EVALUATION OF THE HIGHLY STRESSED VOLUME AS DESIGN PARAMETER FOR FATIGUE STRENGTH OF NOTCHED ALUMINIUM SPECIMENS

B. Van Hooreweder, R. Boonen, D. Moens, P. Sas
Department of Mechanical Engineering (PMA), K.U.Leuven, Belgium

Abstract. Stress concentrations are always present in real-life mechanical components, often in the form of grooves, fillets, keyways or holes. Although a variety of methods are available to analyze the influence of these stress raisers on the fatigue life of materials, it is still very difficult to account for this effect. This paper covers the fatigue analysis of notched aluminium components loaded in reversed bending using the highly stressed volume method. Specimens with different notch geometries are tested at high frequency by means of a straightforward test rig. The highly stressed volume is precisely calculated using finite element software in combination with Matlab for post-processing. The endurance limit is determined using a minimum amount of samples and the application of the staircase method. The results indicate that the highly stressed volume is a relevant and efficient design parameter to account for notch-effects in aluminium specimens.

1 INTRODUCTION

Mechanical components that are subjected to fluctuating loading conditions can fail at low stress levels due to irreversible microplastic deformation followed by crack growth and final failure. Crack initiation is most likely to start at places with high local stress amplitudes e.g. in the vicinity of stress raisers or notches. Unfortunately, macroscopic notches are always present in real-life applications, often in the form of grooves, holes, keyways and fillets.

It is generally agreed that the use of the theoretical stress concentration factor (k_t) yields non-conservative results for fatigue data of ductile materials or when sharp notches are present. In these cases, the fatigue notch factor ($k_f < k_t$) is used. Over the last years, many methods were developed to calculate k_f based on statistics, crack growth or reversed yielding [1-4]. However, none of these methods is suitable for all fatigue cases, often leading to the use of empirical formulas to cope with the influence of stress raisers on the fatigue life of different materials. One of these empirical concepts is the highly stressed volume method, originally developed by Kuguel in 1960 [5]. The highly stressed volume is defined as the small volume which is subjected to more than 90% of the maximum stress occurring in the notch. Kuguel's equation shows a linear relationship between the logarithm of this critical volume and the logarithm of the local endurable stress amplitude. This power law is shown below for two specimens (0) and (1) with different notch geometries.

$$\sigma_{a(1)} = \sigma_{a(0)} \times \left(\frac{V_{90\%(1)}}{V_{90\%(0)}} \right)^m \quad \text{with} \quad m < 0 \quad (1)$$

This empirical concept based on a critical material volume can be clarified as follows. Fatigue crack initiation is associated with cyclic slip in the material. In order for this slip to occur, the shear stress on the active plane should reach a certain critical value. Consequently, not only the peak stress at the root of the notch is important but also the stress acting on lower material particles, i.e. in a small volume close to the stress raiser. In the case of a sharp notch or high stress concentration, a steep stress gradient exists, leading to a small critical volume. Whereas in the case of a mild notch, a less steep stress gradient is present, leading to a larger volume subjected to more than 90% of the maximum stress. The larger the critical volume, the higher the probability for crack initiation to start ($k_f \sim k_t$). The smaller the critical volume, the lower the chance for crack initiation ($k_f < k_t$). Because of this logical reasoning and its general simplicity, the highly stressed volume method can be an adequate design tool for fatigue design of mechanical components. Moreover, the use of finite element software to calculate the critical volume can increase the accuracy and relevance of this method considerably. This is also shown and illustrated in a recent paper of G. Härkegård et al. based on test data by Böhm and Magin [6]. In this study, the $V_{90\%}$-method yields better results than a variety of other methods for the prediction of notch effects in low-alloy steel specimens. However, very little has been reported in literature on the application of the highly stressed volume for fatigue analysis of aluminium alloys. In the present work, a first attempt is made to address this by analyzing the fatigue strength of notched aluminium components loaded in reversed bending. The endurable local stress amplitude and the highly stressed volume are determined with high accuracy as described in the following subsections. Furthermore, micrographs are presented and the fractured surfaces of the specimens subjected to constant amplitude bending loading are analysed. To account for variable amplitude loading, a numerical fatigue calculation is performed using fe-safe software.

