DYNAMIC TRAFFIC ASSIGNMENT UNDER EQUILIBRIUM AND NON-EQUILIBRIUM: DO WE NEED A PARADIGM SHIFT?

Chris M.J. Tampère: Centre for Industrial Management, Traffic & Infrastructure, Katholieke Universiteit Leuven, Belgium Chris.Tampere@cib.kuleuven.be
Francesco Viti: Centre for Industrial Management, Traffic & Infrastructure, Katholieke Universiteit Leuven, Belgium Francesco.Viti@cib.kuleuven.be

Abstract
In this discussion paper we present to the DTA community our view of the future development of what is currently referred to as non-equilibrium DTA approaches. We propose the concept of equilibrium user strategies in congested networks as an extension of the well-known Wardrop equilibrium. Equilibrium user strategies provide a theoretical framework that should bring focus in the development of dynamic process or day-to-day DTA models.

The DTA community is asked to reflect upon the novel concept, whether it is theoretically sound, whether it agrees with how travellers behave in stochastically perturbed congested networks, if it is feasible etcetera. Would the community support the new equilibrium paradigm, it commits itself to a research agenda that affects virtually every aspect of DTA researched so far.

1 Introduction
Since decades already, demand in most transportation networks is rising faster than the development of capacity (supply). This trend has induced three effects to travel costs for which users are highly sensitive:

- expected travel costs have increased, more specifically in periods where *demand exceeds capacity* so that bottlenecks activate and congestion emerges;
- expected travel costs have become *time dependent* because (i) demand is not homogeneously spread over time and (ii) delays at saturated bottlenecks are a function of the time integral of excess demand;
- travel costs in congested networks have become *unreliable*, fluctuating highly around (and primarily above) expected travel times because the highly loaded network is extremely susceptible to perturbations in demand or capacity with little fallback options left.

In the analysis of transportation networks, this has led to the development of dynamic traffic assignment (DTA) models replacing traditional static assignment models[1], because of their inherent capability to model within-day time dependent demands and activation/deactivation of bottlenecks. The introduction of the time dimension into traffic assignment has required a whole series of theoretical and algorithmic innovations to network traffic modeling, including: generalization of static Wardropian equilibrium to dynamic user equilibrium (and all related issues like existence, uniqueness, stability and attraction domain), generalization of day-to-day transient models to doubly dynamic models, dynamic network loading models for both links and nodes, more refined models of user behavior (e.g. combined route and departure time choice). To date, not all issues in DTA have settled into a widely accepted DTA theory and corresponding algorithms. Moreover, given the fact that, due to the complexity of DTA some application- dependent simplifications are usually inevitable, such unified theory is not likely to come forward shortly.

Yet, despite significant progress being made in DTA since its emergence in the late 1970's, the majority of DTA research only addresses the first two concerns of congested transportation networks that necessitated its development, i.e. within-day time- dependent travel costs due to bottlenecks (de)activating. However, even if traffic would tend towards a long-term equilibrium corresponding to expected (unperturbed) traffic conditions, in practice travelers are faced with a wide variety of operational conditions caused by accidents,

[1] Note that the terms 'transient' versus 'stationary' are in fact more appropriate and consistent with the difference in physical flows in the networks than the traditional terms 'dynamic' and 'static' respectively.

adverse weather conditions or demand fluctuations that may cause unexpected peaks in travel costs. Current DTA approaches are not yet capable of quantifying and predicting the resulting *ensembles* of travel costs under stochastic traffic conditions, even though there is significant scientific evidence that travelers value (un)reliability of travel cost comparable to (sometimes even higher than) the mere increase in travel costs. They may as a consequence adapt choice of the route, departure time, travel mode and even location of activities or residence. This holds equally (or even more) for professional traffic and freight flows, because of their high time valuation and more stringent time constraints (urgent services, just-in-time delivery).

We argue in this paper that the need to quantify the impact of various perturbations in demand and supply around equilibrium in an existing or future congested transportation network calls for a *paradigm shift* in DTA towards *true stochastic* modeling. Currently DTA models incorporate randomness in different ways, i.e. in the demand variations due to daily (or weekly) activity-travel patterns, in the supply system due to weather conditions, incidents, etc. From the individual users' perspective randomness has been introduced to model choice and preference heterogeneity, imperfect and incomplete valuation of travel costs, limited rationality etc. However, common requisite in all these models is that demand and supply tend to some equilibrium state, simplifying the true stochastic nature of the problem into a somewhat separate problem of modeling demand (users' decision making) variability and supply (users' costs) reliability around the dynamic user equilibrium state.

