Time optimal MPC for mechatronic systems

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1 Introduction

Model predictive control (MPC) [1] is an advanced control methodology that determines the control action by solving on-line, at every discrete time step, an open-loop optimal control problem taking into account bounds on system variables such as input, outputs and internal state variables. MPC is applied mainly to slow processes, such as chemical and oil refinement plants, where minimizing input costs is usually one of the main control objectives. The application of MPC to fast systems such as mechatronic systems is emerging due to improved computing power and the development of fast numerical optimization algorithms [2]. For these systems, achieving minimal settling time is often the main concern, while the input cost is usually of less importance. Hence, this talk presents a new type of MPC; time optimal MPC (TOMPC) which minimizes the settling time of the system.

2 Time Optimal MPC

TOMPC is developed for point-to-point motion, i.e. a desired endpoint of motion is defined but no intermediate trajectory. Hence, TOMPC has to define the trajectory with the lowest settling time, i.e. deadbeat behavior, while respecting all constraints on inputs and outputs. Hence, the following two layer optimization problem is proposed: a traditional MPC problem with endpoint constraints is formulated, of which the length N has to be optimized, taking into account a minimal length $N_{\min}$, to avoid deadbeat behavior on noise corrupted measurements close to the endpoint.

Low level Problem A:

$$V_A^\star(\bar{x}_l, N) = \min_{\substack{x_0,\ldots,x_N \\ u_0,\ldots,u_{N-1}}} \sum_{k=0}^{N-1} \|u_k - u_{\text{ref}}\|_R^2 + \|x_k - x_{\text{ref}}\|_Q^2,$$

subject to the constraints:

$$\begin{aligned} x_0 &= \bar{x}_l, \\ x_{k+1} &= f(x_k, u_k), \\ g(x_k, u_k) &\geqslant 0 \quad k \in [0, N-1], \\ x_N &= x_{\text{ref}} \end{aligned}$$

Then, an admissible set $\mathbb{X}(N)$ is defined as:

$$\mathbb{X}(N) = \{\bar{x}_l | P_A(\bar{x}_l, N) \quad \text{is} \quad \text{feasible}\}$$

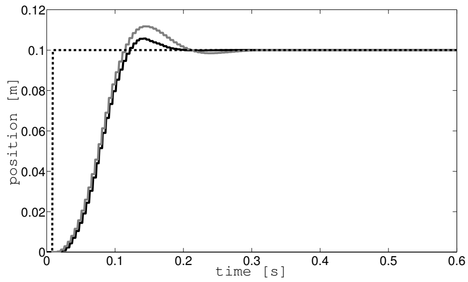

Figure 1: System output with TOMPC (black line) and traditional MPC (grey line) for a given reference (dashed line)

This admissible set allows to define high level Problem B:

$$V_B^\star(\bar{x}_l) = \min_{N \in \mathbb{N}} N$$

subject to the constraints:

$$\begin{aligned} N &\geq N_{\min}, \\ N &\leq N_{\max}, \\ \bar{x}_l &\in \mathbb{X}(N). \end{aligned}$$

3 Results

TOMPC is numerically and experimentally validated on a linear motor drive, with a sampling period of 5ms, and compared with regular MPC. In order to achieve this sampling rate, similarities between subsequent optimization problems are fully exploited. Figure 1 demonstrates the benefits of TOMPC with respect to regular MPC: the settling time with TOMPC is considerably lower.

Acknowledgment

Lieboud Van den Broeck is funded by a Ph.D. fellowship of the Research Foundation - Flanders (FWO - Vlaanderen). This work benefits from K.U.Leuven-BOF EF/05/006 Center-of-Excellence Optimization in Engineering, the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office (DYSCO), research project FP7-HD-MPC, and FWO research projects G.0377.09, G0.0320.08 and G.0558.08. Also, ETEL is gratefully acknowledged for their support.

References

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