Nash bargained consumption decisions:
A revealed preference analysis

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Abstract

We present a revealed preference analysis of the testable implications of the Nash bargaining solution. Our specific focus is on a two-player game involving consumption decisions. We consider a setting in which the empirical analyst has information on both the threat points bundles and the bargaining outcomes. We first establish a revealed preference characterization of the Nash bargaining solution. This characterization implies conditions that are both necessary and sufficient for consistency of observed consumption behavior with the Nash bargaining model. However, these conditions turn out to be nonlinear in unknowns and therefore difficult to verify. Given this, we subsequently present necessary conditions and sufficient conditions that are linear (and thus easily testable). We illustrate the practical usefulness of these conditions by means of an application to experimental data. Such an experimental setting implies a most powerful analysis of the empirical goodness of the Nash bargaining model for describing consumption decisions. To our knowledge, this provides a first empirical test of the Nash bargaining model on consumption data. Finally, we consider the possibility that threat point bundles are not observed. This obtains testable conditions for the Nash bargaining model that can be used in non-experimental (e.g. household consumption) settings, which often do not contain information on individual consumption bundles in threat points.

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1 Introduction

Bargaining models describe decision processes that simultaneously involve multiple players. They define the outcome of such a process by using information on a bargaining set, which includes all attainable utility levels for every player, and a set of threat (or disagreement) points. In the literature, the Nash bargaining model is by far the most popular one.[1] For example, it has been used for describing household decision making, firm-union wage negotiations, job-matching and job-search models, international trade and oligopolistic competition. The Nash bargaining solution is then mainly used for its theoretical convenience: studies usually assume the model without empirical motivation. Somewhat surprisingly given its widespread use, relatively few studies have actually focused on the testable implications of the Nash bargaining solution.

In this paper, we concentrate on the testable implications of the Nash bargaining model for a two-player game involving consumption decisions on bundles of goods. The distinguishing feature of our study is that we build on the revealed preference characterization of the model. As we will discuss below, this revealed preference approach has some particularly attractive features for empirically testing a specific behavioral model. We demonstrate the practical usefulness of the approach by an application to experimental data. We conduct a specially tailored experiment that implies a most powerful analysis of the Nash bargaining model as a tool for describing decisions on consumption bundles. To our knowledge, this provides a first empirical test of this model in a consumption setting. Next, we also discuss the applicability of our revealed preference approach to observational (or non-experimental) data, which can be useful for household consumption analysis on the basis of the Nash bargaining model.

Testable implications of the Nash bargaining solution. Starting with Manser and Brown (1980)'s seminal contribution, a few studies have focused on the testable implications of the Nash bargaining solution for consumption decisions.[2] A common feature of these studies is that they follow a differential approach, which concentrates on properties of functions representing the primitives of the decision process (e.g. individual preferences).[3] Empirical applications of this approach then usually require some (non-verifiable) a priori specification of these functions. And, thus, testing consistency of observed behavior with the Nash bargaining model is always conditional upon this specification. This will imply a basic difference with our further analysis, which follows a revealed preference approach rather than a differential one.

[1]Other frequently used models are the generalized Nash bargaining model Harsanyi and Selten (1972), the Raiffa-Kalai-Smorodinsky model (Raiffa (1953), Kalai and Smorodinsky (1975)), the egalitarian model (Kalai (1977), Roth (1979)) and the equal sacrifice model (O'Neill (1982), Aumann and Maschler (1985)). In this respect, see also our discussion in the concluding section.

[2]See, for example, Manser and Brown (1980), McElroy and Horney (1981), Ulph (1988), McElroy and Horney (1990), McElroy (1990), Lundberg and Pollak (1993), Konrad and Lommerud (2000) and Chen and Woolley (2001).

[3]The term 'differential' then refers to the fact that this approach focuses on properties obtained by integrating and/or differentiating these functions.

Another notable difference pertains to the fact that existing studies typically do not present a characterization of the Nash bargaining model. Rather, they focus on explaining deviations between behavior consistent with the Nash-bargaining model and behavior consistent with maximizing a single utility function (which follows the so-called 'unitary' consumption model, with the well-known Slutsky conditions as a differential characterization). In this respect, one notable exception is the study of Chiappori, Donni, and Komunjer (2011) . These authors do provide a characterization of the Nash bargaining solution. But, again, their analysis differs from ours in that it follows a differential approach. In addition, Chiappori, Donni, and Komunjer focus on a slightly different setting than we do: contrary to most of the above mentioned studies, they consider testable implications of the Nash bargaining solution for the problem of sharing a pie (e.g. budget sharing) rather than for consumption decisions involving bundles of goods.

When it comes to testable implications of the Nash bargaining solution, threat points always play a crucial role.[4] These threat points are the outcomes of individual players in the case no agreement is reached, and are also referred to as disagreement points. In this study, we carry out a specific experiment that naturally allows for obtaining information on threat point consumption bundles. As we will indicate in Section 4, this entails a very powerful analysis of the empirical goodness of the Nash bargaining model. Importantly, in Section 5 we also discuss the possible extension of our basic framework to situations in which the threat point bundles are not observed. This will obtain testable conditions for the Nash bargaining model that can be used in non-experimental (e.g. household consumption) settings, which often do not contain information on individual consumption bundles in threat points.[5]

Revealed preferences. We adopt a revealed preference approach in the tradition of Samuelson (1938), Houthakker (1950), Afriat (1967), Diewert (1973) and Varian (1982). In contrast to the differential approach, this approach does not require any functional specification prior to the analysis. By contrast, it obtains testable conditions that can be verified by (only) using a finite set of consumption observations (i.e. prices and quantities). This avoids that a particular behavioral model (such as the Nash bargaining model) is rejected because of an erroneous specification (while the actual consumption behavior is consistent with the model).

Another advantage of the revealed preference approach is that it can be meaningfully

[4]See Chiappori (1988), McElroy and Horney (1990), McElroy (1990), Chiappori (1990) and Xu (2007) for a thorough discussion of this point. In the sequel, we will follow the most common practice to consider threat points as the outcome of individual players when they can spend some individually assigned budget (under disagreement). Some authors adopt a slightly different viewpoint and assume that players reach a noncooperative Nash equilibrium in the disagreement case . See, for example, Ulph (1988), Lundberg and Pollak (1993), Konrad and Lommerud (2000) and Chen and Woolley (2001).

[5]In some real life settings, information on the position of threat points (and corresponding testable implications) can be retrieved from environmental variables (e.g. prices, incomes and the so called extra-environmental parameters (EEP) as termed by McElroy and Horney (1981) or distribution factors in the terminology of Browning, Bourguignon, Chiappori, and Lechene (1994)). But we do not follow this route here.

applied to small data sets. For our setting, this means that we can fruitfully use our revealed preference conditions for testing the Nash bargaining model even with only a few consumption observations. As such, we avoid (often debatable) preference homogeneity assumptions across individual players. Specifically, in Section 4 we will show that our revealed preference tests have satisfactory discriminatory power for (only) 9 consumption observations per dyad (i.e. two-player group).

Our study also complements a recent strand of literature that focuses on revealed preference analysis of decision processes with multiple players. More specifically, Cherchye, De Rock, and Vermeulen (2007, 2011a) derived a revealed preference characterization of the collective model, which assumes a Pareto optimal solution, and Cherchye, Demuynck, and De Rock (2011b) provided a revealed preference characterization of noncooperative behavior, which assumes a noncooperative Nash equilibrium. In fact, an important focus in our following analysis will be on comparing the testable implications of the Nash bargaining model with the ones of the collective model. Indeed, we believe that the collective model provides a natural comparison partner for the Nash bargaining model, because it imposes less prior structure.[6] Our analysis shows that this structural difference effectively translates into different empirical restrictions.

One preliminary remark is in order with respect to our following revealed preference analysis. Our specific focus will be on the characterization of the Nash bargaining model, and testing consistency of observed behavior with the model. If observed behavior is consistent with a particular model, then a natural next question pertains to recovering/identifying the primitives of the underlying decision model (e.g. individual preferences). For compactness, we will not consider such recovery here. However, it is worth emphasizing that our revealed preference characterization does allow for subsequent recovery analysis. For example, Varian (1982) and, more recently, Blundell, Browning, and Crawford (2008) and Cherchye, De Rock, and Vermeulen (2011a) studied such recovery (based on revealed preferences) for closely related consumption models. The analysis of these authors can be extended to the current setting when starting from the revealed preference characterization established below.

Paper outline. Let us summarize our main points developed further on. Section 2 sets the stage by introducing the revealed preference approach on which we focus here. Specifically, it briefly recaptures the revealed preference characterizations of individual rationality (i.e. individual utility maximization) and collective rationality (i.e. rational dyad behavior in terms of the collective model). This will be instrumental for our discussion in the following sections.

Section 3 then derives the revealed preference characterization of the Nash bargaining model for the case with observed threat point bundles. As we will show, verifying consistency of observed consumption behavior with this characterization requires solving a set

[6] Specifically, the collective model only assumes Pareto efficiency, whereas the Nash bargaining model additionally assumes symmetry, invariance with respect to affine transformations of the utility functions, and contraction independence. See also our discussion in Section 3.

of inequalities that are nonlinear in unknowns. Such nonlinear conditions are difficult to use in empirical applications. Therefore, we establish (separate) necessary and sufficient conditions that are linear in the unknowns, which are easily testable.

In Section 4, we demonstrate the practical usefulness of our theoretical results by means of an application to experimental data. It has been argued before that revealed preference testing tools are especially useful within an experimental context; see, for example, Sippel (1997), Harbaugh, Krause, and Berry (2001), Andreoni and Miller (2002) and Bruyneel, Cherchye, and De Rock (2010).[7] Moreover, the controlled environment of the lab allows us to obtain data on threat point consumption bundles as well as on the bargaining outcomes. As such, this provides an ideal setting to verify consistency of dyad consumption behavior with the Nash equilibrium solution.

As far as we know, this experimental analysis is the first one that actually tests the validity of the Nash bargaining solution for decisions on consumption bundles. Existing tests in the experimental literature typically do not consider consumption decisions. In addition, they often imply a double hypothesis that imposes a particular preference structure in addition to the Nash bargaining solution.[8] By its very nature, our revealed preference analysis does not require such additional preference assumptions.

In Section 5, we focus on situations in which threat point bundles are not observed. Our main argument here will be that the Nash bargaining solution may have stronger testable implications than the collective consumption model even in such situations. Specifically, we show that this is the case as soon as either threat points are assumed to be the same in different decision situations or if individual incomes (rather than individual consumption bundles) at the disagreement points are known. As also indicated above, these findings may be relevant for applications to observational (e.g. household) data.

