 to 3. The honeycomb panels on which the research discussed in this article is based are Monopan panels. Panels of this type have a cylindrical honeycomb core made of polypropylene (PP). The core is welded to a Twintex skin by means of a welding foil. The Twintex skin consists of a symmetric glass fibre woven fabric with a polypropylene matrix and a theoretical thickness of 0.7 mm. To smoothen the outer surfaces of the panel, a polypropylene finishing foil is welded there. The structure of a typical Monopan panel is shown in fig. 2.

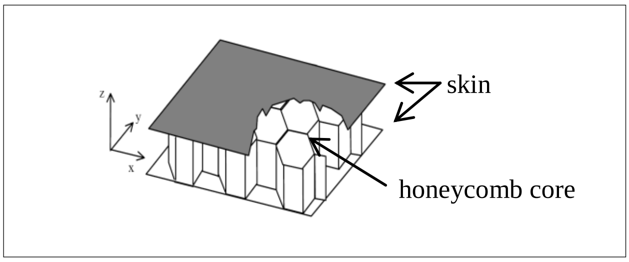


Figure 1. Honeycomb sandwich panel

In this study, Monopan panels of different sizes (length/width and thickness) are used. A first set of 7 panels have an overall thickness of 15 mm and in-plane dimensions 594 x 420 mm (A2 size). A second set of 7 are 25 mm thick and are also A2 size. The third set of 7 panels have a thickness of 25 mm and larger dimensions: 2500 x 1200 mm. All panels have the same structure, only the core height is different.

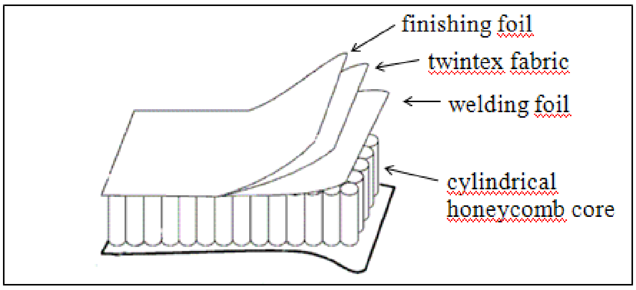


Figure 2. Structure of a Monopan panel

The elastic mechanical properties of a typical honeycomb core are described and analytically calculated by Gibson & Ashby [1]. They propose formulas for calculation of the in-plane and out-of-plane elastic moduli and Poisson ratios of the core.
As honeycomb sandwich panels become more and more important as structural parts in the automotive and aerospace industry, the need for accurate modelling of the dynamic behaviour of such panels increases. Accurate modelling requires knowledge of the different design parameters that determine the dynamic behaviour, which in this case is described by natural frequencies and mode shapes of panels with totally free boundary conditions.
The main work on the dynamics of sandwich panels is related to conventional foam-core structures. Little work has been carried out on honeycomb panels. Nilsson & Nilsson [2] tried to analytically predict natural frequencies of a honeycomb sandwich plate with free boundary conditions using Blevins [3] formula in which areal mass and equivalent bending stiffness are frequency dependent.

Another, more practical way to predict natural frequencies and mode shapes of a honeycomb panel is by means of finite element analysis. In the past years, different new approaches have been developed which incorporate high order shear deformation of the core. Work in this area has been carried out by Topdar [4] and Qunli Liu [5][6][7]. The latter stated that the shear moduli of the core are important factors in the determination of the values of the natural frequencies and the sequence of mode shapes, especially at high frequencies. At low frequencies natural frequencies are mostly determined by the bending stiffness of the panel.
The present analysis identifies parameter variability, with the definition of variability as given in [8]. In this paper a preliminary research is described. Design parameter variability of Monopan panels is studied, along with its influence on the dynamic behaviour of such panels subjected to totally free boundary conditions. For 1D laminated structures an approach to this problem is addressed in [9]. This work mainly focuses on the inverse problem; identifying material properties for layered materials by experimentally determining the vibration behaviour.