2 MATERIALS AND METHOD

The geometry of the test specimens is indicated in Figure 1. All specimens are milled from one block of aluminium EN AW 7075-T7351. The longitudinal direction of the test pieces is chosen in the rolling direction of the original block and special tooling is used to mill the V-type notch on one side of the specimen. The left part (12mm) of the geometry is used to clamp the specimen while the end of the specimen (8mm) is used to impose a fluctuating force in the vertical direction. To satisfy Saint-Venant's principle, the notch is located at sufficient distance from the clamping area. Three different radii (Rx) are used for the V-notches in order to create three different stress concentration factors. The notch radii and the stress concentration factors as calculated by Peterson’s formulas [7] are indicated in Table 1.

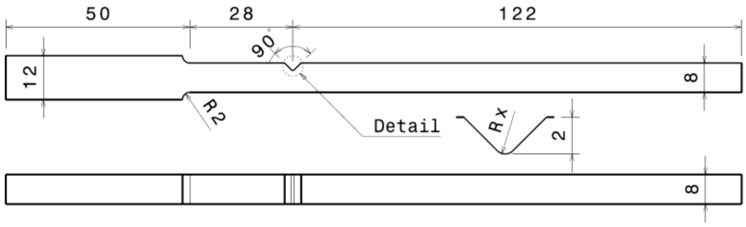

Figure 1. Test specimen geometry with varying radius at the root of the notch

Rx (mm)	$k_{t,analytical}$
0.4	3
0.6	2.5
1.2	2

Table 1. Notch radii and respective stress concentration factors

All specimens are tested using the experimental setup as illustrated in Figure 2 using a clamping system as shown in Figure 3. A permanent magnetic vibration exciter (1) is connected to the test specimen (5) by means of a stinger (2) and a force cell (3). The stinger is stiff in the vertical direction but allows for deflection in the horizontal direction to permit bending of the test piece. A concrete block with steel upper plate (8) is used to connect the clamping plates (7) by means of four M12 bolts pretightened at 50Nm. Both clamping plates have a double groove as indicated at the bottom of Figure 2. As a result, the connection between the test specimen and the steel plates is made over four small and accurate surfaces and a very rigid connection is made. A laser is used to align the exciter with the test specimen in order to avoid secondary torsional loading during operation.

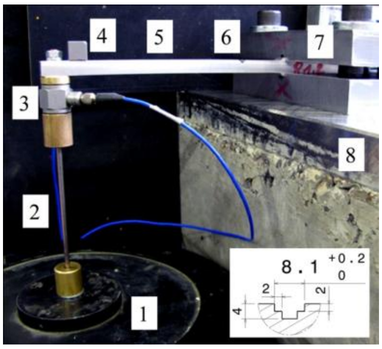

Figure 2. Experimental setup

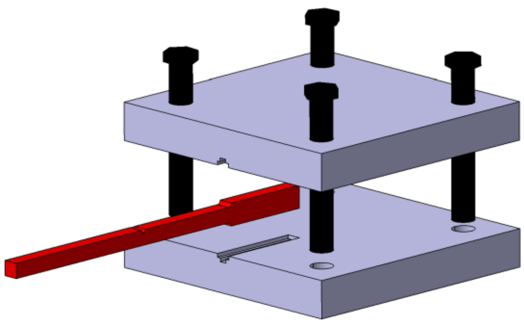

Figure 3. Specimen clamping tools

The control and monitoring of the test is done with LMS Test.Lab software in combination with the Scadas III data acquisition system. All specimens are subjected to force controlled constant amplitude sinusoidal loading in bending. No mean stress is applied (R = -1) and a test frequency of 75Hz is used. This is close to the first eigenfrequency of the system. As a result, high bending stresses are achieved with low power requirements for the electrodynamic shaker. However, if a fatigue crack starts to grow in the notch of the sample, the global bending stiffness of the specimen decreases leading to a decrease in the eigenfrequencies of the system. Since the excitation frequency is constant, a shift in eigenfrequencies will cause the resulting force on the test specimen to lower significantly. To avoid this, closed loop single axis waveform replication (SAWR) is used [8].