In Section 6 we conclude and suggest some avenues for further research. The Appendix contains our proofs.

2 Revealed preference characterization of individual and collective rationality

This section introduces notation and some basic concepts and results that will be useful for our following discussion. We first define individual rationality and present the corresponding revealed preference characterization and, subsequently, we do the same for collective rationality.

[7]See also Cox (1997) for an extensive discussion on the use of revealed preference methodology in combination with experimental data. In particular, this author indicates the implicit assumption that decisions in the experiment are separable from other decisions of the same decision makers. Given our experimental design (see Section 4), we believe that we can reasonably assume that this condition of separability is met (at least by approximation).

[8]For example, two most frequently cited tests of the Nash bargaining model are reported by Siegel and Fouraker (1960) and Roth and Malouf (1979). Siegel and Fouraker work with linear utility functions and the results of Roth and Malouf rely on the assumption that individuals are expected utility maximizers.

2.1 Individual rationality

Throughout, we will consider consumption decisions on bundles with $|N|$ goods. Our analysis starts from a finite set of $|T|$ decision situations, with $T = \{1, ..., |T|\}$. Each situation $t \in T$ is characterized by prices $\mathbf{p}_t \in \mathbb{R}^{|N|}_{++}$ and income Y_t. In the sequel, we will assume utility functions that are continuous, concave, non-satiated and non-decreasing in their arguments. As for now, suppose the individual is endowed with a utility function U. This individual is rational if, for each t, (s)he selects a bundle $\mathbf{q}_t \in \mathbb{R}^{|N|}_{+}$ that solves the following problem (**OP-IR:**)

$$\mathbf{q}_t \in \arg\max_{\mathbf{q}} U(\mathbf{q}) \text{ s.t. } \mathbf{p}_t\mathbf{q} \leq Y_t.$$

Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t\}_{t \in T}$. We obtain the following condition for individual rationality.

Definition 1. *Let $S = \{\mathbf{p}_t, \mathbf{q}_t\}_{t \in T}$. We say that S is individually rationalizable if there exists a utility function U such that, for all $t \in T$, we have that $\mathbf{q}_t$ solves **OP-IR** given the utility function U, prices $\mathbf{p}_t$ and income $Y_t = \mathbf{p}_t\mathbf{q}_t$.*

Varian (1982), based on Afriat (1967), provided the revealed preference characterization of individual rationality. It is contained in the next theorem.

Theorem 1. *Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t\}_{t \in T}$. The following conditions are equivalent:*

(i) *S is individually rationalizable.*

(ii) *For all $t \in T$, there exist numbers $U_t \in \mathbb{R}_+$ and $\lambda_t \in \mathbb{R}_{++}$ such that, for all t, $v \in T$,*

$$U_t - U_v \leq \lambda_v \mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v).$$

In this result, the equivalence between statements (i) and (ii) means that there exists a rationalizing utility function U if and only if the set S satisfies a number of inequalities defined in the unknowns U_t and λ_t. These inequalities are commonly referred to as Afriat inequalities. Intuitively, these Afriat inequalities allow for an explicit construction of the utility levels (U_t) and the marginal utilities of income (λ_t) associated with each observation t. We remark that these inequalities are linear in unknowns. Thus, we can use standard linear programming techniques to verify if S is individually rationalizable.

2.2 Collective rationality

Consider a dyad (or two-player group) consisting of A and B, with utility functions U^A and U^B. Like before, in each decision situation t the dyad spends an income Y_t on a set of

$|N|$ goods. We will assume that all goods are privately consumed and that each individual only cares for her/his own consumption.[9]

Collective rationality means consistency with the collective consumption model, which assumes a Pareto optimal solution of the multi-player (in casu two-player) game. Based on the second welfare theorem, Chiappori (1988, 1992) has shown that a collectively rational consumption decision can be represented as if it were the outcome of a two-step procedure. At each observation t, the first step divides the income Y_t into individual incomes Y_t^A and Y_t^B (with $Y_t = Y_t^A + Y_t^B$). In the second step, the individuals A and B subsequently choose consumption bundles ($\mathbf{q}_t^A$, $\mathbf{q}_t^B \in \mathbb{R}_+^{|N|}$) that solve the following optimization problems (**OP-CR):**

$$\mathbf{q}_t^A \in \arg\max_{\mathbf{q}} U^A(\mathbf{q}) \text{ s.t. } \mathbf{p}_t\mathbf{q} \leq Y_t^A;$$
$$\mathbf{q}_t^B \in \arg\max_{\mathbf{q}} U^B(\mathbf{q}) \text{ s.t. } \mathbf{p}_t\mathbf{q} \leq Y_t^B.$$

Now consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t \in T}$. We get the following condition for collective rationality.

Definition 2. *Let $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t \in T}$. We say that S is collectively rationalizable if there exist utility functions U^A and U^B such that, for all $t \in T$, we have that $\mathbf{q}_t^A$ and $\mathbf{q}_t^B$ solve **OP-CR** given the utility functions U^A and U^B, prices $\mathbf{p}_t$ and incomes $Y_t^A = \mathbf{p}_t\mathbf{q}_t^A$ and $Y_t^B = \mathbf{p}_t\mathbf{q}_t^B$.*

Using the result in Theorem 1, this definition directly obtains a characterization of collective rationality, which is given by the next theorem.

Theorem 2. *Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t \in T}$. The following conditions are equivalent:*

(i) *S is collectively rationalizable.*

(ii) *For all $t \in T$, there exist numbers $U_t^A, U_t^B \in \mathbb{R}_+$ and $\lambda_t^A, \lambda_t^B \in \mathbb{R}_{++}$ such that, for all t, $v \in T$,*

$$U_t^A - U_v^A \leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A); \quad \text{(CR-i)}$$
$$U_t^B - U_v^B \leq \lambda_v^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B). \quad \text{(CR-ii)}$$

[9] For simplicity, we will abstract from modeling public goods or consumption externalities and we will focus on two-player groups. However, our following analysis can readily be extended to groups with more than two players. It is also fairly easy to extend the theoretical models and the corresponding revealed preference characterizations presented in this and the following sections to account for public goods and externalities. For example, see Cherchye, De Rock, and Vermeulen (2011a) for dealing with public goods and externalities in revealed preference analysis of the collective consumption model.

Just like for individual rationality, collective rationality requires finding a solution for Afriat inequalities, which are linear in unknowns. In this case, we obtain a set of inequalities for both A and B. As before, these inequalities allow for an explicit construction of (in casu player-specific) utilities (U_t^A and U_t^B) and marginal utilities of income (λ_t^A and λ_t^B). It will be interesting to compare this characterization with the revealed preference characterization that applies to the Nash bargaining model. As indicated in the Introduction, the Nash bargaining model differs from the collective model by assuming more than just Pareto efficiency for the within-dyad decision process.

3 Nash bargaining model

This section first defines the Nash bargaining solution and introduces the corresponding revealed preference characterization. As we will discuss below, this characterization is non-linear in unknown variables, which makes it difficult to use in practical applications. Given this, we subsequently present necessary conditions and sufficient conditions for consistency with the Nash bargaining model that are linear in unknowns. These conditions will be used in our following application for empirical verification of the Nash bargaining model.

3.1 Revealed preference characterization

We again consider a setting with two players (A and B) who, in each situation t, spend the income Y_t on a set of $|N|$ private goods. Like before, each individual only cares for her/his own consumption. However, as is standard in the literature, we assume that the individuals' preferences are (possibly) different under agreement and disagreement. Specifically, A and B have utility functions V^A and V^B if no agreement can be reached, while they have utilities U^A and U^B in case of agreement (which means that the Nash bargaining solution is implemented).

Let us first consider the within-dyad allocation when no agreement is reached. In this case, total income Y_t is replaced by two individual incomes Y_t^A and Y_t^B. Importantly, the sum of individual incomes at the disagreement point should not necessarily equal the available income under agreement (i.e. we may have $Y_t^A + Y_t^B < Y_t$). This reflects the possibility that disagreement can be costly, which actually implies an additional incentive for effectively obtaining an agreement. Under disagreement, the individual players A and B then select the threat point bundles ($\mathbf{x}_t^A, \mathbf{x}_t^B \in \mathbb{R}_+^{|N|}$) that solve the following problems (**OP-TP**):

$$\mathbf{x}_t^A \in \arg\max_{\mathbf{x}} V^A(\mathbf{x}) \text{ s.t. } \mathbf{p}_t\mathbf{x} \leq Y_t^A,$$
$$\mathbf{x}_t^B \in \arg\max_{\mathbf{x}} V^B(\mathbf{x}) \text{ s.t. } \mathbf{p}_t\mathbf{x} \leq Y_t^B.$$

Next, if the players come to an agreement, then the dyad allocation coincides with the Nash bargaining solution. For the given income Y_t and prices $\mathbf{p}_t$, this solution maximizes the product of the individuals' excess utility (i.e. utility under agreement minus utility under disagreement). As shown by Nash (1950), this is the unique bargaining outcome that satisfies the axioms of Pareto optimality, symmetry, invariance with respect to affine transformations of the utility functions, and contraction independence. We remark that these four axioms usually lead to a unique outcome of the decision process. This implies an important difference with the collective consumption model. As indicated above, this last model only assumes Pareto efficiency, which generally characterizes a continuum of possible outcomes.

Formally, the Nash bargaining solution defines individual consumption bundles ($\mathbf{q}_t^A, \mathbf{q}_t^B \in \mathbb{R}_+^{|N|}$) that solve the next problem (**OP-NB**):

$$\begin{aligned}\{\mathbf{q}_t^A, \mathbf{q}_t^B\} \in \arg\max_{\mathbf{q}^A, \mathbf{q}^B} & \left(U^A(\mathbf{q}^A) - V^A(\mathbf{x}_t^A)\right)\left(U^B(\mathbf{q}^B) - V^B(\mathbf{x}_t^B)\right) \\ \text{s.t. } & \mathbf{p}_t(\mathbf{q}^A + \mathbf{q}^B) \leq Y_t, \\ & U^A(\mathbf{q}^A) > V^A(\mathbf{x}_t^A), \\ & U^B(\mathbf{q}^B) > V^B(\mathbf{x}_t^B).\end{aligned}$$

To obtain our testable implications of the Nash bargaining solution, let us assume that we have a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$. We note that this set S includes consumption information on both the bargaining outcomes and the threat points. We will get back to this below. Using the set S, we can define the following Nash bargaining rationality condition.