2. FINITE ELEMENT CALCULATION OF THE DYNAMIC BEHAVIOUR

2.1 Design parameters

Honeycomb panels are complex structures with a high number of design parameters. It is therefore difficult to accurately predict their dynamic behaviour, certainly when some of the parameters are very difficult or even impossible to measure in a direct way.
The Monopan honeycomb panel structure shown in fig. 2 has a high number of design variables. Table 1 gives an overview of the different parameters considered with these panels. They can be divided into two groups, geometric and material parameters. The abbreviations, used for the different parameters in table 1 will be used throughout the article.
A reliable Finite Element model requires accurate values of the design parameters. The experimental determination of model parameters is not straightforward. In addition, the number of parameters is high, 29 in this case. The skin of the panel, for instance, is in itself a complicated structure. Figure 3 shows a microscopic section view of the skin of a Monopan panel. It is evident that simply recognizing the three constituting layers, Twintex fabric, welding foil and finishing foil is not straightforward. During the production process of the panels skin faces are welded to the honeycomb core. This implies that the Twintex matrix (polypropylene) material, the welding foil and the finishing foil are melting into one resulting layer at some places. As figure 3 is a close up of the upper part of 1 honeycomb cell with a distance of about 8 mm between the cell walls, it is clear that skin dimensions and skin elastic properties will show a large spatial variability.

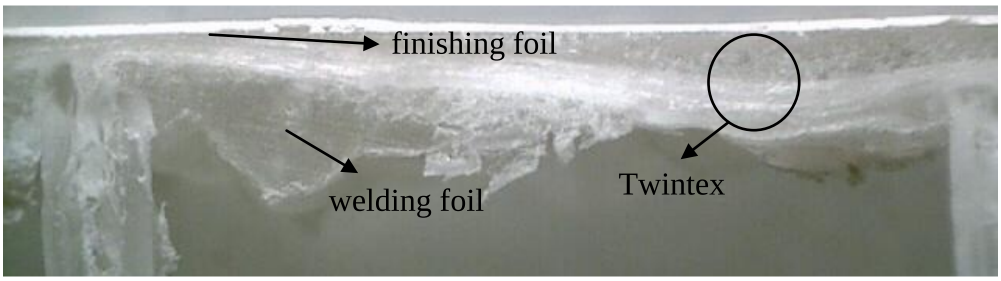


Figure 3. Section view of Monopan skin

Scatter on experimentally measured thicknesses of these three layers is high, because of variability that is physically inherent to the production process and because of measurement uncertainty. In this study these thicknesses are determined by analysis of microscopic views, similar to the one shown in figure 3.

	parameter description	symbol	unit
geometric	overall panel width	w	mm
	overall panel length	l	mm
	skin thickness	t_s	mm
	core thickness	t_c	mm
	core cell inner diameter	d_{ci}	mm
	core cell wall thickness	t_{cw}	mm
	finishing foil thickness	t_{ff}	mm
	welding foil thickness	t_{wf}	mm
material	core material elastic modulus	E_{cm}	MPa
	core material poisson ratio	μ_{cm}	-
	core material shear modulus	G_{cm}	MPa
	core material mass density	ρ_{cm}	kg/m³
	welding foil elastic modulus	E_{wf}	MPa
	welding foil poisson ratio	μ_{wf}	-
	welding foil shear modulus	G_{wf}	MPa
	welding foil mass density	ρ_{wf}	kg/m³
	finishing foil elastic modulus	E_{ff}	MPa
	finishing foil poisson ratio	μ_{ff}	-
	finishing foil shear modulus	G_{ff}	MPa
	finishing foil mass density	ρ_{ff}	kg/m³
	Twintex matrix elastic modulus	E_{tm}	MPa
	Twintex matrix poisson ratio	μ_{tm}	-
	Twintex matrix shear modulus	G_{tm}	MPa
	Twintex matrix mass density	ρ_{tm}	kg/m³
	Twintex fibre elastic modulus	E_{tf}	MPa
	Twintex fibre poisson ratio	μ_{tf}	-
	Twintex fibre shear modulus	G_{tf}	MPa
	Twintex fibre mass density	ρ_{tf}	kg/m³
	Twintex fibre volume fraction	v_{tf}	%

Table 1: Design parameters of a Monopan panel.