3 ENDURANCE LIMITS

The endurance limit is determined in purely reversed bending using the setup as described in the previous section. In the finite fatigue region (10^4 - $2.5x10^6$) three specimens are tested at three or four stress levels for each specimen. In the infinite fatigue region ($2.5x10^6$ - 10^7) the staircase method, also known as the up-and-down method is used. This method was originally developed by Dixon and Mood in 1948 for explosives research and is now a commonly used method for determining statistical characteristics of the fatigue strength based on the maximum-likelihood estimation principle [9-12]. The primary advantage of this method is that it automatically concentrates testing near the mean. According to Dixon and Mood, 30 to 40% fewer tests are required with respect to ordinary methods of testing groups of equal size at preassigned stresses. To start, the fatigue endurance limit is estimated using available fatigue data in literature [3,13,14] and the first test is executed. If the specimen fails prior to 10^7 cycles, the following experiment is performed at lower stress amplitude. If the specimen survives 10^7 cycles, the test is stopped and the next experiment is run at higher stress amplitude. By doing so, each experiment is dependent on the result of the previous test. Assuming the variation of the applied fatigue limit to be normally distributed and using a fixed interval between the stress levels, the mean (M) and standard deviation (S_{dev}) of the fatigue limit are estimated using the following equations:

$$M = S_0 + S_d\left(\frac{A}{N} \pm \frac{1}{2}\right) \quad (2)$$

$$S_{dev} = 1.620 \times S_d\left(\frac{BN - A^2}{N^2} + 0.029\right) \quad if \quad \frac{BN - A^2}{N^2} \geq 0.3 \quad (3)$$

$$S_{dev} = 0.530 \times S_d \quad if \quad \frac{BN - A^2}{N^2} < 0.3 \quad (4)$$

$$with \quad A = \sum_{i=\text{à}}^{i_{max}} i \times n_i \;,\; B = \sum_{i=\text{à}}^{i_{max}} i^2 \times n_i \;,\; N == \sum_{i=\text{à}}^{i_{max}} n_i \quad (5)$$

For calculating the mean, the plus sign is used if the more frequent event is survival and the minus sign is used if the more frequent event is failure. Furthermore, 'i' represents the stress level, where i = 0 for the lowest stress level 'S_o', 'n_i' indicates the number of events recorded at each stress level and 'S_d' represents the constant stress increment [9]. The counted events represent either the failures or the survivals, depending on which has the smaller total. If survival or run out is the less frequent event, then the lowest stress level at which this occurs corresponds to the i = 0 level and n_i corresponds to the number of specimens which survives stress level i. If failure is the less frequent event, then the lowest stress level at which a failure was observed is the i = 0 level and n_i corresponds to the number of specimens which failed at stress level i. One of the conditions of the Dixon and Mood theory is that the sample size should be in the order of 40-50 specimens in order to use large sample theory on which the analysis is based. Additional research by Brownlee et al. for the application of the Dixon and Mood theory to the mean endurance limit has shown that the method is reliable even in samples as small as 5-10 [10]. In this study, a sample size of 8-11 specimens is used.

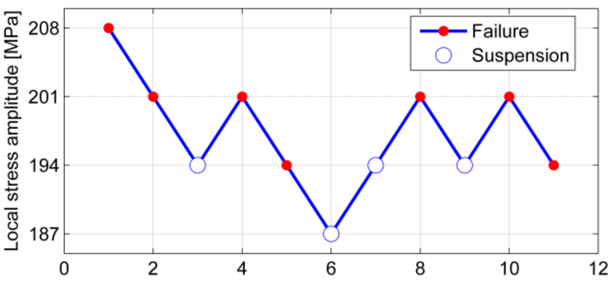

Figure 4. Staircase method for k_t = 2

$k_{t,analytical}$	A	B	N	$M_{loc.}$ (MPa)	$S_{dev,loc.}$ (MPa)
3	4	8	3	243	10.4
2.5	3	5	3	216.2	7.9
2	3	3	4	189	3.7