Definition 3. *Let $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$. We say that S is Nash bargaining rationalizable if there exist utility functions V^A, V^B, U^A and U^B such that, for all $t \in T$, we have that*

(i) *$\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ solve **OP-TP** for the utility functions V^A and V^B, prices $\mathbf{p}_t$ and incomes $Y_t^A = \mathbf{p}_t \mathbf{x}_t^A$ and $Y_t^B = \mathbf{p}_t \mathbf{x}_t^B$, and*

(ii) *$\mathbf{q}_t^A$ and $\mathbf{q}_t^B$ solve **OP-NB** for the utility functions U^A and U^B, prices $\mathbf{p}_t$, income $Y_t = \mathbf{p}_t(\mathbf{q}_t^A + \mathbf{q}_t^B)$ and threat points $V^A(\mathbf{x}_t^A)$ and $V^B(\mathbf{x}_t^B)$.*

As shown in the Appendix, we get the next revealed preference characterization of Nash bargaining rationalizability.

Theorem 3. *Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$. The following conditions are equivalent:*

(i) *S is Nash bargaining rationalizable.*

(ii) *For all $t \in T$, there exist numbers U_t^A, U_t^B, $V_t^A, V_t^B \in \mathbb{R}_+$ and λ_t^A, λ_t^B, $\delta_t^A, \delta_t^B \in \mathbb{R}_{++}$*

such that, for all t, $v \in T$,

$$U_t^A - U_v^A \leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A), \quad \text{(NB-i)}$$
$$U_t^B - U_v^B \leq \lambda_v^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B), \quad \text{(NB-ii)}$$
$$V_t^A - V_v^A \leq \delta_v^A \mathbf{p}_v(\mathbf{x}_t^A - \mathbf{x}_v^A), \quad \text{(NB-iii)}$$
$$V_t^B - V_v^B \leq \delta_v^B \mathbf{p}_v(\mathbf{x}_t^B - \mathbf{x}_v^B), \quad \text{(NB-iv)}$$
$$U_t^A > V_t^A \qquad U_t^B > V_t^B, \quad \text{(NB-v)}$$
$$\frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A - V_t^A}{U_t^B - V_t^B}. \quad \text{(NB-vi)}$$

Similar to Theorems 1 and 2, the inequalities (NB-i)-(NB-iv) are Afriat inequalities. Like before, these inequalities allow us to construct (player-specific) utilities and marginal utilities of income (in casu for both the bargaining outcomes and the threat points). Moreover, it follows from Theorem 2 that the inequalities (NB-i)-(NB-ii) guarantee that the bargaining outcome is Pareto efficient. Next, the constraints (NB-v) correspond to the last two constraints of **OP-NB**.

Finally, constraint (NB-vi) essentially captures the requirement that each bargaining outcome must maximize the product of the individuals' excess utility; see the objective function of **OP-NB**. This constraint (NB-vi) is the crucial one for obtaining testable implications that are particular to the Nash bargaining solution. More specifically, as indicated above, the constraints (NB-i)-(NB-v) imply the existence of utility functions V^A, V^B, U^A and U^B as well as Pareto efficiency. Thus, constraint (NB-vi) guarantees consistency with the remaining axioms underlying the Nash bargaining solution.

Unfortunately, the constraint (NB-vi) is nonlinear in the unknowns (U_t^A, U_t^B, V_t^A, V_t^B, λ_t^A and λ_t^B). This makes it difficult to verify this constraint in practical applications. In the next subsection, we will introduce necessary conditions and sufficient conditions for consistency with the characterization in Theorem 3. These conditions will be linear in unknowns and, thus, do allow for empirical verification.

To conclude, two remarks are in order with respect to our use of the data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$. Firstly, this implicitly assumes that individuals always reach an agreement, since we observe the corresponding bargaining outcomes. We could relax this constraint by introducing additional notation. However, this would only complicate our exposition without really adding new insights. Secondly, for each decision situation t we need to observe not only the consumption bundles in the bargaining outcomes ($\mathbf{q}_t^A$ and $\mathbf{q}_t^B$) but also the threat point bundles ($\mathbf{x}_t^A$ and $\mathbf{x}_t^B$). Obviously, this may seem to be a stringent data requirement. Still, as we will show in Section 4, such threat point information can fairly easily be obtained in an experimental setting. Next, in Section 5 we will provide testable implications of the Nash bargaining solution that imply weaker data requirements and, therefore, can be useful in (non-experimental) settings where exact information about the threat point bundles is lacking.

3.2 Empirical verification

The characterization in Theorem 3 implies conditions that are both necessary and sufficient for consistency of observed consumption behavior with the Nash bargaining model. However, because the constraint (NB-vi) turns out to be nonlinear in unknowns, these conditions are difficult to apply. In what follows, we will present necessary conditions and sufficient conditions for Nash bargaining rationality that are linear and, thus, easily testable. As we will indicate, these necessary and sufficient conditions do not coincide, which means that a particular data set may pass the necessary conditions but not the sufficient conditions. However, in Section 4 we will show that the conditions do obtain a conclusive answer for most data sets in our application. In our opinion, this suggests that these conditions constitute a useful starting point for empirically assessing Nash bargaining rationality. In general, we may expect their empirical implications to be fairly close to each other.

To obtain the conditions, we start from an equivalent reformulation of the constraint (NB-vi) in Theorem 3. Specifically, consider $\alpha_t \in]0,1[$ such that

$$\frac{1-\alpha_t}{\alpha_t} = \frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A - V_t^A}{U_t^B - V_t^B}.$$

Then, for every $t \in T$, there exist $U_t^A, U_t^B, V_t^A, V_t^B$, λ_t^A and λ_t^B that meet (NB-vi) if and only if there exist an $\alpha_t \in]0,1[$ that satisfies the following two constraints:

$$\alpha_t(U_t^A - V_t^A) - (1-\alpha_t)(U_t^B - V_t^B) = 0 \text{ and } \alpha_t\lambda_t^A - (1-\alpha_t)\lambda_t^B = 0 \qquad \text{(NB-vi-a)}$$

Sufficient conditions. Evidently, the constraints (NB-vi-a) remain nonlinear in the unknowns (U_t^A, U_t^B, V_t^A, V_t^B, λ_t^A, λ_t^B and α_t). However, they do suggest a natural sufficient condition for Nash bargaining rationality. Essentially, this sufficient condition implies a grid search on a finite set A that contains a series of possible values for the variable α_t in the above constraints. Specifically, consider a finite set $A = \{a_1, a_2, \ldots, a_K\}$ containing K numbers from the unit interval $]0,1[$. Then, we get the next result.

Proposition 1. *Let* $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t\in T}$ *and* $A = \{a_1, \ldots, a_K\} \in\ ]0,1[^K$. *The set* S *is Nash bargaining rationalizable if, for all* $t \in T$, *there exist numbers* U_t^A, U_t^B, $V_t^A, V_t^B \in \mathbb{R}_+$, λ_t^A, λ_t^B, $\delta_t^A, \delta_t^B \in \mathbb{R}_{++}$ *and* $\alpha_t \in A$ *that satisfy* (NB-i)-(NB-v) *and, in addition,*

$$\alpha_t(U_t^A - V_t^A) - (1-\alpha_t)(U_t^B - V_t^B) = 0, \qquad \text{(NB-vi-b)}$$

$$\alpha_t\lambda_t^A - (1-\alpha_t)\lambda_t^B = 0. \qquad \text{(NB-vi-c)}$$

Thus, for a given set A, this result provides sufficient conditions for Nash bargaining rationality, which replace the nonlinear constraint (NB-vi) in Theorem 3 by the constraints (NB-vi-b)-(NB-vi-c). Clearly, for a given specification of $\{\alpha_1, \ldots, \alpha_{|T|}\}$ the constraints

(NB-vi-b)-(NB-vi-c) are linear in the unknowns (U_t^A, U_t^B, V_t^A, V_t^B, λ_t^A and λ_t^B). The practical implementation of these sufficient conditions requires checking these linear constraints (together with (NB-i)-(NB-v)) for each possible specification of $\{\alpha_1, \ldots, \alpha_{|T|}\}$. In our empirical application in Section 4 we use $K = 9$ and $A = \{0.1, 0.2, \ldots, 0.9\}$.

Necessary conditions. Our necessary conditions again start from a finite set A as defined above. Still, unlike the sufficient conditions in Proposition 1, which focus on specific values $\alpha_t \in A$ for each t, the necessary conditions consider all $a_k \in A$. At the outset, it is worth indicating that these necessary conditions will be rather technical ones, which have a less obvious intuition in terms of Nash bargaining rationality than our starting characterization in Theorem 3. However, our empirical application in Section 4 will show that they do have substantial practical usefulness. Actually, as also mentioned before, a main result will be that the empirical implications of these necessary conditions are situated fairly closely to those of the sufficient conditions in Proposition 1.

The starting point of our necessary conditions is that, given (NB-vi-a), for each $a_k \in A$ and $t \in T$ we must have

$$\frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A - V_t^A}{U_t^B - V_t^B} \leq \frac{1 - a_k}{a_k} \text{ or } \frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A - V_t^A}{U_t^B - V_t^B} > \frac{1 - a_k}{a_k}.$$

Clearly, the equality constraints in these expressions are nonlinear in the unknowns (U_t^A, U_t^B, V_t^A, V_t^B, λ_t^A and λ_t^B).Therefore, our necessary conditions distinguish between the following two cases for each $a_k \in A$ and $t \in T$:

$$\frac{U_t^A - V_t^A}{U_t^B - V_t^B} \leq \frac{1 - a_k}{a_k} \quad \text{and} \quad \frac{\lambda_t^A}{\lambda_t^B} \leq \frac{1 - a_k}{a_k}, \tag{NB-vi-d}$$

$$\text{or} \quad \frac{U_t^A - V_t^A}{U_t^B - V_t^B} > \frac{1 - a_k}{a_k} \quad \text{and} \quad \frac{\lambda_t^A}{\lambda_t^B} > \frac{1 - a_k}{a_k}. \tag{NB-vi-e}$$

Now consider a binary variable $R(k, t) \in \{0, 1\}$. Let $R(k, t) = 0$ correspond to scenario (NB-vi-d) and $R(k, t) = 1$ to scenario (NB-vi-e). Then, we can show that the constraints (NB-vi-a) are met only if there exist $R(k, t) \in \{0, 1\}$ such that, for $C \geq \max\{(U_t^A - V_t^A), (U_t^B - V_t^B), \lambda_t^A, \lambda_t^B\}$,[10]

$$a_k(U_t^A - V_t^A) - (1 - a_k)(U_t^B - V_t^B) \leq R(k, t)C, \tag{NB-vi-d1}$$

$$a_k\lambda_t^A - (1 - a_k)\lambda_t^B \leq R(k, t)C, \tag{NB-vi-d2}$$

$$a_k(U_t^A - V_t^A) - (1 - a_k)(U_t^B - V_t^B) > (R(k, t) - 1)C, \tag{NB-vi-e1}$$

$$a_k\lambda_t^A - (1 - a_k)\lambda_t^B > (R(k, t) - 1)C, \tag{NB-vi-e2}$$

If $R(k, t) = 0$, then (NB-vi-d1)-(NB-vi-e2) comply with scenario (NB-vi-d). Else, if $R(k, t) = 1$, then (NB-vi-d1)-(NB-vi-e2) comply with scenario (NB-vi-e). See the proof of Proposition 2 for a detailed argument.