For example, from an analysis of 30 views, the thickness of the polypropylene filled Twintex fabric is determined. The corresponding histogram is shown in figure 4. A mean value of 0.703 mm is obtained from the measurements but the histogram clearly shows that there is indeed large scatter, which results in a wide probability interval for this parameter. Other parameters can be measured more easily. Figure 5 shows the histogram of the inner diameter of the cylindrical honeycomb core cells. This was experimentally determined with a 3D CNC measurement bench; 240 cells were measured, each determined diameter resulting from the least squares best fit through 10 measurement points on the cell circumference. For this parameter a mean value of 7.84 mm is found from the measurement. Figure 5 clearly shows that the relative uncertainty on this parameter is very small in comparison to the uncertainty on the previously determined Twintex thickness. In the context of determining design parameter values and their variability, it has to be mentioned that at this stage of the research only global parameter variability is studied.

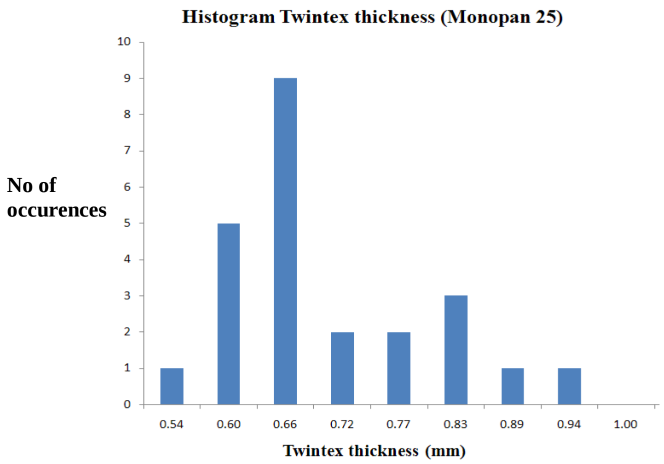


Figure 4. Histogram of measured Twintex thickness

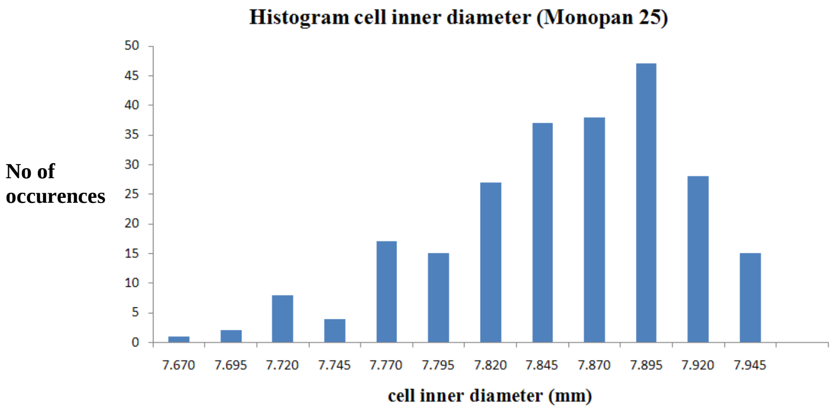


Figure 5. Histogram of measured cell inner diameter

At this stage the focus is on the determination of mean parameter values, together with their respective probability intervals. Both are needed to build a good mean computational model and to be able to update the model by tuning the input design parameters within their specific probability interval. The spatial parameter distribution within a certain panel (intra-variability) or the distribution between different panels of the same kind (inter-variability) has not been studied yet.

2.2 The finite element model

A physically realistic FE model of a honeycomb sandwich panel is very large. Even for a small size panel this approach yields a very high number of finite elements and nodes, making these models computationally hard to solve. However, on a unit cell of the honeycomb panel this method is applicable.

A more suitable method of modelling a honeycomb sandwich panel is to use some degree of homogenisation. A full homogenisation of the panel is computationally attractive but this approach yields unsatisfactory results as the differences between calculated and measured natural frequencies increase dramatically with increasing frequency.

The so called SVS–concept is a very good compromise between accuracy and computational efficiency; SVS stands for ‘Shell – Volume – Shell’. This method homogenises the honeycomb core and the skin. As the core height is about the same order as the total panel

thickness, the core is meshed with volume elements. The skin thickness on the other hand, is much smaller than the total panel thickness, so shell elements can be used here.
The honeycomb core with a repetitive cylindrical shape is homogenised as an orthotropic material. To characterise the elastic behaviour of an orthotropic material, 9 independent constants have to be determined. For the honeycomb core of the Monopan panels studied in this article, these 9 constants are determined by modelling a unit cell of honeycomb core material and by loading it with tension and shear along the 3 planes of symmetry. The unit cell is modelled using shell elements and experimentally determined mean parameter values for e.g. core height, cell inner diameter, cell wall thickness and mass density are used. It is shown in figure 6. Note the axes used.