The existing concept of dynamic user equilibrium (whether deterministic or stochastic[2]) is insufficient: congested network traffic flows under perturbed conditions are by definition non-equilibrium because travelers do not have full predictive information on all travel alternatives under stochastically perturbed conditions. However, this raises the problem that there is a myriad of potential non-equilibrium behaviors, with little or no theoretical or empirical evidence that allows distinguishing relevant from irrelevant non-equilibria (which is why equilibrium models – though never fully corresponding to real life – have such success in theory and practice: at least the equilibrium concept "gives model developers something to hold on to[3]").

In this discussion paper it is proposed to expand the dynamic equilibrium concept to the stochastically perturbed dynamic context by considering *user equilibrium strategies*. The main idea is that under perturbations, travelers do not longer select a preferred travel option that they stick to irrespective of the network conditions; neither would travelers apply any arbitrary

The remainder of this discussion paper is structured as follows. Section 2 gives a state of the art review of DTA approaches. In section 3, the concept of a user strategy and the extension of Wardrop equilibrium to user equilibrium strategies are proposed. Section 3 raises some topics for discussion on this newly introduced paradigm.

2 State of the art on Dynamic Traffic Assignment

The equilibrium strategy paradigm proposed in this discussion paper frames within the area of dynamic traffic demand and supply modeling, which is commonly indicated as Dynamic Traffic Assignment (DTA). DTA refers to a set of criteria through which the demand for mobility is distributed over time and space on a transport network. Underneath this synthetic definition, there is a wide range of models and theories, which have been developed with the aim of solving this fundamental transportation problem. Accordingly, it has been interpreted and solved in many ways, among which the most popular is the *fixed-point* approach derived from the two equilibrium principles of Wardrop (1952) and formulated mathematically as optimization problems by Beckmann et al. (1956) (see also: Boyce et al, 2005). The first principle, which is more relevant for this research proposal, states that at equilibrium no traveler will be better off by shifting to another alternative of travel, i.e. all travelers' utilities are maximal within the actual current traffic conditions.

[2] SDUE or 'stochastic' dynamic user equilibrium is a misleading name in this sense: it is a *deterministic* equilibrium condition in which the addition *stochastic* merely accounts for differences between individual users from which only aggregate statistics are known.
[3] This observation is restated in the authors' words after John Polak, 1st DTA Congress, Leeds, June 2006

Prior to the seminal works of Merchant and Nemhauser (1978a,b), the research has focused on static assignment, which is still a widely accepted approach in planning and design problems, or in general when congestion dynamics are of minor importance. The need for models able to capture in a more realistic way the dynamic features of traffic has been acknowledged since the growing application of dynamic management strategies, real-time adaptive traffic control, information and guidance systems etc., and also because of the increasing congestion levels worldwide. The simplifying assumptions characterizing static assignment approaches (e.g., steady-state conditions, time independency of the demand and the travel costs) are under these conditions unacceptable. The goal of Dynamic Traffic Assignment is therefore to represent more appropriately the dynamic character of traffic and to capture the temporal variations of the demand for mobility and of daily congestion.

The role of DTA is, in essence, to provide a functional relationship between the *demand for mobility* and the *network supply*. To specify this functional relationship, two elements are fundamental: (1) how travelers' perceive, and respond to, the costs related to a trip, i.e. the *travel choice process* and (2) how costs for mobility are generated, i.e. the *network loading process*. Travelers' choices are results of individuals' perception of travel costs and utilities.

It is not surprising that DTA has many more theoretical and computational challenges and shares many limitations characterizing static assignment approaches. For this reason DTA theory is still relatively undeveloped, as thoroughly discussed in the excellent literature review of Peeta and Ziliaskopoulos (2001). Many approaches, models and algorithms have been proposed, e.g., to better suit the various application domains, or to incorporate dynamisms and behaviours that cannot be explained with classical principles of equilibrium, and ultimately to trade off computational tractability with real observations.

Often in traffic assignment models drivers are assumed to be fully rational, and to have perfect information and perception of the costs in the network, for any possible alternative for traveling and distribution of the travel demand. These are the basic assumptions of the *Deterministic User Equilibrium*, DUE. The assumption of perfect information and perception of travel costs has been addressed as a major weak point in this theory from the early developments and, although computationally advantageous, it has been criticized as being rather unrealistic. To incorporate errors in drivers' perception Daganzo and Sheffi (1977) introduced the principle of *Stochastic User Equilibrium* (SUE). Thereby the principle of perceived costs was adopted successfully in the Dynamic Traffic Assignment context (Ran and Boyce, 1996). Despite the clear modeling advantage introduced with the SUE approach, it is widely recognized that this is simply a deterministic approach with a stochastic component in the users' utility function and it is therefore only an extension of DUE with heterogeneous drivers' perceptions.[4]