The following proposition captures these necessary conditions for (NB-vi) to hold.

[10] By rescaling the Afriat inequalities (NB-i)-(NB-iv) it is always possible to find a suitable value for C.

Proposition 2. *Consider a data set* $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$ *and* $A = \{a_1, \ldots, a_k\} \in]0,1[^K$*. The data set* S *is Nash bargaining rationalizable only if, for every* $s \in T$*, there exist numbers* U_s^A*,* U_s^B*,* $V_s^A, V_s^B \in \mathbb{R}_+$ *and* λ_s^A*,* $\lambda_s^B \in \mathbb{R}_{++}$ *with*

$$U_s^A - V_s^A = U_s^B - V_s^B, \tag{NB–vi-f1}$$

$$\lambda_s^A = \lambda_s^B, \tag{NB–vi-f2}$$

such that, for all $t \in T \backslash \{s\}$*, there exist numbers* U_t^A*,* U_t^B*,* $V_t^A, V_t^B \in \mathbb{R}_+$ *and* λ_t^A*,* λ_t^B*,* $\delta_t^A, \delta_t^B \in \mathbb{R}_{++}$ *that satisfy* (NB-i)-(NB-v) *and, in addition, for all* $k \leq K$ *there exist binary numbers* $R(k,t) \in \{0,1\}$ *for which*

$$a_k(U_t^A - V_t^A) - (1 - a_k)(U_t^B - V_t^B) \leq R(k,t)C, \tag{NB–vi-d1}$$

$$a_k\lambda_t^A - (1 - a_k)\lambda_t^B \leq R(k,t)C, \tag{NB–vi-d2}$$

$$a_k(U_t^A - V_t^A) - (1 - a_k)(U_t^B - V_t^B) > (R(k,t) - 1)C, \tag{NB–vi-e1}$$

$$a_k\lambda_t^A - (1 - a_k)\lambda_t^B > (R(k,t) - 1)C, \tag{NB–vi-e2}$$

The constraints (NB–vi-d1)-(NB–vi-e2) have been explained before. The additional constraints (NB–vi-f1) and (NB–vi-f2) imply a normalization that is required for the necessary conditions to have bite (i.e. to be rejectable); see the proof of Proposition 2 for a more detailed discussion. As we require the test to be independent of the identity of s, we verify this set of inequalities for each possible $s \in T$. In the end, our necessary conditions imply constraints that are linear in unknowns, with some binary integer variables (i.e. the variables $R(k,t)$). These conditions are easily tested by mixed integer programming solvers. In general, the finer the grid that defines the set A (i.e. the larger K), the more stringent this necessary test will be.

4 Experimental analysis

We conducted an experiment to illustrate the practical usefulness of the testable implications in Propositions 1 and 2. This experiment obtained a collection of data sets $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t \in T}$. We will first provide details on our experimental design. Subsequently, we will present our main empirical findings.

4.1 Experimental design

We conducted our experiment at the University of Leuven (a Belgian University). Participants of our experiment were first year business economics students (116 in total, 39 females). The experiment consisted of three sessions, which each contained around 40 participants. In each session, participants were divided over two computer rooms with 20 PCs each. Every participant was seated in front of a computer. Decision problems were presented on the computer. Before the actual experiment, each participant had to fill in a short questionnaire. The most important question was to choose between three kinds of beverage items (a soda, a light version of the same soda and orange juice) and three kinds of food items (potato chips, chocolate and grapes). All items were shown in front of the class room. We asked the participants to pick their preferred beverage and food items. This should avoid that, during the experiment, participants had to choose between one or more items they actually did not like: participants had to make allocation decisions that involved (only) the selected beverage and food items (i.e. $|N| = 2$).

The actual experiment began after filling out the questionnaire. It consisted of two parts. In a first part, each participant had to make 9 ($= |T|$) individual consumption decisions, which defined the threat point bundles $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$.[11] Each decision situation involved a number of tokens (defining the individual incomes Y_t^A or Y_t^B) and prices ($\mathbf{p}_t$) for the food and beverage items they had selected before. Prices were expressed per 10 centiliters for the beverage item and per 10 grams for the food item. Participants could select their consumption quantities by using a scroll-bar, which implies a high degree of accuracy. They had to spend the full budget, i.e. savings were not allowed. Table 1 presents the prices ($\mathbf{p}_t$) and individual income levels (Y_t^A and Y_t^B) for the 9 decision situations.[12] We note that the price-income situations in our experiment imply a high discriminatory power of our rationality tests (i.e. a high probability of detecting irrational behavior), because there is little variation in income but a lot of variation in prices.[13] Below, we will provide empirical measures for the power of our tests. Table 2 provides summary statistics on the budget shares of the food and beverage items for the individual decisions that we observed.

For the second part of the experiment, participants were matched randomly 2 by 2. For our sample, this obtained 7 female-female, 26 male-male and 25 male-female dyads. Each dyad again had to make 9 (in casu joint) consumption decisions. Each such joint decision corresponded to an individual decision in the first part of the experiment. Specifically, if the individual decision was associated with incomes Y_t^A and Y_t^B and prices $\mathbf{p}_t$, then the dyad decision was characterized by a joint income $Y_t = Y_t^A + Y_t^B + 10$ and the same prices $\mathbf{p}_t$; see again Table 1. (We gave the dyads 10 extra tokens to provide them with an incentive to effectively find an agreement.) In every joint decision, participants had to account for the consumption quantities $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ that were chosen in the associated individual decision situations: these individual choices could no longer be changed and figured as threat (or disagreement) points for the joint decisions. In the case of a dyad decision, the subjects were asked to agree on a division of the joint income Y_t over bundles $\mathbf{q}_t^A$ and $\mathbf{q}_t^B$. In addition, for each consumption decision both participants had the possibility to default on the agreement (by clicking a radio button). This resulted in 48 dyads that always found an agreement. Below, we will only report results for these 48 dyads (and 96 individual players). Table 2 provides summary statistics on the dyads' choices.

[11] In particular, this assumes that the disagreement utility functions V^A and V^B coincide with the utility functions under individual decision making. We believe this to be a plausible assumption for our experimental setting.

[12] The order of the decision problems was randomized over the participants.

[13] For example, Blundell, Browning, and Crawford (2003) apply a similar idea in their 'maximum power sequential path' procedure for maximizing the power of their revealed preference tests.

Table 1: The 9 price regimes and corresponding (individual and joint) income levels

Observation	price good 1	price good 2	Y_t^A	Y_t^B	Y_t
1	3	5	12	22	44
2	4	4	14	24	48
3	5	3	13	23	46
4	3	5	18	18	46
5	4	4	17	17	44
6	5	3	19	19	48
7	3	5	24	14	48
8	4	4	23	13	46
9	5	3	22	12	44

Table 2: Summary statistics for the budget share spent on the beverage

	mean	var	min	1st quartile	median	3rd quartile	max
individual decisions	0.483	0.052	0	0.35	0.5	0.62	1
collective decisions	0.475	0.039	0	0.4	0.5	0.58	1

To enhance the external validity of our experiment, we told the participants beforehand that they would actually receive one of the consumption bundles they selected. The knowledge that each choice ostensibly had the same chance of being implemented was supposed to give economic significance to otherwise merely hypothetical decisions, thus providing participants with an incentive for making choices that truly represented their preferences. More specifically, at the beginning of the second part of the experiment we explained that, if we picked a decision exercise from this second set of (joint) decisions, we would first check whether each player preferred it to the default option. If this was effectively the case, then participants received the bargaining outcome $\mathbf{q}_t^A$ and $\mathbf{q}_t^B$. In the other case, if at least one player preferred the disagreement option, then we gave the threat point bundles $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$. The goods were handed over in a separate room immediately after the experiment, and they were given in packages that induced immediate consumption.

4.2 Results

We present results for the testable conditions in Theorem 1 (individual rationality), Theorem 2 (collective rationality) and Propositions 1 and 2 (Nash bargaining rationality). A specific focus will be on comparing the empirical performance of the Nash bargaining model with that of the collective model. In this comparison, we will consider goodness-of-fit (captured by pass rates) as well as discriminatory power (measured as rejection probability for the given data) of the corresponding conditions. Indeed, we believe a fair comparison of different behavioral models must complement a goodness-of-fit analysis with a power analysis: favorable goodness-of-fit results, indicating few violations of the testable conditions, have little meaning if the behavioral implications have low power, i.e. the conditions could hardly be violated for the data at hand.

Specifically, our following analysis will quantify the adequacy of different models by adopting a recent proposal of Beatty and Crawford (2010). These authors propose a predictive success measure to evaluate the overall empirical performance of specific behavioral models in the context of revealed preference analysis. As we will explain, this measure simultaneously accounts for goodness-of-fit and discriminatory power.

One further remark is in order. The tests defined above are 'sharp' tests: they only tell us whether behavior is exactly optimizing in terms of the behavioral model that is under evaluation. Clearly, this is a demanding premise. In fact, one may argue that exact optimization is not a very interesting hypothesis, but that we rather want to know whether the behavioral model under study provides a reasonable way to describe observed behavior. Therefore, in our empirical analysis we will also consider extended versions of the basic (sharp) tests that account for optimization error; these extended tests focus on nearly optimizing behavior rather than exactly optimizing behavior. See also Varian (1990) for a general discussion on the usefulness of considering such nearly optimizing behavior in empirical revealed preference analysis.

To deal with optimization error, we adapt an original proposal of Afriat (1973) (for revealed preference tests in a unitary setting). In particular, we capture optimization error by a so-called Afriat index $e \in [0, 1]$. For a given value of e, the extended tests then replace the observed quantity bundles $\mathbf{q}_v^A, \mathbf{q}_v^B, \mathbf{x}_v^A, \mathbf{x}_v^B$ in our above rationality conditions by the adjusted bundles $e\mathbf{q}_v^A, e\mathbf{q}_v^B, e\mathbf{x}_v^A, e\mathbf{x}_v^B$. For example, in the extended test of Nash bargaining rationality the inequality (NB-i) becomes

$$U_t^A - U_v^A \leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - e\mathbf{q}_v^A).$$

(The other rationalizability constraints have a straightforwardly similar construction.) Clearly, if the Afriat index $e = 1$, then the extended tests coincide with the original sharp tests. Lower values for e account for optimization error, which generally implies weaker conditions to be tested. Considering $e < 1$ allows us to analyze the impact of optimization error on our goodness-of-fit, power and predictive success results.