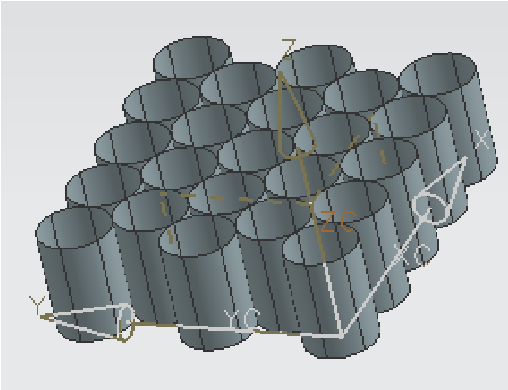


Figure 6. Unit cell of Monopan cylindrical honeycomb material

To validate the results of the core homogenisation, two sets of experiments are carried out. In a first test the elastic modulus along the Z – axis E_3 is determined experimentally by carrying out a set of 50 compression tests. These tests yield 124.1 MPa as a mean value for the homogenised through-the-thickness modulus E_3, while the calculated value is 130 MPa.
In a second test the out-of-plane shear modulus G_{13} is determined. This is done by carrying out 3–point bending tests on Monopan beam samples. Equation 1 [10] expresses the relation between the pure bending stiffness of a sandwich beam and the shear modulus.

$$\frac{d}{FL} = \frac{L^2}{48D} + \frac{1}{4AG} \tag{1}$$

In equation 1, d is the deflection under the centre load F, L is the span length, D is the pure bending stiffness and G is the core shear modulus. A is $\frac{b(t_c + t_s)^2}{t_c}$ where b is the width of the beam and t_c and t_s are respectively the core height and the skin thickness (see table 1). Two sets of 20 bending tests are carried out for span lengths 300 and 500 mm, giving a mean value of 61 MPa for G. The calculated value for G_{13} is 64 MPa.
For the two kinds of tests little difference between calculation and measurement occurs, so the homogenization of the honeycomb material is justified.
The skin properties are determined using a similar procedure. According to Ishai [11], materials reinforced with a woven fabric can be approximated by a laminate structure. If the woven fabric is symmetric, as in the Twintex case, the elastic behaviour of this laminate can be modelled as an orthotropic material. The two other layers in the skin, the welding and finishing foils, are treated as isotropic materials. Eventually the whole skin is homogenised as an orthotropic material as illustrated in figure 7. To model the skin, again 9 independent elastic constants have to be determined.

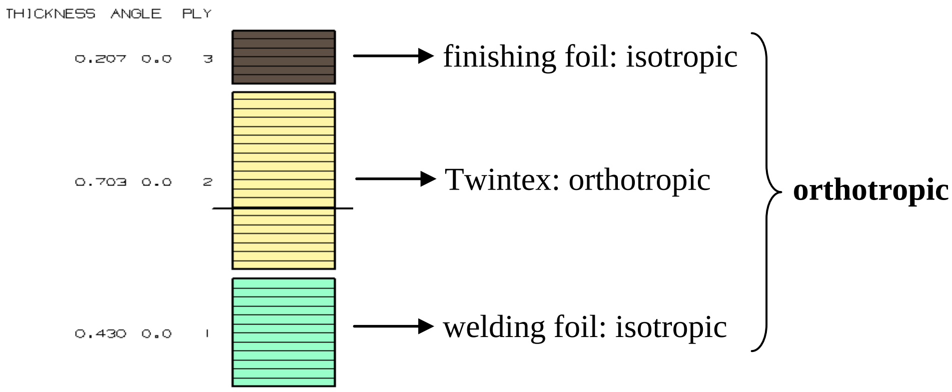


Figure 7. Homogenization of the skin

As the skin is relatively thin in comparison to the thickness of the whole panel and since the skins are located on the outer sides of the sandwich panel, only the in – plane (xy - plane) elastic properties of the skin seem are important for the elastic behaviour of the whole panel, especially the elastic moduli E_{s1} , E_{s2} and the shear modulus G_{12}. Since the Twintex fabric is symmetric, equal moduli in directions 1 and 2 can be expected. To determine this modulus of elasticity a series of 30 tensile tests have been carried out on beam samples (25 mm wide and 200 mm long). The experimental results show a large amount of scatter. This is illustrated by the histogram shown in figure 8. A mean value of 10.8 GPa is found while the manufacturer of the Twintex reinforced Polypropylene specifies a value of 14 GPa.