The way travel utilities are perceived and calculated is crucial for determining which DTA solution is determined. Apart from how travelers perceive utilities and the value they associate to each utility component, a fundamental difference between DTA models is in the calculation of the expected travel time. Three approaches can be distinguished: (1) models based on instantaneous travel time, which are commonly referred to Dynamic User Optimal (DUO), where drivers make choices on the basis of travel time information at each instant in time (e.g, Ran et al., 1993), (2) models based on actual travel costs, i.e. those that drivers will actually experience (e.g., Chen and Hsueh, 1998) and (3) models based on expected/predicted travel times, which are calculated using past experienced costs and present information. It is easy to understand that the adopted cost updating method yields to different equilibria. Focusing on this last category, which is most relevant for this discussion paper a number of studies have proposed methods to incorporate learning from experience and information in the choice process in a dynamic environment. Horowitz (1984) made one of the first contributions to this field by modelling the mean perceived travel cost by a weighted average of the realized costs experienced in past days. Information was integrated later in this framework by, e.g., van Berkum and van der Mede (1992), Jha et al. (1998), Chen and Mahmassani (2004).

Apart from more sophisticated learning mechanisms and models for including imperfect information in decision-making processes, questions are to what extent road users perceive uncertainty and differences in

[4] This was clarified in several presentations (the ones of Martin Hazelton and David Watling among others) during the Seminar on Day-to-day models (DADDY) held in Salerno, in December 2009.

utility between travel alternatives, and in particular when they can be considered to be satisfied with their choice and stop learning to seek a more convenient alternative. For this reason theories of *habit* and *bounded rationality* have been introduced (e.g., Mahmassani and Jou (1998)). Other studies have proposed to include a cost component for travel time uncertainty. Small (1982) and Mirchandani and Soroush (1987) proposed to include the standard deviation of experienced travel times in the utility function as a measure of uncertainty. Luo and Lo (2003) proposed the use of the *budget time*, which depends not only on the standard deviation of travel times but also on the probability of being late weighted by the individual risk attitude. Lam and Small (2001) showed however that the 90th percentile is a better measure for reliability than the standard deviation. Some other studies criticize the use of standard probabilistic methods, and suggest a more radical shift towards new utility concepts. Non-linear utility theories, which deal with the influence of risk and uncertainty in travellers' decision-making, have been recently proposed (e.g. Avineri and Prashker, 2004, de Palma and Picard, 2005, Avineri, 2006) inspired by renowned psychological theories (e.g., Von Neumann and Morgenstern, 1944, Kahneman and Tversky, 1979). However, first steps have highlighted the substantial complexity of tuning such models (Avineri and Bovy, 2008).

The majority of past studies, although obviating many limitations and introducing novelties in the users' travel choice criteria and in the (dynamic) network loading aspects, have been founded on Wardrop's equilibrium principles. Recently, a number of studies have questioned the validity of this principle, arguing that equilibrium in dynamic traffic networks is rarely observable and non-equilibrium/transient behaviour might be a more important aspect to incorporate in DTA. Recent research prefers to view equilibrium as an *attractor*, i.e. a point that could be achieved if all conditions are kept constant or at least they remain such that convergence to the attractor is possible. The *Doubly Dynamic DTA* formulation, or *Dynamic Process models,* originally introduced by Horowitz (1984), adopt this view. The main innovation brought by dynamic process models is the possibility to model the behavior towards more than one attractor, and therefore to identify the relevant conditions for which a certain state can or cannot be achieved. In this approach the evolution over time towards anthe attraction point is important, as it may determine its convergence and stability properties as shown in various studies. An extensive review of dynamic process models is found in Watling and Hazelton (2003) and in Viti and Tampère (2010), together with a thorough discussion of equilibrium and non-equilibrium approaches.

The question whether it is necessary to assume non-equilibrium or transient traffic behaviour is currently one of the main topics of discussion among the DTA research arena[5]. Equilibrium analysis has obvious advantages in terms of computational and mathematical tractability, and they are found more interpretable and understandable for many applications. Nevertheless it should be discussed whether it makes always sense to look for an equilibrium point in all applications of DTA, especially in those where changes in the demand and supply systems and perturbations prevent (unique) equilibrium conditions to ever occur. Better understanding is needed on how to deal with recurrent, observable traffic variability (e.g, daily fluctuations of demand and supply) and with non-recurrent, unpredictable traffic variations (incidents, special events) and incorporate these elements in the utility of the road users. Moreover better insight is needed into how drivers perceive and value this uncertainty and how they respond to it (e.g., by rerouting, or by day-to-day route alternative shifts, or by choosing earlier departure times).
In conclusion research into DTA is far to be on an ending point. The importance of this theory in traffic modeling and forecasting, the need for models that more sensitive to changes in congested traffic systems where many sources of dynamics are involved (information, day-to-day traffic patterns, rerouting strategies) and for models able to capture the travelers' behavior under uncertain conditions motivates the research proposed in the following of this document.

3 Towards a new paradigm: equilibrium strategies

3.1 Need for a new paradigm

[5] This has been indeed one of the hottest topics in the recent seminar DADDY, on day-to-day dynamic for transportation network analysis held in Salerno, Italy.