Pass rates. Figure 1 presents the pass rates for the different tests under consideration: Individual Rationality (IR in what follows), Collective Rationality (CR) and Nash Bargaining Rationality (NBR). For each test, pass rates are measured as the fraction of participants or dyads that meet the associated rationalizability conditions. The figure shows pass rates as a function of the Afriat index e. We note that the figure contains 2 curves for Nash bargaining rationality: the lower curve corresponds to the sufficient conditions in Proposition 1 and the upper curve to the necessary conditions in Proposition 2. Table 3 provides exact pass rates for selected values of e.

Let us first consider individual rationality. The IR curve in Figure 1 pertains to the 96 individuals (in 48 dyads) that found an agreement for all decisions (see the discussion of our experimental design). For each individual, we verify if the associated data set $\{\mathbf{p}_t, \mathbf{x}_t^M\}_{t\in T}$ ($M = A$ or B) satisfies the conditions in Theorem 1. Generally, we find that individual rationality is well supported: pass rates are close to 1 even for high values of e. We conclude that individual rationality seems to be a reasonable assumption.

Next, the CR curve pertains to the 48 dyads that reached agreement. For each dyad, we checked if the data set $\{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t\in T}$ meets the conditions in Theorem 2. As explained above, this verifies whether Pareto efficiency is a tenable assumption for the joint decisions that we observe. The CR curve displays a similar pattern as the IR curve: pass rates are high, also when e gets close to 1. Similar to before, we can argue that collective rationality (or Pareto efficiency) appears to be a plausible assumption.

These findings for individual and collective rationality make it interesting to consider Nash bargaining rationality. In this case, our rationality tests apply the conditions in Propositions 1 and 2 to the sets $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{x}_t^A, \mathbf{x}_t^B\}_{t\in T}$. As for the associated NBR curve in Figure 1, we observe three remarkable facts. Firstly, the pass rates for the necessary and sufficient conditions are generally close to each other. This suggests that the empirical implications of the two conditions almost coincide. Also, when we decrease e, the pass rates for the two tests increase at a similar pace. In our opinion, this suggests that combining the two sets of conditions does form a useful basis for empirical analysis. This seems all the more true when taking into account that we considered a fairly basic grid search (with $K = 9$; see Section 3); a finer grid search can only bring the necessary and sufficient conditions closer to each other.

Secondly, we find that pass rates are quite low if we consider the 'sharp' Nash bargaining rationality test: for an Afriat index $e = 1$, we get a pass rate between (only) 0.25 (sufficient conditions) and 0.27 (necessary conditions). However, pass rates increase very rapidly if we allow for some optimization error. For example, for $e = 0.90$ we obtain that no less than 92% of all dyads in our sample pass the Nash bargaining conditions (both necessary and sufficient). This suggests that the Nash bargaining model effectively does provide an adequate description of observed dyad behavior as soon as we account for nearly optimizing behavior instead of exactly optimizing behavior.

Our final observation is directly related to the second one. Specifically, if we exclude optimization error, then pass rates for the Nash bargaining test are substantially below those for the collective rationality test: for $e = 1$, the difference in pass rates is no less than 50%. However, and in line with our previous observation, this difference decreases rapidly with the Afriat index e. For example, if we consider $e = 0.90$, the difference is no more than 4%. Thus, when allowing for small optimization error, the goodness-of-fit of the Nash bargaining model (almost) coincides with the one of the collective model.

At this point, it is important to remark that lower pass rates for the Nash bargaining test can be expected a priori. Indeed, because the Nash bargaining solution imposes considerable structure on top of Pareto efficiency (see Section 3), pass rates for Nash bargaining rationality will always be situated below the pass rates for collective rationality. As such, the lower pass rates for the Nash bargaining model may also signal more discriminatory power rather than a worse model per se. This directly motivates our following exercises, which consider power of the different tests in addition to the mere pass rates.

Figure 1: Pass rates

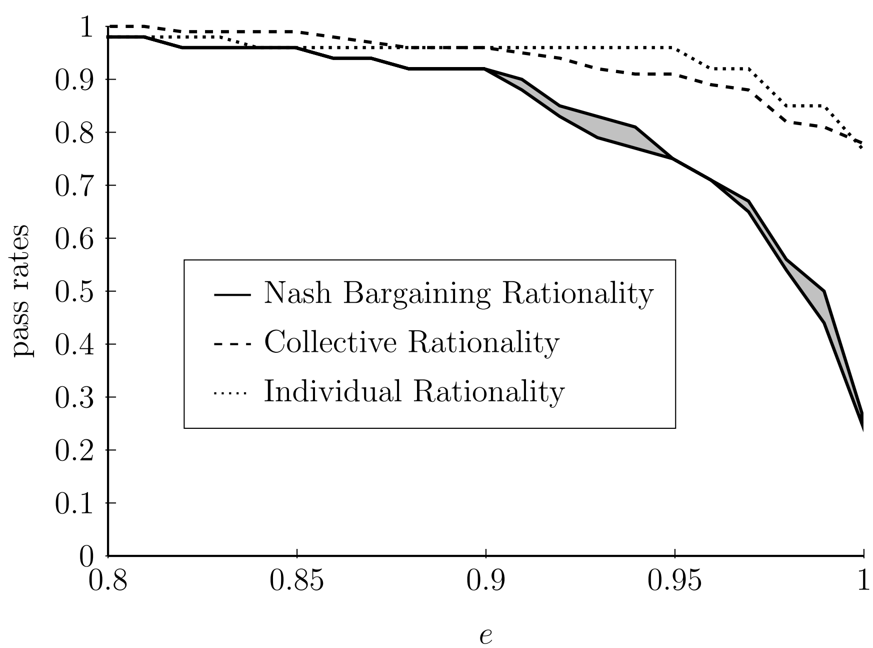


Table 3: Pass rates for different values of optimization error

	Afriat Index (e)				
	1	0.95	0.9	0.85	0.8
Nash Bargaining Rationality					
Lower bound	0.25	0.75	0.92	0.96	0.98
Upper bound	0.27	0.75	0.92	0.96	0.98
Collective Rationality	0.77	0.96	0.96	0.96	0.98
Individual Rationality	0.78	0.91	0.96	0.99	1

Power and predictive succes. As argued before, pass rates are only one part of the story. Generally, favorable goodness-of-fit results (i.e. high pass rates) for a specific behavioral model provide convincing support for the model only if the associated test has high discriminatory power, i.e. a high probability of detecting behavior that is inconsistent with the model (which we will call irrational behavior).[14] Therefore, Beatty and Crawford (2010) proposed a predictive success measure that combines the pass rate and power of a test into a single metric. It is defined as follows:[15]

$$\textit{predictive success} = \textit{power - (1 - pass rate).}$$

Because pass rates and power values lie between 0 and 1, the value of this predictive success measure is always situated between -1 and 1. A value close to 1 indicates a model with approximately 100% power and 100% fit, i.e. the best possible scenario. This means that (almost) all data pass the rationality test, even though the test effectively detects (almost) any deviating (i.e. irrational) behavior. By contrast, a value close to -1 implies a model with approximately 0% power and 0% fit, i.e. the worst possible scenario. In this case, the test effectively allows for (almost) any observed behavior and yet the data fail to pass. Finally, a value of 0 corresponds to a model with a rejection rate for the observed behavior (= 1 - pass rate) that exactly equals the expected rejection rate if behavior were irrational (= power). Essentially, this means that the rationality test does not allow for distinguishing observed behavior from irrational behavior.

To compute the predictive success rate of a particular behavioral model, we first need to measure the power of the model. As mentioned above, power stands for the probability of detecting irrational behavior (i.e. behavior that is inconsistent with the model). Following Bronars (1987), we model irrational behavior by randomly drawing a quantity bundle for each price regime. In particular, in our application we conducted Monte Carlo simulations with 1000 iterations, which obtained 1000 random data sets of 9 observations. Our power measure then equals the probability that our tests reject rational behavior for this simulated random/irrational behavior.

Before presenting the predictive success rates of the different models, we have a quick look at the power results. For the four models under evaluation, Figure 2 sets out power as a function of the Afriat index e; Table 4 gives power estimates for a selection of values for e. Not surprisingly, we find that power decreases with e for all models under evaluation.

Next, and more importantly, we observe a substantial difference between the NBR and CR curves.[16] In general, the discriminatory power of the Nash bargaining test is much

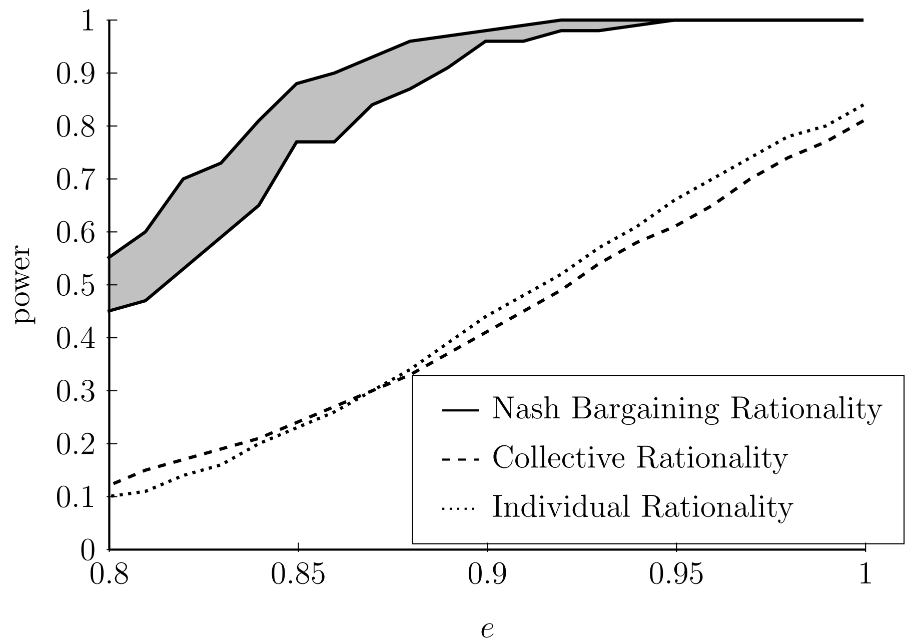
[14] See Beatty and Crawford (2010), Bronars (1987) and Andreoni and Harbaugh (2008) for a detailed discussion of this point.