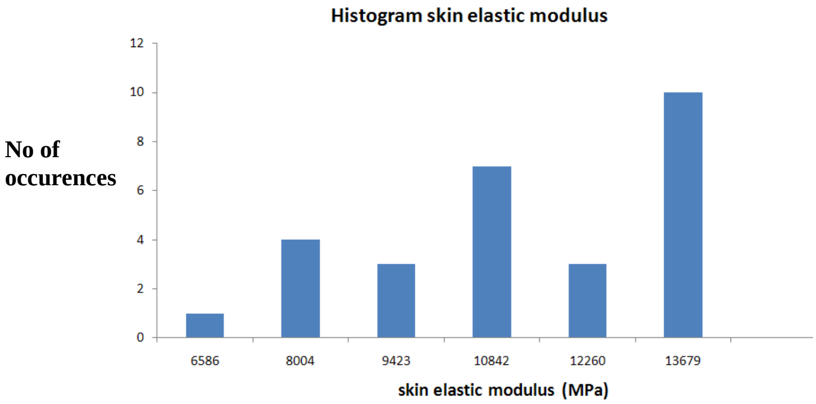


Figure 8. Histogram of measured skin elastic modulus

The final SVS – finite element model thus consists of 3 orthotropic layers. This model still has a rather large number of design variables: 18 elastic constants, 2 mass densities and 4 dimensions (panel dimensions and layer thicknesses). Sensitivity analysis is helpful to find out which of these 24 parameters are dominant for the dynamic behaviour of the honeycomb panels (see section 2.3).
The minimum number of elements in the FE model is determined by checking the convergence of the solutions when increasing the number of elements in the FE model For the A2 size Monopan 15 panel for instance, convergence to a steady solution is reached when 80 x 48 x 9 elements are used.
As mentioned earlier, the finite element model is used to calculate natural frequencies and mode shapes of panels with free-free boundary conditions. These boundary conditions are simple to model and also relatively easy to realise in experiments. For each of the three types of panels 20 modes are calculated. The results are discussed when compared with the experimental results in section 3.

2.3 Sensitivity analysis

As mentioned earlier the current finite element model has 24 input parameters. It is relevant to know which parameters are dominant and which are negligible. Sensitivity analysis is used here for this purpose.

The relative and normalised sensitivity coefficients are determined by varying the design parameters within their probability interval and by calculating the corresponding natural frequencies. For every parameter the probability interval is symmetric and has a width of 4 σ (standard deviation). In this study the sensitivity analysis is restricted to the first 15 natural frequencies of the free vibrating panels. After the sensitivity analysis the original list of 24 model parameters is shortened to a list of 13 (see table 2).

	parameter description	symbol	unit
geometric	overall panel width	W	mm
	overall panel length	L	mm
	skin thickness	t_s	mm
	core thickness	t_c	mm
material	core elastic modulus direction 1	E_{c1}	MPa
	core elastic modulus direction 2	E_{c2}	MPa
	core shear modulus 13 plane	G_{c13}	MPa
	core shear modulus 23 plane	G_{c23}	MPa
	core mass density	ρ_c	kg/m³
	skin elastic modulus direction 1	E_{s1}	MPa
	skin elastic modulus direction 2	E_{s2}	MPa
	skin shear modulus 12 plane	G_{s12}	MPa
	skin mass density	ρ_s	kg/m³

Table 2: Parameters of the finite element model.

The process of homogenisation eliminates some of the design parameters in table 1from the analysis. For instance, the core cell inner diameter d_{ci} is a design parameter from the ‘original’ Monopan panel, while in the homogenised model it has vanished apparently. This cell wall thickness though, has its direct effect on the core mass density ρ_c and the moduli E_{c1}, E_{c2}, G_{c13} and G_{c23} of the homogenised core, which are all parameters of the finite element model. Further sensitivity analyses have to be made to identify the influence between the variation on the original design parameter and the mentioned parameters of the finite element model. Figure 9 for example, shows the relation between the cell inner diameter and the shear modulus G_{c13} of the homogenised core.