It is well-known that users value significantly the variability of travel time of travel alternatives (Bates et al., 2000; Fosgerau et al., 2008 and references therein). It is therefore relevant and pertinent to develop models that are capable of predicting travel time variability, rather than only predicting expected travel cost (Batley et al., 2008). For this purpose, it is not sufficient to assume rigid, equilibrium route choice by the users and to combine it with a set of stochastic network loadings (e.g. with capacities fluctuating due to incidents and adverse weather conditions, Corthout et al., 2010). Not only would this approach contradict with empirical evidence that drivers do make en-route rerouting decisions under incident conditions, it would also lead to overestimation of the travel time fluctuations, including occasional prediction of total gridlocks that are almost never observed in reality. Indeed, adaptive en-route behavior and day-to-day learning and information provision has been shown to be essential for smoothening day-to-day fluctuations of traffic conditions (Bifulco et al., 2009).
Neither is it realistic to randomize demand and/or supply each day e.g. in a Monte Carlo procedure and to calculate user equilibrium for each random draw. Clark & Watling (2005) highlight the key flaw in this reasoning. Namely, since equilibrium is reached on each day, it implies that drivers have perfect predictive knowledge of the travel times they will experience on that day, before they make their journey. This seems a reasonable assumption for slowly-changing or systematic trends where drivers may have an opportunity for repeated experience: for example, if the different samples were to represent the different mean demands arising in school-holidays and term-times. For events that can change on a daily basis, such as demand and incidents, it seems more difficult to justify however, and so even though the resulting problem is tractable it is highly questionable as to whether it is the appropriate approach for representing travel time variability due to perturbed demand and supply (Batley et al., 2008).

However, doubly dynamic traffic assignment models that include adaptive en-route and day-to-day learning behavior are by definition non-equilibrium, because travelers do not have full predictive information on all travel alternatives under stochastically perturbed conditions. Instead of yielding a fixed-point solution (e.g. the traditional DUE or dynamic user equilibrium), potential long-term behavior of the model might range from settling into a stable attractor (DUE), over limit cycle, equilibrium probability distribution of traffic states (Balijepalli et al., 2006) to chaotic sequence of states (Nakayama, 2006), even under plausible behavioral assumptions like DUO (dynamic user optimal) routing. Both from a theoretical and practical point of view, this wide range of potential behaviors with little empirical ground to support either type of solution is a complicating factor. Even if the real en-route and learning behavior of travelers can be empirically studied (e.g. Khattak et al., 1993; Bogers et al., 2005; Abdel-Aty & Abdalla, 2006), such validated models are often obtained in a stated-preference context (which is known to deviate from revealed behavior), reflect only current behavior and may have limited predictive power for future scenarios or for application to other networks.

So as dynamic model developers we are facing a dilemma. On the one hand we are devoted to providing decision makers with models that are capable of quantifying travel time variability through causal relationships. That is, the models should contain explanatory variables and their causal relationship with actual TTV, so that the model can predict the impact of changes that might affect TTV (e.g. faster incident response, better traffic control measures smoothening capacity fluctuations, more flexible travel behavior dependent on current and predicted traffic conditions, like working at home a few hours to avoid the aftermath of an incident during the peak). We are convinced that for such a truly stochastic DTA model with demand and capacity fluctuations it is essential to consider day-to-day learning and en-route decision making. On the other hand, we feel that such models may offer too many degrees of freedom in their specification and quantification. There might be just too many conceivable ways for a congested traffic network to be in non-equilibrium (i.e. ways for travelers to deviate from a long-term equilibrium travel plan), with tremendous empirical efforts required for distinguishing the plausible non-equilibrium states from the irrelevant ones.

3.2 Generalizing Wardrop: equilibrium strategies

We start by recognizing that so far, for our essentially deterministic static and dynamic models, the equilibrium concept of Wardrop and its inherent assumption of rationality in travel choices has limited the degrees of freedom in behavioral modeling to only a plausible subset of all conceivable models. Inspired by this and for the very same reason, we attempt to generalize this equilibrium concept to the truly stochastic case. We argue that the assumption of (at least partial) rationality of drivers is likely to also hold under stochastic demand and capacity. A rational traveler will therefore develop a strategy that enables him to respond optimally (from his point of view) to the uncertainties of everyday travel. Limiting daily travel decisions to only those corresponding to the user optimal strategies may indeed provide a similar kind of focus to the day-to-day and en-route model developer than the (S)DUE concept did so far.

Such user optimal strategy may consists of two parts:

- the selection of *which information to acquire* about current and predicted traffic conditions: a traveler may consult various traffic or travel information services. Usually these services come at a cost, so he will select only those services for which the cost is smaller than the pay-off obtained by making better pre-trip and en-route decisions.
- a way (algorithm) of *making pre-trip and en-route travel decisions* under expected and unexpected conditions, based on past experience and the information provided through the selected services[6].