Table 2. Results of the determination of the local endurable stress amplitude

Figure 4 shows the failures and suspensions for the test pieces with stress concentration k_t = 2. It this case, 11 experiments are performed to determine the mean (M) and standard deviation (S_{dev}) of the local endurance limit using equations (2) and (4). The results for all the specimens are indicated in Table 2 and the Wöhler curves for the three notch geometries are shown in Figure 5. In the finite fatigue region, only the average fatigue life of three samples at a certain stress level is indicated. The least squares method is used to fit three power functions in this region of which the equations are indicated in the figure.

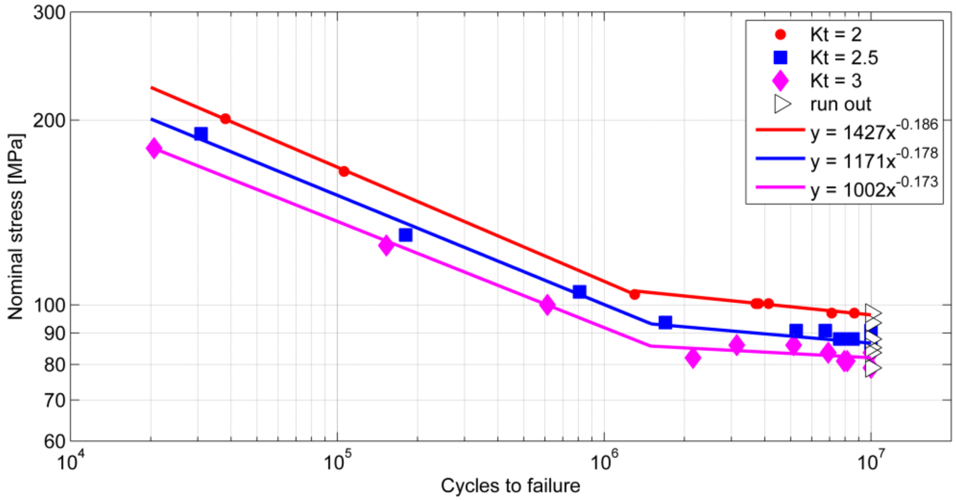

Figure 5. Wöhler curves for notched aluminium 7075 T7351

4 HIGHLY STRESSED VOLUME

Originally, R. Kuguel used a triangular approximation to estimate the highly stressed volume of a notched specimen [5]. The use of numerical finite element software enhances the accuracy of Kuguel's method considerably. In this study, MSC Patran/Nastran software is used for a 2D simulation using quadratic triangular finite elements [15]. First, a two dimensional FE-model is created using symmetrical boundary conditions to reduce the computational effort. A very fine mesh with element size ranging from 0.002mm close to the notch, to 1mm further away from the notch is applied and a linear static finite element analysis is performed. To avoid an incorrect stress field near the notch caused by finite element imperfections, the mesh size is gradually refined while monitoring the maximum stresses near the notch. The maximum principle stress in the centre of the elements is than exported to Matlab together with the area and position of every element. After sorting these parameters, elements close to the clamping- or force acting area are removed and the maximum principal stress (σ_{max}) in the remaining elements is calculated. Then the summation is made of the areas of all elements that are subjected to a stress level between 0.9 σ_{max} and σ_{max}. To calculate the highly stressed volume ($V_{90\%}$), this area is multiplied with the thickness of the specimen. The last step is to plot the elements which are subjected to >0.9 σ_{max} in order to verify their position.