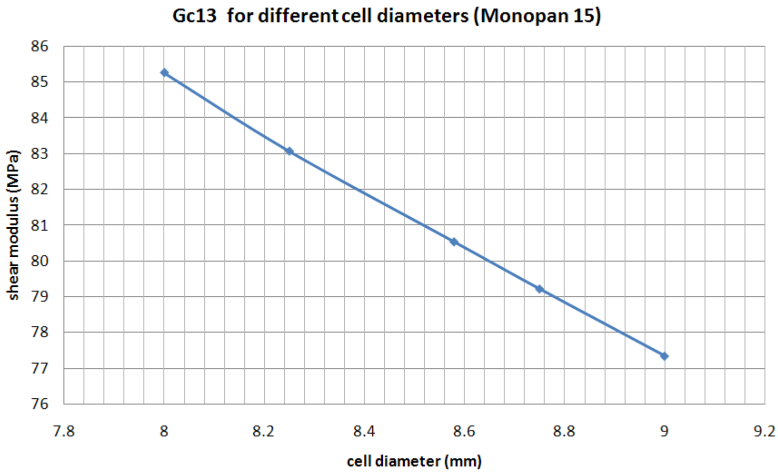


Figure 9. Relation between core cell diameter and G_{c13}

Caution should be made when using the results of a sensitivity analysis because it inherently involves linearisation of the relation between parameters and response. In practice, this means that the interval must not be taken too wide.
It must be mentioned that in this study the finite element model is only used to calculate natural frequencies and mode shapes of panels with free-free boundary conditions in order to obtain a good mean computational model that is useful for studying the influence of parameter variability on the dynamic behaviour of the panels in the first place and to 'filter out' the most important parameters that determine the vibration behaviour of the panels. At this stage of the research parameter spatial variability is not yet included in the FE models.

3. EXPERIMENTAL IDENTIFICATION OF THE DYNAMIC BEHAVIOUR

In this study natural frequencies and mode shapes of Monopan panels with free-free boundary conditions are determined experimentally. Free-free boundary conditions are achieved by suspending the panels by elastic springs. As mentioned earlier, 3 sets of 7 panels each were used for the measurements. At this stage of the research the goal of the measurements is to have an idea of the scatter on the natural frequencies of a set of virtually identical panels. Two measurement techniques are used. In the first technique the suspended panel is acoustically excited with random white noise by means of a loudspeaker and the panel's dynamic response is measured with a laser vibrometer; in this way a fully contactless measurement is obtained. The second technique uses classic hammer excitation. At first sight the contactless method seems preferable because no impact is made to the panel and no transducer mass is added. There is one drawback however. When acoustic excitation is used, the real local excitation force at a certain measurement point is not known. In this case the measured frequency response functions do not represent the correct relation between the structural response and the excitation. This makes that measured mode shapes may be distorted and that measured resonance frequencies are shifted. There for, the classical hammer excitation method is used. Accelerometers with a low mass (< 2 g) are used to measure the response. The two measurement techniques are compared by carrying out a measurement on the same panel. The conclusion is that the contactless method yields resonance frequencies with values between 2 and 10% lower than the corresponding frequencies, determined with the hammer excitation method.
Table 3 gives an overview of the measurement results obtained for the three types of Monopan panels (only results with hammer excitation are shown). In table 3 the mode number indicates a certain mode shape which is the same for the three types of panels. For each mode, the interval width is calculated symmetrically round the measured mean value,

using the minimum and maximum measured frequency. Although only 7 panels of every type have been measured, it becomes clear from table 3 that the interval widths are somewhat related to the mode shape. Further research will provide an explanation for this.