We now assume that a traveler adapts his travel strategy such that, when applied over a relevant period under daily varying travel conditions, his valuation of the resulting travel cost distribution is maximized. This valuation may weight not only expected travel cost but also some measures of travel time variability like variance, 90 percentile etc.

When applied to a congested traffic network under stochastic demand and capacity conditions, this concept of long-term rational user behavior gives rise to a new extension of the idea of Wardrop equilibrium: *user equilibrium strategies*:

> *User equilibrium in stochastic traffic conditions is established if no user can improve his valuation of the* ***set of*** *experienced (perceived)* ***travel costs*** *by unilaterally changing his* ***strategy for*** *pre-trip and en-route* ***decision making*** *on departure time and route choice*[7].

This novel concept may integrate and unify existing equilibrium, en-route and day-to-day approaches. Figure 1 reflects the idea that equilibrium strategies can be considered as being a fixed-point solution of a bi-level problem. Note that the figure suggests an iterative calculation scheme to solve the fixed-point problem, even though there may be different, more appropriate solution algorithms. The iterative representation is merely chosen as we feel it is the most intuitive way to communicate the basic idea of equilibrium strategies.

---

[6] The DUO strategy (possibly under incomplete and/or erroneous information provided by the available travel services) might be a reasonable candidate algorithm. This illustrates that the proposed concept is not necessarily a revolution, but may rather encapsulate existing theories.

[7] The concept can be easily extended to include pre-trip and en-route modal choice.

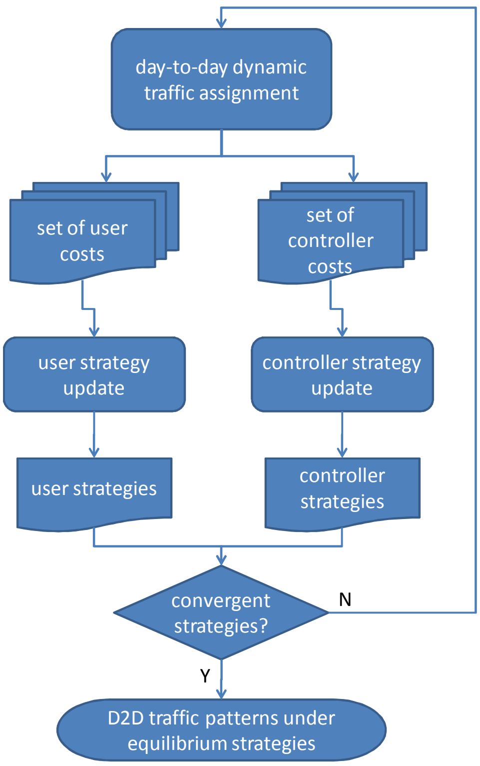

Figure 1: equilibrium strategies as a fixed point in doubly-dynamic stochastically perturbed traffic networks

In the upper level, the users of a transportation network choose a strategy that they will apply in a day-to-day (D2D) stochastically perturbed, within-day dynamic context[8]. With these strategies fixed, a sequence of daily network loadings is performed at the lower level (so-called *doubly dynamic* traffic simulation, see below). Both the pre-trip (departure time, initial route choice) and en-route decisions of the users depend on the chosen strategies. Each day, the actual pre-trip and en-route decisions may be different from the previous day because of two reasons. For one, each day exhibits different exogenously determined fluctuations of capacity and demand in the network (due to weather, seasonal effects, incidents, events,…). Secondly, users learn from previous experiences and use this knowledge when making decisions (e.g. "taking this initial route gave me less options yesterday to avoid unexpected congestion later on, so today I prefer another route"). After a representative sequence of days, users evaluate their set of experienced costs and their perceptions of costs of alternatives that they did not choose. They then choose a strategy that will allow them to increase their valuation of the set of experienced cost, which is then again evaluated in the day-to-day simulation. The system reaches a fixed point if no user can improve on the valuation of his/her experienced set of travel costs by unilaterally changing strategy. With Wardrop, one can expect that only strategies that lead to sets of travel costs with equal and maximum valuation will be retained, whereas strategies leading to travel costs with lower valuation will not be used.

[8] Note that a similar adaptation and equilibration process can be considered for controllers' strategies, who will try to maximize their valuation of the set of system costs by choosing an optimal control strategy. Equilibrium strategies can be defined for controllers given fixed strategies of the users, for users given fixed strategies of the controllers, or for controllers and users simultaneously optimizing their strategies. Likewise, system optimal user and controller strategies are conceivable, as those strategies that maximize some overall social welfare function when applied over a relevant sequence of days.