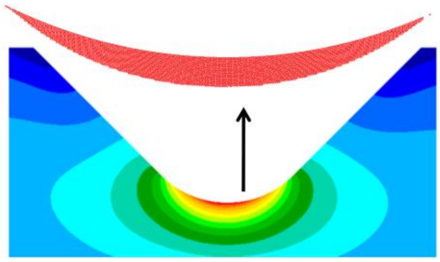

Figure 6. Highly stressed volume

$k_{t,analytical}$	$k_{t,numerical}$	$V_{90\%}$ (mm³)	elements
3	3.05	0.0277	4045
2.5	2.58	0.0610	7512
2	1.98	0.2159	43029

Table 3. Results of the calculation of $V_{90\%}$

Table 3 gives an overview of the critical volumes for the three stress concentration factors. Also the acceptable correlation between the analytical stress concentration factor as calculated by Peterson and the numerical stress concentration factor is indicated. As shown in Table 3 and Figure 6, the FE model contains a sufficient number of elements within the highly stressed volume to approximate the curvature of this volume with acceptable accuracy. The total computational time for the finite element model and post processing is less than 15 minutes on a conventional desktop computer. Finally, the relation between the mean values of the local endurable stress amplitudes and the corresponding highly stressed volumes can be plotted on a double logarithmic scale as shown in Figure 7. The least square method is used to fit a power function of which the equation is indicated in the figure. It is clear that this data can be approximated as a straight line on the double logarithmic scale. Consequently, the presented results can be used for fatigue life estimations of notched aluminium specimens or engineering components and equation (1) can be successfully applied with m = -0.118.

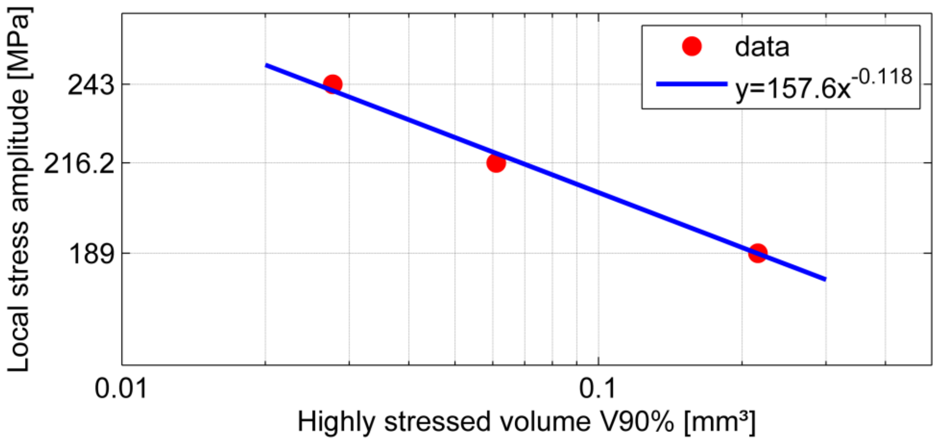

Figure 7. Local stress amplitude vs highly stressed volume

5 MICROGRAPHS

The fracture surfaces of the various specimens where studied using an electron microscope. Figure 8 shows the results of this analysis for one specimen subjected to a constant amplitude load in bending. Figure 8A presents a fractured cross section of the specimen at low magnification with the root of the notch clearly visible in the upper part of the micrograph. A flat fracture surface is observed with a distinctive crack propagation pattern from top to bottom. In all cases, the crack started from the root of the notch and grew further perpendicular to the notch root surface. At higher magnifications (Figure 8B,C), the typical fatigue striations become visible. Consecutive striations are equally spaced as expected due to the constant amplitude loading [16]. Figure 8D shows the finale rupture of the specimen. No dimples can be observed, indicating brittle material behaviour during final fracture.

