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THE FIRST RESOURCE FOR MATHEMATICS

Katzourakis, Nikos; **Vărvărucă, Eugen**

An illustrative introduction to modern analysis. (English) Zbl 1495.26002

Boca Raton, FL: CRC Press (ISBN 978-1-138-71827-2/hbk; 978-0-367-65741-3/pbk; 978-1-351-76533-6/ebook). xv, 541 p. (2018).

This is an introduction to mathematical analysis intended for first year university students who will need some deeper concepts for modern treatments of applied mathematical problems like e.g. partial differential equations, or variational, harmonic, geometric, or numerical analysis.

The necessary toolbox for the applied mathematician is covered: metric, normed, and inner product spaces, continuity, convergence, limits, (linear) operators and functionals, Lebesgue integration and the book concludes with the classical L^p spaces. Most of the analysis is in abstract spaces often having $\mathbb{R}^m$ in mind. When the underlying field is $\mathbb{C}$ instead of $\mathbb{R}$, the reader is sometimes referred to it being similar to $\mathbb{R}^2$.

The reader should be warned that this is not an engineering type of approach, although the authors try to maintain the geometric intuition that the reader may have from an elementary course in univariate real analysis. However to lift this intuition to a more abstract generalization, all the mathematical nuts and bolts of the geometric concept are unscrewed and carefully re-assembled in the abstract setting. As the result depends strongly on the topology that the spaces are equipped with, topology plays a key role throughout the book.

The first 90 pages deal with topics that are probably somewhat familiar (sets, mappings, metric, Euclidean spaces, Banach spaces). Then, more abstract topology becomes central to define continuity and compactness, measure theory (in particular Lebesgue integrals, and Lebesgue functional spaces), and linear operators on normed spaces. Before a full analysis of the classical L^p spaces is discussed, the weak and weak* topologies are introduced.

Almost all the theorems have full elaborated proofs, and the reader is introduced to all the fundamental theorems (Baire, Banach-Steinhause, Carathéodory, Hahn-Banach, Heine-Borel, Picard, Dominated convergence,...). Each chapter has a set of exercises and about 100 pages of solutions are collected at the end of the book.

A distinctive particularity of this book is that the authors take special care to explain why some complication or abstraction is needed, with some examples illustrating the concept or counter examples of what could happen if a condition were not introduced. So every chapter, and many sections start by explaining which questions they want to find answers for, and what the successive steps are to reach these answers. Hence it is not only fit to serve as lecture notes but it is suited for self-study as well.

An index for a book like this is essential to retrieve all the different definitions and theorems when needed. Unfortunately, the pages in the index refer to the true pages plus 14, which is probably due to the first 14 pages numbered i–xiv, before the roman numbering starts. So, it requires a small mental computation each time you want to retrieve something via the index.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

26-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to real functions
46-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to functional analysis

Cited in **5** Documents

Keywords:

compactness; Lebesgue integral; measures; inner product spaces; linear operators; L^p spaces; metric and normed spaces; convergence; topology

Full Text: Link

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