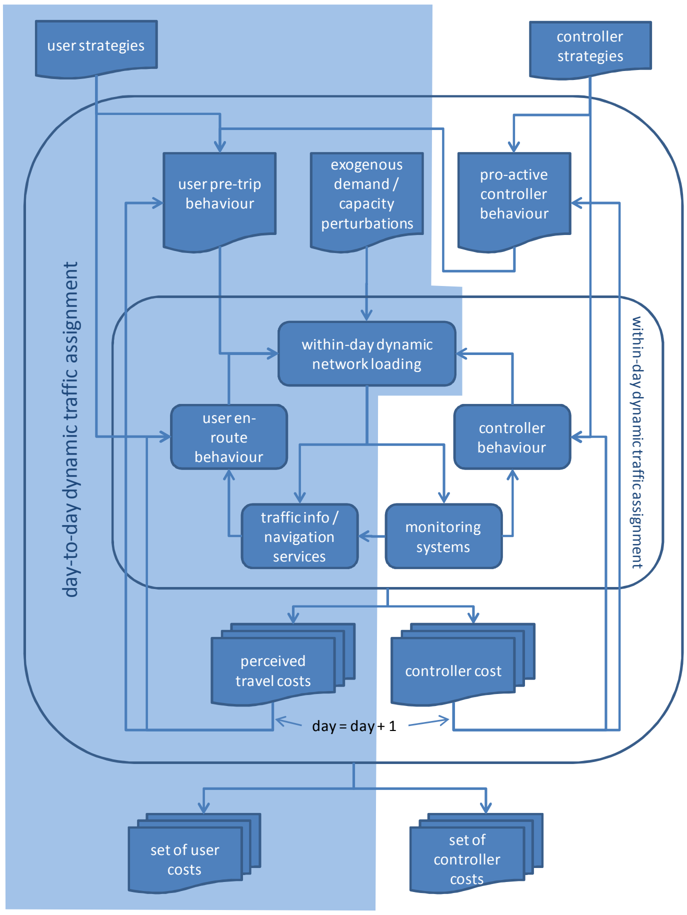

Figure 2: Doubly dynamic simulation of stochastically perturbed network traffic

One of the crucial, non-trivial components of the system in Figure 1 is the doubly dynamic evaluation of user strategies, the structure of which is depicted in Figure 2. The figure shows in some more detail how pre-trip and en-route decisions under exogenously perturbed traffic conditions are influenced by the fixed strategy chosen by the user in the upper level. The user strategy determines which traffic information and navigation services are available for consultation, as well as the way in which this information affects decision making about departure time (e.g. "if the current or predicted cost of route alternatives exceeds my current expectation of travel cost at a later time + schedule delay until then, then I postpone my trip"), initial route or diversions from the current route. At the end of the day, the user updates experienced costs for the travel options that he chose that day and his ex-post perception (if any) of potential alternative travel options he did not actually use. The updated experience is used together with the newly provided information and advices on the next day. This cycle is repeated until a relevant sequence of days is evaluated.

The controllers in the network go to a similar updating process that is also depicted (but not highlighted) in Figure 2 but will not be discussed further in this paper.

3.2 User optimal strategies: is it really new?

It is worthwhile wondering whether the user optimal strategies proposed in this paper are really new. To the best of our knowledge the concept has never been considered in DTA-related literature. However, transportation networks are only one example of the *congestion game* that is well-established in game theory: In a congestion game, the payoff of each player depends on the resources it choose and the number of players choosing the same resource. What is commonly referred to as a Wardrop equilibrium, is the equivalent of a Nash equilibrium in a non-cooperative congestion game where each traveler is a player optimizing his choice among the available resources being the travel options (mode, time of travel, route).

It is therefore interesting to consult the game theoretical definition of a player's strategy. In game theory, a player's strategy is a complete plan of action for whatever situation might arise; this fully determines the player's behaviour. A player's strategy will determine the action the player will take at any stage of the game, for every possible history of play up to that stage. It is a complete algorithm for playing the game, telling a player what to do for every possible situation throughout the game.

The idea of user optimal strategies in a perturbed congested transportation network that we put forward in section 3.2 is perfectly in line with this game theoretical definition of a player's strategy and therefore not really new: it is an algorithm for decision making in each conceivable traffic condition that finally gives the highest pay-off when applied to a representative set of days with fluctuating demand and supply.

However, whereas the generic definition of a player's strategy may encapsulate our proposed approach, it has never been proposed as a generalization of Wardrop's equilibrium to our knowledge. The equilibrium paradigm proposed in this discussion paper may therefore be new for both the transportation and game theory domains:

- In DTA, the *strategy space* from which travelers were allowed to select their optimal strategy has so far been limited to *one single combination* of (possibly multimodal) travel mode, route and departure time in a given situation. In the new paradigm, a traveler chooses an *algorithm for deciding on the currently available travel options* in each conceivable situation.
- In game theory, we are unaware of any previous effort combining in a congestion game time dependent (dynamic) travel conditions, randomly perturbed demand and supply with user strategies in the broadest sense, i.e. as a complete plan of action or algorithm for each conceivable situation.