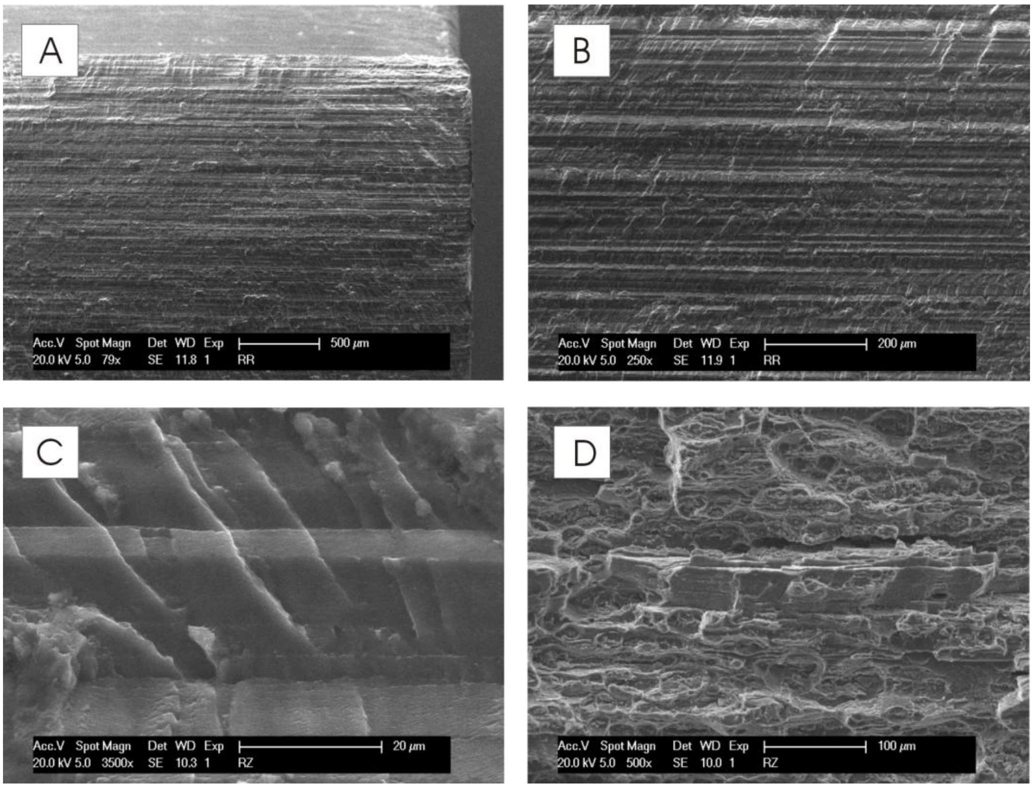

Figure 8. Microscopic analysis of the fracture surfaces

6 VARIABLE AMPLITUDE LOADING

In real-life applications, mechanical components are often subjected to variable amplitude loading conditions. The aluminium 7075 alloy for example is widely used in the design of various parts for airplane wings. These parts are subjected to changing loading amplitudes during takeoff, flight, manoeuvring and landing. For aircraft fatigue design, standardized load spectra can be used to account for the irregular loading histories. In this study, the Falstaff spectrum [17] (Fighter Aircraft Loading Standard for Fatigue) is applied on the test specimens with a stress concentration k_t of 3 using the electrodynamic shaker and SAWR. One load spectrum takes 8.25 seconds and represents one flight consisting of 32 force levels ranging from 3.4N tot -40N. This corresponds to a normal stress of 26.1MPa in compression and 300MPa in tension at the root of the notch. To verify that the applied load spectrum corresponds to the loading at the notch of the test piece, two strain gages are mounted on the specimen near the notch. A Wheatstone bridge is used in ½ tension/compression configuration to eliminate temperature effects and to increase the signal-to-noise ratio. Figure 9 shows the good correspondence between the Falstaff input and the strain output. The resulting fatigue life of five identical specimens equals 61530 ± 8974 flights.