[15] We note that this predictive success measure actually assigns an equal weight to discriminatory power and goodness-of-fit. This equal weighting may seem arbitrary to some. Interestingly, however, Beatty and Crawford (2010) show that this weighting scheme has an interesting axiomatic characterization. We believe this provides a convincing theoretical foundation for our focus on the equally weighted predictive success measure.

[16] In fact, the same applies when comparing the NBR and IR curves, but this difference is less relevant here.

above the one of the collective rationality test, and the difference remains more or less constant for different values of e. In addition, we find that the Nash bargaining test has no less than 100% power for e close to 1. In fact, power remains very high (i.e. close to 100%) for $e = 0.90$. Overall, this suggests that the Nash bargaining model is a very powerful one.

Figure 2: Power



Let us then consider the predictive success rates of the different models. Figure 3 displays predictive success rates as a function of the Afriat index e, and Table 5 gives predictive success rates for specific values of e. These results bring together our earlier goodness-of-fit and power results. Firstly, if we look at the IR and CR curves, we find that predictive success generally increases with e. The best performing model specification corresponds to $e = 0.99$ (collective rationality) or 1 (individual rationality). In both cases, predictive success is about 0.60. Because this is far above 0, we conclude that both models can be categorized as 'good' models for the application at hand.

Next, it is interesting to compare the CR curve with the NBR curve. Here we find that the Nash bargaining model with a little optimization error largely outperforms the collective model. For example, for $e = 0.90$ the predictive success of the Nash bargaining model amounts to no less than 0.90. This is very close to the maximum of 1, which indicates this specification of the Nash bargaining model as a 'very good' one for the setting under study.

At a more general level, we believe that these results provide a convincing empirical argument pro the Nash bargaining model. The model imposes considerable structure on joint decision processes, which gives it substantial discriminatory power. Interestingly,

Table 4: Power for different values of optimization error

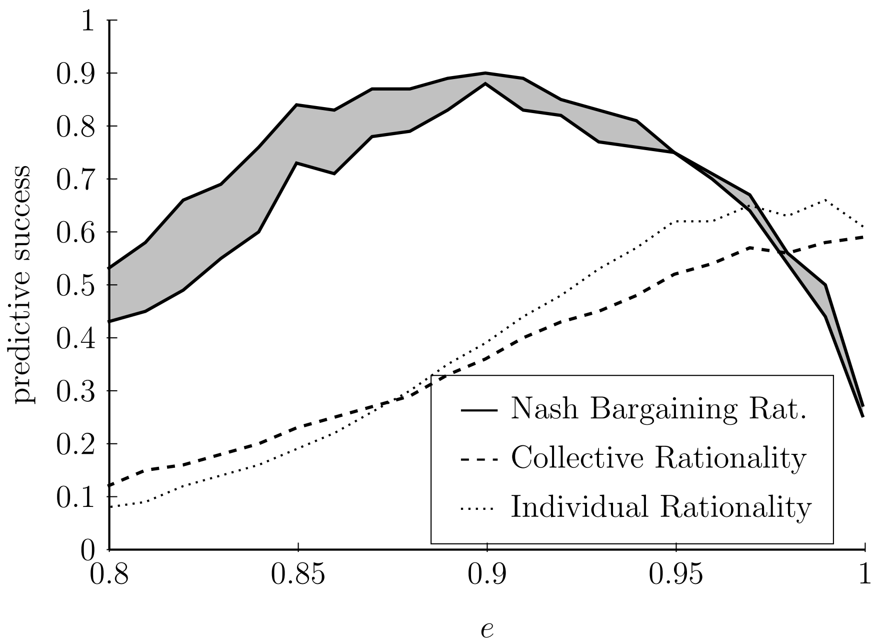
	Afriat Index (e)				
	1	0.95	0.9	0.85	0.8
Nash Bargaining Rationality					
Lower bound	1.00	1.00	0.96	0.77	0.45
Upper bound	1.00	1.00	0.98	0.88	0.55
Collective Rationality	0.84	0.66	0.44	0.23	0.10
Individual Rationality	0.81	0.61	0.41	0.24	0.12

even though it implies much prior structure, the model does provide a good empirical fit of the observed consumption behavior (if we account for a small amount of optimization error). In our opinion, these two attractive features together, which imply a high degree of predictive success, strongly suggest the model as a most valuable alternative for describing consumption decisions involving multiple players.

Table 5: Predictive success for different values of optimization error

	Afriat Index (e)				
	1	0.95	0.9	0.85	0.8
Nash Bargaining Rationality					
Lower bound	0.25	0.75	0.88	0.73	0.43
Upper bound	0.27	0.75	0.90	0.84	0.53
Collective Rationality	0.61	0.62	0.39	0.19	0.08
Individual Rationality	0.59	0.52	0.36	0.23	0.12

Figure 3: Predictive success



5 What if threat point bundles are not observed?

Our above application demonstrates the usefulness of the revealed preference approach for analyzing experimental data in terms of the Nash bargaining model. A natural next question is whether this approach can also be useful for analyzing observational (i.e. non-experimental) data. After all, consumption models are often applied for analyzing household behavior on the basis of observational data.

In this respect, an important concern is that observational data often do not contain information on threat point bundles ($\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ in Theorem 3). For example, as thoroughly discussed by Chiappori (1988), McElroy and Horney (1990), McElroy (1990) and Chiappori (1990), exact information on threat points is usually lacking in household consumption applications. In this section, we consider the extension of our previous characterization towards (e.g. household) settings with unobserved threat point bundles.

Preliminary discussion. As a starting observation, we note that the testable implications of the Nash bargaining model coincide with the ones of the collective model if threat point bundles are not observed (i.e. we have a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t \in T})$ and we make no further assumption. Specifically, this case does not impose any restrictions on the consumption bundles at the threat points (i.e. $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ for all $t \in T$). As such, we can also freely choose the values of V_t^A and V_t^B. Given this, it is easily verified that the corresponding testable implications of Theorem 3 are equivalent to the ones of Theorem 2.

As a result, the Nash bargaining model is empirically indistinguishable from the collective consumption model.[17]

Given this, our following analysis will make particular assumptions about the threat points. Specifically, we will show that the Nash bargaining model obtains specific restrictions if we assume either that the same threat points apply to different decision situations or that the individual incomes (rather than individual consumption bundles) at the disagreement points are known. As we will show, in each case the Nash bargaining model has stronger testable implications than the collective consumption model.

Our focus on these two specific assumptions is motivated by our belief that they may be particularly relevant for analyzing observational data on household consumption. For example, in such a setting it may effectively be a reasonable hypothesis that threat points remain constant over a given period of time. Next, knowledge of the divorce legislation can help to simulate the income distribution in case of disagreement. We will return to possible applications on household data in the concluding Section 6.

Constant threat points. Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t\in T}$, i.e. the threat point bundles $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ ($t \in T$) are not observed. Under the assumption of constant threat points, we have that there exist values $\overline{V}^A$ and $\overline{V}^B$ such that $V^A(\mathbf{x}_t^A) = \overline{V}^A$ and $V^B(\mathbf{x}_t^B) = \overline{V}^B$ for all $t \in T$. As a specific instance, this applies if $\mathbf{x}_t^A = \overline{\mathbf{x}}^A$ and $\mathbf{x}_t^B = \overline{\mathbf{x}}^B$ for some bundles $\overline{\mathbf{x}}^A$ and $\overline{\mathbf{x}}^B$, i.e. the threat point consumption bundles are the same for each observation.

In this case, we get the next condition for Nash bargaining rationalizability.

Definition 4. *Let $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t\in T}$. We say that S is Nash bargaining rationalizable under constant threat points if there exist $\mathbf{x}_t^A, \mathbf{x}_t^B \in \mathbb{R}_+^{|N|}$, utility functions V^A, V^B, U^A and U^B and numbers $\overline{V}^A$ and $\overline{V}^B$ such that, for all $t \in T$, we have that $V^A\left(\mathbf{x}_t^A\right) = \overline{V}^A$, $V^B(\mathbf{x}_t^B) = \overline{V}^B$ and, in addition,*

(i) *$\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ solve **OP-TP** for the utility functions V^A and V^B, prices p_t and incomes $Y_t^A = \mathbf{p}_t\mathbf{x}_t^A$ and $Y_t^B = \mathbf{p}_t\mathbf{x}_t^B$, and*

(ii) *$\mathbf{q}_t^A$ and $\mathbf{q}_t^B$ solve **OP-NB** for the utility functions U^A and U^B, prices $\mathbf{p}_t$, income $Y_t = \mathbf{p}_t(\mathbf{q}_t^A + \mathbf{q}_t^B)$ and threat points $V^A(\mathbf{x}_t^A)$ and $V^B(\mathbf{x}_t^B)$.*

We now obtain the following characterization of Nash bargaining rationality under constant threat points.

Proposition 3. *Consider a data set $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B\}_{t\in T}$. The following conditions are equivalent:*

(i) *S is Nash bargaining rationalizable under constant threat points.*

[17] See also Chiappori, Donni, and Komunjer (2011) for a similar conclusion.

(ii) *For all $t \in T$, there exist numbers U_t^A, $U_t^B \in \mathbb{R}_+$ and λ_t^A, $\lambda_t^B \in \mathbb{R}_{++}$ such that, for all t, $v \in T$,*

$$U_t^A - U_v^A \leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A), \tag{NBfix-i}$$

$$U_t^B - U_v^B \leq \lambda_v^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B), \tag{NBfix-ii}$$

$$U_t^A > 0 \qquad U_t^B > 0, \tag{NBfix-iii}$$

$$\frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A}{U_t^B}. \tag{NBfix-iv}$$

Two observations apply to this result. Firstly, this proposition includes the conditions for Pareto efficiency (or collective rationality); see (NBfix-i) and (NBfix-ii). But it imposes the additional constraint (NBfix-iv). This constraint makes that the Nash bargaining model is empirically distinguishable from the collective rationality model under constant threat point bundles. Secondly, the constraint (NBfix-iv) is nonlinear in the unknowns (U_t^A, U_t^B, λ_t^A and λ_t^B). This nonlinearity parallels the one of constraint (NB-vi) in Theorem 3. As such, it can be solved in a similar manner (using analogues of Propositions 1 and 2).