4 Topics for discussion

Any newly proposed theoretical concept needs critical evaluation and discussion before it may possibly be adopted by researchers in the field. Minimally, the following non-exhaustive list of topics needs to be clarified:

- Does the DTA community agree that there is indeed a need for a new paradigm in congested transportation networks?
- Does the paradigm of equilibrium strategies have added value over existing equilibrium and non-equilibrium approaches? Will the concept of equilibrium strategies indeed provide modelers with something to hold on to, just like equilibrium link/route flows do now?
- Are there alternative theories that have comparable (or even more?) advantages than equilibrium user strategies and that may be simpler (following Occam's razor)?
- Is the proposed equilibrium strategy concept compatible with existing research? Could it serve as a unified framework that encapsulates most existing DTA research?
- Can we ever get the data? Inevitably a model for optimal response in daily fluctuating traffic conditions introduces more parameters than a standard (S)DUE approach. Despite the focus that the equilibrium strategy paradigm may bring, it still offers substantial degrees of freedom to DTA modelers. Will we ever dispose of the data to validate the resulting models?
- Can we ever do the calculations? Inevitably a model considering stochastic demand and supply requires a higher calculation effort than a deterministic one (e.g. Monte Carlo techniques). Yet, even for deterministic DTA models, analyzing realistically sized networks is a challenge. Will we ever be capable of doing dynamic Monte Carlo simulations for such networks, and even more so if we need to do them in an iterative way in pursue of equilibrium strategies? This provides a challenge on the algorithmic and programming sides to say the least.

5 Research agenda

Suppose that the DTA community would support the newly proposed paradigm, then this would actually set a complete research agenda before the concept is ever applicable. The following topics may serve as an onset for such research agenda:

- Elaborate exact definition of user equilibrium strategies and its extensions (controller equilibrium strategies, system optimal strategies, including modal choice and other choice levels,…)
- Explore mathematical properties of the novel equilibrium concept (incl.: what can we learn from game theory that is rapidly evolving, e.g. congestion games)
- Elaborate solution approaches and other algorithmic challenges
- Behavioural research: defining the strategy space and how (ir)rationale users select from it
- Empirical research and research into calibration techniques for stochastic dynamic network traffic simulation
- Generalization of the equilibrium strategy concept and applicability to other domains (e.g. other congestion games with stochastic boundary conditions, like supply chains)

6 Conclusions

In this discussion paper we have presented to the DTA community our view of the future development of what is currently referred to as non-equilibrium DTA approaches. We proposed the concept of equilibrium user strategies in congested networks as an extension of the well-known Wardrop equilibrium. Equilibrium user strategies provide a theoretical framework that should bring focus in the development of dynamic process or day-to-day DTA models.

The DTA community is asked to reflect upon the novel concept, whether it is theoretically sound, whether it agrees with how travellers behave in stochastically perturbed congested networks, if it is feasible etcetera. Would the community support the new equilibrium paradigm, it commits itself to a research agenda that affects virtually every aspect of DTA researched so far.

7 References

Abdel-Aty, M. A. & M. F. Abdalla (2006). Examination of multiple mode/route-choice paradigms under ATIS. IEEE Transactions on Intelligent Transportation Systems, 7(3):332–348

Avineri, E. (2006), ‘The Effect of Reference Point on Stochastic Network Equilibrium’, Transportation Science 40(4), 409–20.

Avineri, E. and J.N. Prashker (2004), ‘Violations of Expected Utility Theory in Route-choice Stated Preferences: The Certainty effect and Inflating of Small Probabilities’, Transportation Research Record 1894, 222–9.

Avineri, E. and P.H.L. Bovy (2008) ‘Parameter Identification of Prospect Theory Model for Travel Choice Analysis’, Proceedings of the 87th TRB Meeting, Washington D.C., CD–ROM.

Balijepalli, N.C., D.P. Watling & R. Liu (2006), Doubly Dynamic Simulation Model For Traffic Assignment, 1st International Symposium on Dynamic Traffic Assignment, 21-23 June 2006, Leeds, UK

Bates, Jj., J.W. Polak, P.M. Jones, & A.J. Cook (2000), The Valuation of Reliability, Transportation Research E, 37, pp.191-229

Batley, R., S. Grant-Muller, J. Nellthorp, G. de Jong, D. Watling, J. Bates, S. Hess & J. Polak (2008), Multimodal Travel Time Variability: Final Report, Institute for Transport Studies, University of Leeds, Imperial College and John Bates Services, November 11, 2008

Beckmann M, McGuire CB, Winsten CB (1956) Studies in the economics of transportation. Yale University Press, New Haven, Connecticut; also published as Rand-RM-1488-PR, Rand Corporation, Santa Monica, CA, May 12, 1955

Bifulco, G., F. Simonelli, and R. D. Pace (2009): The Role of the Uncertainty in ATIS Applications, Springer Berlin / Heidelberg, vol. 52 of Applications of Soft Computing, 230–239.