Mode number	Monopan 15 594 x 420 mm		Monopan 25 594 x420 mm		Monopan 25 2500 x 1200 mm	
	mean f	interval	mean f	interval	mean f	interval
	Hz	± %	Hz	± %	Hz	± %
1	81.67	2.14	126.21	2.18	10.49	2.38
2	140.39	5.70	214.36	5.60	11.46	6.54
3	211.97	2.12	317.04	3.51	23.71	3.16
4	276.78	5.28	430.21	4.45	31.93	5.87
5	314.97	3.93	478.75	3.08	45.50	4.12
6	361.83	4.21	521.82	4.14	57.93	3.86
7	418.17	2.75	598.21	1.86	61.89	0.61
8	437.95	1.23	625.63	1.64	92.00	1.90
9	613.14	0.96	858.08	1.76	118.21	1.34
10	675.42	4.66	893.50	0.50	181.64	0.89

Table 3: Overview of the measurement results.

4. NUMERICAL VERSUS EXPERIMENTAL RESULTS AND MODEL UPDATING

The experimentally and numerically determined natural frequencies are compared. With the results of the sensitivity analysis, discussed in 2.3, the finite element model is updated by changing the initial input parameter values for the finite element model within their probability interval. At this stage in the research the specific probability density function for every parameter is not taken into account; a uniform distribution is thus considered along each interval. Incorporating real (spatial) probability functions into the finite elements is subject for future research. When updating the finite element model, the optimisation problem expressed by 2 has to be solved.

$$\text{Minimise:} \quad f_{num,i} - f_{\exp,i} \qquad for\ i = 1 \rightarrow n \tag{2}$$

In equation 2, n is the number of modes used for model updating. In this study 20 modes are considered. The total error for this set of natural frequencies is minimised in a least squares sense. The optimisation constraint requires that the difference between the model panel mass and the real panel mass should be minimum and that design parameters should only be updated within their probability interval.

The results for the first 8 modes of the Monopan 15 panels are given in table 4. The natural frequencies calculated by the finite element model approach the measured frequencies very well. At this stage of the research it is not clear yet to what extent the remaining deviations originate from model inaccuracy, measurement errors or scattered dynamic behaviour between the various panels. Future research will investigate this further.

Calculated and measured mode shapes are also compared using the Modal Assurance Criterion (MAC). This criterion [12] expresses the correlation between two vectors or matrices. Table 4 clearly shows that calculated and measured mode shapes have a very good resemblance.

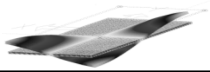
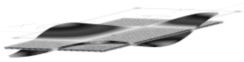
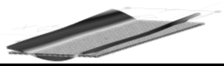
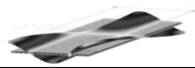
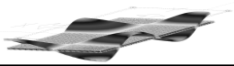
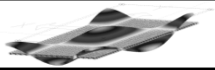
Mode shape	Calculated natural frequency (Hz)	Measured natural frequency (Hz)	MAC	Relative frequency deviation (%)
	80.2	81.7	0.999	-1.8
	137	140.4	0.979	-2.41
	210.5	212	0.997	-0.69
	286.8	276.8	0.983	3.6
	308.2	315	0.997	-2.15
	384.5	361.8	0.936	6.29
	429.4	418.2	0.971	2.69
	450.2	438	0.964	2.8

Table 4: Experimental versus numerical results for Monopan 15 panels.

5. CONCLUSIONS AND OUTLOOK TO FUTURE RESEARCH

This paper presents a first phase of an extensive research on the influence of design parameter variability on the dynamic behaviour of honeycomb sandwich panels. A simple and reliable finite element model, with a minimum number of parameters is presented. With the finite element model the relations between design parameter variation and dynamic response variation are studied. Correlation between real panel design parameters and finite element model parameters is studied.

Future research will focus on the inclusion of realistic parameter variations in the finite element models. Especially the effect of spatial distribution of parameters [13] will be studied. The implementation of design parameter variability will result in a stochastic analysis of the honeycomb panel dynamic behaviour using a random field description (see [14] and [15]), based on variability analysis of the various parameters. Correlation between the various design parameters will have an important effect. Describing design parameters with a certain unknown variability by random fields is an appropriate method to determine real parameter uncertainty, even when only limited experimental data, for example sets of measured frequency response functions, are available. Spatial parameter distribution as well as parameter uncertainty due to lack of sufficient measurement data is dealt with. Research in this area is carried out by Desceliers [16] and Soize [17] and by Perrin and Sudret [18]. So far, only 1D applications and single random parameter problems have been addressed. In future research the use of these methods will be extended to 2D applications with several random parameters taken into account.

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