Known individual incomes under disagreement. Let us then consider the case in which we know the individual income levels Y_t^A and Y_t^B under disagreement (but not the bundles $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$). The relevant data set now becomes $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, Y_t^A, Y_t^B\}_{t\in T}$. Then, we can obtain testable implications by considering indirect indirect utility functions W^A and W^B that apply under disagreement (instead of the direct utility functions V^A and V^B). Formally, for any prices $\mathbf{p}$ and incomes Y^A and Y^B, we have the following relations between the functions W^A and W^B and the corresponding functions V^A and V^B:

$$W^A(\mathbf{p}, Y^A) = \max\{V^A\left(\mathbf{x}^A\right)|\mathbf{x}^A \in \mathbb{R}_+^{|N|} : \mathbf{p}\mathbf{x}^A \leq Y^A\},$$
$$W^B(\mathbf{p}, Y^B) = \max\{V^B\left(\mathbf{x}^B\right)|\mathbf{x}^B \in \mathbb{R}_+^{|N|} : \mathbf{p}\mathbf{x}^B \leq Y^B\},$$

In what follows, we will use that the functions W^A and W^B are convex.

We now get the following condition of Nash bargaining rationality.

Definition 5. *Let $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, Y_t^A, Y_t^B\}_{t\in T}$. We say that S is Nash bargaining rationalizable for known individual incomes under disagreement if there exist direct utility functions U^A and U^B and, in addition, indirect utility functions W^A and W^B that correspond to direct utility functions V^A and V^B such that, for all $t \in T$, we have that*

(i) *there exist $\mathbf{x}_t^A$, $\mathbf{x}_t^B \in \mathbb{R}_+^{|N|}$ that solve **OP-TP** for given V^A and V^B, prices $\mathbf{p}_t$ and incomes $Y_t^A = \mathbf{p}_t\mathbf{x}_t^A$ and $Y_t^B = \mathbf{p}_t\mathbf{x}_t^B$, and*

(ii) *$\mathbf{q}_t^A$ and $\mathbf{q}_t^B$ solve **OP-NB** given the functions U^A and U^B, prices $\mathbf{p}_t$, income $Y_t = \mathbf{p}_t(\mathbf{q}_t^A + \mathbf{q}_t^B)$ and threat points $V^A(\mathbf{x}_t^A)$ and $V^B(\mathbf{x}_t^B)$.*

We can establish the next characterization.

Proposition 4. *Consider a data set* $S = \{\mathbf{p}_t, \mathbf{q}_t^A, \mathbf{q}_t^B, Y_t^A, Y_t^B\}_{t \in T}$. *The following conditions are equivalent:*

(i) *S is Nash bargaining rationalizable for known individual incomes under disagreement.*

(ii) *For all $t \in T$, there exist numbers U_t^A, U_t^B, $W_t^A, W_t^B \in \mathbb{R}_+$, λ_t^A, $\lambda_t^B \in \mathbb{R}_{++}$ and $\mathbf{z}_t^A, \mathbf{z}_t^B \in \mathbb{R}_{++}^{|N|}$ such that, for all t, $v \in T$,*

$$U_t^A - U_v^A \leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A), \tag{dual-i}$$
$$U_t^B - U_v^B \leq \lambda_v^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B), \tag{dual-ii}$$
$$W_t^A - W_v^A \geq \mathbf{z}_t^A(\mathbf{p}_t/Y_t^A - \mathbf{p}_v/Y_v^A), \tag{dual-iii}$$
$$W_t^B - W_v^B \geq \mathbf{z}_t^B(\mathbf{p}_t/Y_t^B - \mathbf{p}_v/Y_v^B), \tag{dual-iv}$$
$$U_t^A > W_t^A \qquad U_t^B > W_t^B, \tag{dual-v}$$
$$\frac{\lambda_t^A}{\lambda_t^B} = \frac{U_t^A - W_t^A}{U_t^B - W_t^B}. \tag{dual-vi}$$

It is useful to compare this proposition with Theorem 3. The main difference is that the inequalities (dual-iii) and (dual-iv) replace the original inequalities (NB-iii) and (NB-iv) in our earlier theorem. These new inequalities are so-called dual Afriat inequalities. Similar to the Afriat inequalities that we considered before, these dual inequalities allow us to provide an explicit construction of the indirect utility levels (W_t^A and W_t^B) associated with each observation t. The vectors $\mathbf{z}_t^A$ and $\mathbf{z}_t^B$ are then collinear with the consumption bundles at the threat points. See Brown and Shannon (2000) for a more detailed discussion of these dual Afriat inequalities.

The same two observations apply as to Proposition 3. In particular, an analogous argument as before obtains that the Nash bargaining model again imposes stronger empirical restrictions than the collective consumption model. Next, the constraint (dual-vi) is nonlinear in the unknowns (U_t^A, U_t^B, W_t^A, W_t^B, λ_t^A and λ_t^B), but this nonlinearity can be resolved similarly as before (for constraint (NB-vi) in Theorem 3).

One final remark is in order. Because the characterization in Proposition 4 implies a weaker data requirement than the characterization in Theorem 3, it has a wider applicability. However, the counterpart is that its empirical implications generally have less discriminatory powerful. Therefore, if the threat point bundles are effectively observed, we recommend focusing on the characterization in Theorem 3 rather than the one in Proposition 4.

6 Concluding discussion

We have studied the testable implications of the Nash bargaining model for a two-player game involving consumption decisions on bundles of goods. The distinguishing feature of our study is that we followed a revealed preference approach. We have argued that this approach is particularly useful for verifying the empirical validity of the Nash bargaining model. Specifically, we have derived a revealed preference characterization of the Nash bargaining model both when threat point bundles are observed and when threat point bundles are not observed. We have shown that this can be used for practical tests of consistency of observed behavior with the Nash bargaining model. We also demonstrated the usefulness of these tests by means of an application to experimental data. This provided a first empirical test of the validity of the Nash bargaining model as a tool for describing consumption decisions. In addition, it showed that a specially tailored experiment can obtain a very powerful analysis of the Nash bargaining model as a tool for describing consumption decisions.

Our analysis also allows us to draw some further theoretical and empirical conclusions. From a theoretical point of view, our results shed light on the different testable implications of the Nash bargaining model and the collective consumption model. In this respect, a first observation is that the Nash bargaining model has stronger empirical implications than the collective model if we can observe the threat point bundles. These additional implications reflect the fact that the Nash bargaining model imposes more prior structure on the consumption decisions than the collective model, which only maintains Pareto efficiency as an assumption. More interestingly, however, we have also demonstrated that the Nash bargaining model can have stronger implications even if the threat point bundles are not observed. Specifically, we have shown that this is the case as soon as threat points are assumed to be constant over different decision situations or if individual incomes at the disagreement point are known by the empirical analyst. As discussed in Section 5, we believe that these last findings may have practical usefulness for analyzing observational data (e.g. on household consumption) in terms of the Nash bargaining model.

At an empirical level, our application to experimental data has shown that the Nash bargaining model may effectively provide a good description of multi-player consumption decisions. In particular, we obtained that the testable implications of the model have much discriminatory power (e.g. when compared to collective consumption model). Importantly, even though it has considerable power, the model also provides a very good empirical fit of the observed consumption decisions in our experiment. In our opinion, these two attractive features together strongly suggest the Nash bargaining model as a most valuable alternative for empirically analyzing joint consumption decisions.

We see different avenues for follow-up research. Firstly, given the favorable results for the Nash bargaining model in our experimental setting, we believe a natural next step consists of bringing the testable implications developed in this paper to household consumption data. Indeed, multi-player consumption models are often applied for the empirical analysis of household behavior. As indicated above, such an analysis can start from our revealed preference characterization that does not require observed threat point bundles. In this respect, one important remark pertains to the fact that all our testable conditions need that individual consumption bundles in bargaining outcomes are observed. This is often problematic in a household context: household data sets usually only contain information on the aggregate household consumption and not on the individual consumption. Interestingly, however, data sets with individual consumption information are increasingly available in the literature. See, for example, Browning and Gortz (2006), Bonke and Browning (2009), and Cherchye, De Rock, and Vermeulen (2010). For such data sets our testable conditions are directly applicable, which may thus obtain a powerful revealed preference analysis of household consumption behavior.

Next, follow-up research can also focus on other bargaining solutions that are frequently considered in the literature, such as the generalized Nash bargaining solution, the Raiffa-Kalai-Smorodinsky solution, the egalitarian solution and the equal sacrifice solution. Essentially, these models differ from each other in terms of the axioms they impose on the bargaining solution. In fact, by adopting a similar reasoning as in this paper, it is possible to derive the revealed preference characterizations of these alternative bargaining models. One can then use these characterizations to compare the empirical performance of the different models (and the underlying axioms). For example, such a comparison may carry out an experimental analysis similar to ours.

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Appendix A: proofs

Proof of Theorem 3

We will use the following lemma.

Lemma 1. *Let $U^A, U^B, \bar{U}^A, \bar{U}^B \in \mathbb{R}$. Then for any $V^A, V^B \in \mathbb{R}$, for which $V^A < \min\{U^A, \bar{U}^A\}$ and $V^B < \min\{U^B, \bar{U}^B\}$, we have that*

$$U^A + \left(\frac{U^A - V^A}{U^B - V^B}\right) U^B \geq \bar{U}^A + \left(\frac{U^A - V^A}{U^B - V^B}\right) \bar{U}^B \text{ implies} \tag{1}$$

$$(U^A - V^A)(U^B - V^B) \geq (\bar{U}^A - V^A)(\bar{U}^B - V^B). \tag{2}$$

Proof. We prove this by contradiction. Assume that (2) does not hold, i.e.

$$(U^A - V^A)(U^B - V^B) < (\bar{U}^A - V^A)(\bar{U}^B - V^B). \tag{3}$$

We can rewrite (1) to obtain the following equivalence statements:

$$
\begin{aligned}
& U^A + \left(\frac{U^A - V^A}{U^B - V^B}\right) U^B \geq \bar{U}^A + \left(\frac{U^A - V^A}{U^B - V^B}\right) \bar{U}^B \\
\Leftrightarrow \quad & U^A(U^B - V^B) + U^B(U^A - V^A) \geq \bar{U}^A(U^B - V^B) + \bar{U}^B(U^A - V^A) \\
\Leftrightarrow \quad & (U^B - V^B)(U^A - V^A - \bar{U}^A + V^A) + (U^A - V^A)(U^B - V^B - \bar{U}^B + V^B) \geq 0 \\
\Leftrightarrow \quad & 2(U^B - V^B)(U^A - V^A) \geq (\bar{U}^A - V^A)(U^B - V^B) + (\bar{U}^B - V^B)(U^A - V^A) \\
\Leftrightarrow \quad & 2 \geq \frac{\bar{U}^A - V^A}{U^A - V^A} + \frac{\bar{U}^B - V^B}{U^B - V^B}.
\end{aligned}
$$

Next, (3) implies

$$
\frac{\bar{U}^A - V^A}{U^A - V^A} > \frac{U^B - V^B}{\bar{U}^B - V^B}, \tag{1}
$$

so that we obtain

$$
\begin{aligned}
& 2 > \frac{U^B - V^B}{\bar{U}^B - V^B} + \frac{\bar{U}^B - V^B}{U^B - V^B} \\
\Leftrightarrow \quad & 2 > \frac{(U^B - V^B)^2 + (\bar{U}^B - V^B)^2}{(\bar{U}^B - V^B)(U^B - V^B)} \\
\Leftrightarrow \quad & 0 > \frac{\big((\bar{U}^B - V^B) - (U^B - V^B)\big)^2}{(\bar{U}^B - V^B)(U^B - V^B)}.
\end{aligned}
$$

By assumption the right hand side in this last inequality is positive, which yields the wanted contradiction. This proves the lemma. □

We can now prove Theorem 3.