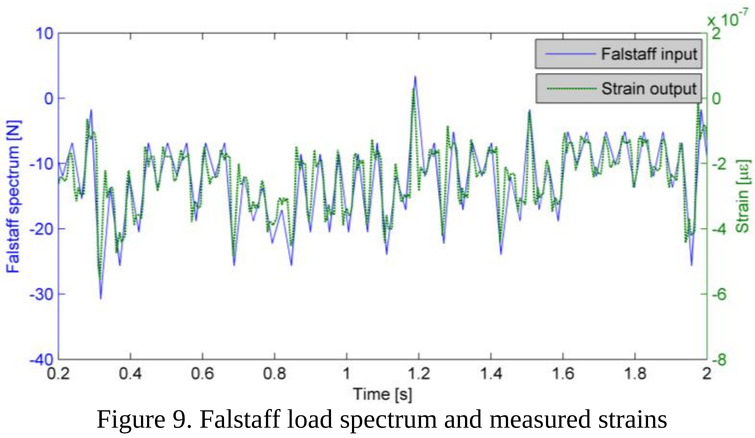

Figure 9. Falstaff load spectrum and measured strains

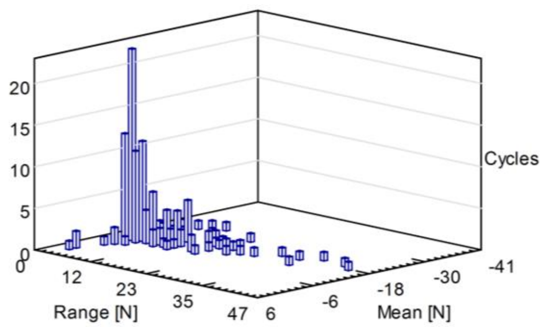

Figure 10. Rainflow cycle histogram

It is clear that for these experiments in the finite life region, the highly stressed volume method is not applicable for fatigue life predictions. Since the notch-effect is included in the experimental data of the constant amplitude tests, numerical fatigue software can be used to account for the variable amplitude loading. In this study, the fe-safe solver is used in combination with a linear elastic MSC Patran/Nastran finite element model of the test specimen and the SN-data as described in previous sections. Since limited fatigue data is available, only a few stress based analysis algorithms can be used. The 3D linear elastic finite element model consists of quadratic tetrahedron elements and a unit load that represents the bending force applied by the electrodynamic shaker. This loading is uni-axial but near the notch a multi-axial stress state is present. Therefore, the biaxial analysis algorithm based on principal stress is used for fatigue life calculations. The principal stresses of each finite element node as calculated from the Patran model are imported in fe-safe and multiplied with the Falstaff time history. The rainflow cycle counting principle is applied (Figure 10) and the Palmgren Miner rule is used for linear damage accumulation. Furthermore, the theory of Walker is used to account for the effect of the mean stress. The fatigue life is calculated at each finite element node on eighteen planes perpendicular to the surface and spaced at 10 degree increments [18]. The life of the most critical planes is exported to a Patran op2 results file which can be post-processed in Patran as indicated in Figure 12. The calculated fatigue life equals 75100 flights which is within acceptable range of the experimentally determined fatigue life of 61530 ± 8974 flights.

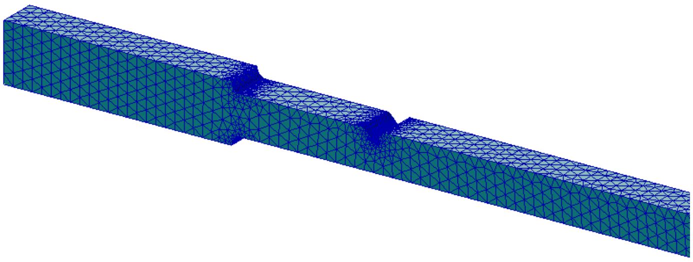

Figure 11. Finite element model

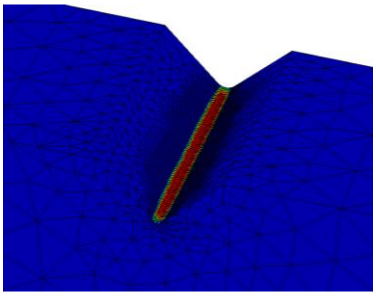

Figure 12. Results of numerical fatigue life calculation

Since aluminium is a ductile material and the applied load causes relatively high stresses, it would be more appropriate to use a biaxial strain based analysis algorithm such as the Brown Miller method for calculating the fatigue life. This algorithm uses critical planes perpendicular to the surface and at 45° to the surface. On each of these three planes, fatigue lives are calculated on subsidiary planes, spaced at 10 degree increments. Both the normal and the shear strain are used to calculate the fatigue life. However, in order to be able to use this algorithm, strain-based material data is required. Future work is needed to verify if the simulation with the Brown Miller method yield better results for fatigue life predictions of the 7075 T7531 aluminium alloy.