Proof. Necessity. Take any $t \in T$. The first order conditions of the optimization programs **OP-NB** and **OP-TP** are given by:

$$
\begin{aligned}
U^A_{\mathbf{q}_t^A} &= \frac{\lambda_t}{U^B(\mathbf{q}_t^B) - V^B(\mathbf{x}_t^B)} \mathbf{p}_t, \\
U^B_{\mathbf{q}_t^B} &= \frac{\lambda_t}{U^A(\mathbf{q}_t^B) - V^A(\mathbf{x}_t^A)} \mathbf{p}_t, \\
V^A_{\mathbf{x}_t^A} &= \delta_t^A \mathbf{p}_t, \\
V^B_{\mathbf{x}_t^B} &= \delta_t^B \mathbf{p}_t,
\end{aligned}
$$

with λ_t, δ_t^A and δ_t^B the respective Lagrange multipliers. Note that $U^C_{\mathbf{q}_t^C}$ ($V^C_{\mathbf{x}_t^C}$) is a suitable subdifferential for the function U^C (V^C) at the bundle $\mathbf{q}_t^C$ ($\mathbf{x}_t^C$), with $C = A, B$. The functions U^A, U^B, V^A and V^B are concave and, thus, for all $t, v \in T$, we have

$$\begin{aligned} U^A(\mathbf{q}_t^A) - U^A(\mathbf{q}_v^A) &\leq U^A_{\mathbf{q}_v^A}(\mathbf{q}_t^A - \mathbf{q}_v^A), \\ U^B(\mathbf{q}_t^B) - U^B(\mathbf{q}_v^B) &\leq U^B_{\mathbf{q}_v^B}(\mathbf{q}_t^B - \mathbf{q}_v^B), \\ V^A(\mathbf{x}_t^A) - V^A(\mathbf{x}_v^A) &\leq V^A_{\mathbf{x}_v^A}(\mathbf{x}_t^A - \mathbf{x}_v^A), \\ V^B(\mathbf{x}_t^B) - V^B(\mathbf{x}_v^B) &\leq V^B_{\mathbf{x}_v^B}(\mathbf{x}_t^B - \mathbf{x}_v^B). \end{aligned}$$

For all $t \in T$, let $U_t^A = U^A(\mathbf{q}_t^A)$, $U_t^B = U^B(\mathbf{q}_t^A)$, $V_t^A = V^A(\mathbf{x}_t^A)$, $V_t^B = V^B(\mathbf{x}_t^B)$. This ensures that the constraint (NB-v) is satisfied. Next, take

$$\lambda_t^A = \frac{\lambda_t}{U_t^B - V_t^B} \quad \text{and} \quad \lambda_t^B = \frac{\lambda_t}{U_t^A - V_t^A},$$

which implies that the constraint (NB-vi) is satisfied. Substituting all this in the above conditions gives

$$\begin{aligned} U_t^A - U_v^A &\leq \lambda_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A), \\ U_t^B - U_v^B &\leq \lambda_t^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B), \\ V_t^A - V_v^A &\leq \delta_v^A \mathbf{p}_v(\mathbf{q}_t^A - \mathbf{q}_v^A), \\ V_t^B - V_v^B &\leq \delta_v^B \mathbf{p}_v(\mathbf{q}_t^B - \mathbf{q}_v^B). \end{aligned}$$

This shows that the remaining constraints (NB-i)-(NB-iv) are also satisfied.
Sufficiency. Similar to Varian (1982), we define the following utility functions:

$$\begin{aligned} U^A(\mathbf{q}^A) &= \min_{t \in T} U_t^A + \lambda_t^A \mathbf{p}_t(\mathbf{q}^A - \mathbf{q}_t^A), \\ U^B(\mathbf{q}^B) &= \min_{t \in T} U_t^B + \lambda_t^B \mathbf{p}_t(\mathbf{q}^B - \mathbf{q}_t^B), \\ V^A(\mathbf{x}^A) &= \min_{t \in T} V_t^A + \delta_t^A \mathbf{p}_t(\mathbf{x}^A - \mathbf{x}_t^A), \\ V^B(\mathbf{x}^B) &= \min_{t \in T} V_t^B + \delta_t^B \mathbf{p}_t(\mathbf{x}^B - \mathbf{x}_t^B). \end{aligned}$$

Varian (1982) showed that the utility functions V^A and V^B make sure that, for all $t \in T$, $\mathbf{x}_t^A$ and $\mathbf{x}_t^B$ solve **OP-TP**. Moreover he obtained that $U_t^A = U^A(\mathbf{q}_t^A)$, $U_t^B = U^B(\mathbf{q}_t^B)$, $V_t^A = V^A(\mathbf{x}_t^A)$ and $V_t^B = V^B(\mathbf{x}_t^B)$.

It only remains to show that $\mathbf{q}_t^A$ and $\mathbf{q}_t^B$ are a solution for **OP-NB** for the utility functions U^A, V^B, V^A and V^B. Take any $t \in T$ and consider any $\mathbf{q}^A, \mathbf{q}^B \in \mathbb{R}_+$ such that $\mathbf{p}_t(\mathbf{q}^A + \mathbf{q}^B) \leq \mathbf{p}_t(\mathbf{q}_t^A + \mathbf{q}_t^B)$. Observe that we need $\mathbf{q}^A, \mathbf{q}^B$ with $U(\mathbf{q}^A) > V_t^A$ and $U(\mathbf{q}^B) > V_t^B$. By construction, we have

$$\begin{aligned} U^A(\mathbf{q}^A) + \frac{\lambda_t^A}{\lambda_t^B} U^B(\mathbf{q}^B) &\leq U_t^A + \frac{\lambda_t^A}{\lambda_t^B} U_t^B + \lambda_t^A \left(\mathbf{p}_t(\mathbf{q}^A - \mathbf{q}_t^A) + \mathbf{p}_t(\mathbf{q}^B - \mathbf{q}_t^B) \right) \\ &\leq U_t^A + \frac{\lambda_t^A}{\lambda_t^B} U_t^B. \end{aligned}$$

The constraint (NB-v) guarantees that $U_t^A > V_t^A$ and $U_t^B > V_t^B$. Given this, the constraint (NB-vi) and Lemma 1, imply

$$(U_t^A - V_t^A)(U_t^B - V_t^B) \geq (U^A(q^A) - V_t^A)(U_t^B(q^B) - V_t^B).$$

□

Proof of Proposition 1

The result follows directly from our argument in the main text.

Proof of Proposition 2

Proof. If $0 < a_k(U_t^A - V_t^A) - (1 - a_k)(U_t^B - V_t^B)$, then (NB–vi-d1) implies $R(k,t) = 1$ and, because of (NB–vi-e2), $0 < a_k\lambda_t^A - (1 - a_k)\lambda_t^B$. As such, we obtain

$$\frac{U_t^A - V_t^A}{U_t^B - V_t^B} > \frac{1 - a_k}{a_k} \Rightarrow \frac{\lambda_t^A}{\lambda_t^B} > \frac{1 - a_k}{a_k}. \tag{2}$$

A similar reasoning implies

$$\frac{U_t^A - V_t^A}{U_t^B - V_t^B} \leq \frac{1 - a_k}{a_k} \Rightarrow \frac{\lambda_t^A}{\lambda_t^B} \leq \frac{1 - a_k}{a_k}. \tag{3}$$

Given this, the constraint (NB-vi) in Theorem 3 can only hold if the constraints (NB–vi-d1)-(NB–vi-e2) are met for any $k \leq K$: if the constraints were violated for some k, then we can never obtain (NB-vi-a) (or, equivalently, (NB-vi)). However, the constraints (NB–vi-d1)-(NB–vi-e2) have no bite (i.e. are not rejectable) by themselves: without additional conditions, it is always possible to rescale the Afriat numbers U_t^A, U_t^B, V_t^A, V_t^B and λ_t^A, λ_t^B such that (2) (or, similarly, (3)) is met for any value of a_k.

To obtain necessary conditions that are rejectable, it suffices to normalize these Afriat numbers for some observation s. This is guaranteed by the constraints (NB–vi-f1) and (NB–vi-f2). One can easily verify that such a normalization does not interfere with feasibility of the constraints (NB-i)-(NB-v). In fact, if the set S satisfies the characterization in Theorem 3, then feasibility of the constraints (NB–vi-d1)-(NB–vi-e2) and (NB–vi-f1) and (NB–vi-f2) (in addition to (NB-i)-(NB-v)) must be independent of the identity of s. Therefore, we have to check the same constraints for each possible $s \in T$. □

Proof of Proposition 3

Proof. Without loss of generality, we can use $V_t^A = V_t^B = \overline{V}$ and $\mathbf{x}_t^A = \mathbf{x}_t^B = \overline{\mathbf{x}}$ for all observations $t \in T$ (i.e. threat point bundles are always the same). Then, because $\mathbf{x}_t^A = \mathbf{x}_v^A$ and $V_t^A = V_v^A$, any $\delta_t^A > 0$ automatically solves (NB-iii) in Theorem 3, i.e. we can drop the corresponding constraints as redundant in Proposition 3. Of course, the same applies to individual B and condition (NB-iv). Finally, observe that the empirical implications of conditions (NB-i) and (NB-ii) remain unaffected if we add a common term to all U_t^A or U_t^B. Hence, redefining U_t^A and U_t^B by subtracting the common term $\overline{V}$ for all $t \in T$ effectively gives conditions (NBfix-i) - (NBfix-iv). □

Proof of Proposition 4

Proof. Theorem 1 of Brown and Shannon (2000) shows that (dual-iii) and (dual-iv) provide a revealed preference characterization of the indirect utility functions W_t^A and W_t^B for the given set S. Using this, the proof of the result is directly similar to the one of Theorem 3. □