7 CONCLUSION

This research indicates that the highly stressed volume can be used as an efficient fatigue design parameter for notched aluminium 7075 T7531 specimens and engineering components. The applied method is straightforward and only requires a limited amount of fatigue experiments of at least two different notch geometries. The critical volume of these specimens can be determined using commercially available finite element software and straightforward post processing in Matlab. The fatigue endurance limits are determined using a minimum amount of samples, the staircase method and the Dixon en Mood theory for statistical analysis. The result is a linear correlation between the highly stressed volume and the local endurable stress amplitude on a double logarithmic scale. Hence, the fatigue limit for intermediate stress concentration factors can easily be determined by calculating the highly stressed volume. Furthermore, the experimental stress based fatigue data is successfully used as starting point for a numerical fatigue life prediction of notched specimens subjected to variable amplitude loading.

8 ACKNOWLEDGEMENTS

This research is funded by a Ph.D grant of the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen). The author also gratefully acknowledges Ir. F. Vandeput from Sabca for the material and manufacturing of the test specimens and ing. L. Vanton and ing. W. Eraly for their effort in the experimental procedure.

9 REFERENCES

1- N.E. Dowling, Mechanical behaviour of materials: Engineering Methods for Deformation, Fracture, and Fatigue, Pearson Prentice Hall, 2007.
2- E. Zahavi, V. Torbilo, Fatigue Design: Life Expectancy of Machine Parts. CRC press, 1996.
3- FKM-guideline, Analytical Strength Assessment of Components in Mechanical Engineering, VDMA, 5th revised edition, 2003.
4- J. Schijve, Fatigue of structures and materials, Springer, 2009.

5- R. Kuguel, The highly stressed volume of material as a fundamental parameter in the fatigue strength of metal members, T.A.M. report no. 169, 1960.
6- G. Härkegård, G. Halleraker, Assessment of methods for prediction of notch and size effects at the fatigue limit based on test data by Böhm and Magin, Int. J. Fatigue 32 (2010) 1701-1709.
7- Peterson RE., Stress concentration factors, London, Chapman & Hall, 1974.
8- LMS International, The LMTest.Lab Single Axis Waveform Replication manual, Rev 10B, 2010.
9- Dixon WJ, Mood AM., A method for obtaining and analyzing sensitivity data. J. Amer. Stat. Assoc. 43 (1948) 109–126.
10- Brownlee, K.A., Hodges Jr., J.L., Rosenblatt, M., The up-and-down method with small samples, J. Amer. Stat. Assoc. 48 (1953) 262-277.
11- Lin, S.K., Lee, Y.L., Evaluation of the staircase and the accelerated test methods for fatigue limit distributions, Int. J. Fatigue 23 (2001) 75–83
12- R. Pollak, A. Palazotto, T. Nicholas. A simulation-based investigation of the staircase method for fatigue strength testing, Mechanics of Materials 38 (2006) 11701-1181.
13- H. E. Boyer, Atlas of fatigue curves, ASM International, 1986.
14- A. Buch, Fatigue data handbook, CRC press, 1998, pp 85.
15- MSC Software Corporation, USA, Sana Ana, CA92707, MSC Fatigue user's manual, version 2005.
16- B.M. Strauss, W.H. Cullen, Fractography in failure analysis, ASTM, Philadelphia, 1978.
17- G.M. Van Dijk, Introduction to a Fighter Aircraft Loading Standard for Fatigue Evaluation, Falstaff:, NLR MP 75017U, National Aerospace Laboratory, Amsterdam, The Netherlands, 1975.
18- Safe technology, Sheffield, UK, fe-safe user's manual, volume 1, issue 13, 2011.