Bogers E.A.I, Viti F., Hoogendoorn S.P., Van Zuylen H.J.(2006), Valuation of Different Types of Travel Time Reliability in Route Choice: Large-Scale Laboratory Experiment. Transportation Research Record, 1985, 162-170.

Boyce, D. E., H. S. Mahmassani, A. Nagurney (2005). A retrospective on Beckmann, McGuire and Winsten’s Studies in the Economics of Transportation. Papers Regional Sci. 84 85–103.

Chen H.K., Hsueh C.F. (1998). A model and an algorithm for the dynamic user-optimal route choice problem. Transportation Research Part B 32(3), 219-234.

Chen, R.B. and H.S. Mahmassani (2004), ‘Travel Time Perception and Learning Mechanisms in Traffic Networks’, Proceedings of the 83rd TRB Meeting, Washington D.C.

Clark SD & Watling DP (2005) Modelling network travel time reliability under stochastic demand. Transportation Research 39B, 119-140

Corthout, R., Tampère, C.M.J. & L.H. Immers (2010), Stochastic Dynamic Network Loading for Travel Time Variability due to Incidents. In: Tampère C.M.J., Viti F. & Immers L.H. (eds): New Developments in Transport Planning: Advances in Dynamic Traffic Assignment. Edward Elgar, Cheltenham, UK - Northampton, MA, USA.

Daganzo, C. and Y. Sheffi (1977). ‘On Stochastic Models of Traffic Assignment’, Transportation Science, 11, 253-74.

De Palma, A., N. Picard (2005), ‘Route Choice Decision under Uncertainty’, Transportation Research Part A 39(4), 295–324.

Fosgerau, M., K. Hjorth, C. Brems & D. Fukuda (2008), Travel time variability: definition and valuation, report 1:2008, DTU Transport, Denmark

Horowitz, J.L. (1984), The stability of stochastic equilibrium in a two-link transportation network, Tr. Res. B, Vol;18, No. 1, pp.13-28

Jha, M., Madanat S. and S. Peeta (1998), ‘Perception Updating and Day-to-day Travel Choice Dynamics in Traffic Networks with Information Provision’, Transportation Research Part C 6(3), 189–212.

Kahneman, D. and A. Tversky (1979), ‘Prospect Theory: An Analysis of Decision Under Risk’, Econometrica 47(2), 263–91.

Khattak A.J., Schofer J.L., Koppelman F.S. (1993). Commuters' enroute diversion and return decisions: Analysis and implications for advanced traveler information systems, Transportation Research Part A, 27 (2), pp. 101-111

Lam, T.C. and K.A. Small (2001), ‘The Value of Time and Reliability: Measurement from a Value Pricing Experiment’, Transportation Research Part E 37(2–3), 231–51.

Luo, X. and H.K. Lo (2003), ‘Travel Time Budget in Transportation Networks with Stochastic Degradations’, Proceedings of the 8th Conference of Hong Kong Society for Transportation Studies, Hong Kong, China.

Mahmassani H.S. and R.–C. Jou (1998), ‘Bounded Rationality in Commuter Decision Dynamics: Incorporating Trip Chaining in Departure Time and Route Switching Decisions’, Transportation Research Part B 9, 201–29.

Merchant, D.K. and G.L. Nemhauser (1978a), ‘A Model and an Algorithm for the Dynamic Traffic Assignment Problems’, Transportation Science 12(3), 183–99.

Merchant, D.K. and G.L. Nemhauser (1978b), ‘Optimality Conditions for a Dynamic Traffic Assignment Model’, Transportation Science 12(3), 200–07.

Mirchandani, P. and H. Soroush (1987), ‘Generalized Traffic Equilibrium with Probabilistic Travel Times and Perceptions’, Transportation Science 21, 133–52.

Nakayama, S. (2006), Stability Of Network Flows With Bounded Rational Route Choice, 1st International Symposium on Dynamic Traffic Assignment, 21-23 June 2006, Leeds, UK

Peeta, S. and Ziliaskopoulos, A. (2001). Foundations of Dynamic Traffic Assignment: The Past, the Present and the Future, Networks and Spatial Economics, Vol. 1, No. 3/4, pp. 233-266.

Ran B. and D.E. Boyce (1996), ‘Dynamic Urban Transportation Network Models’. Springer-Verlag, Berlin, Germany.

Ran, B., David E. Boyce & Larry J. Leblanc (1993). A New Class of Instantaneous Dynamic User-Optimal Traffic Assignment Models, Operations Research, Vol. 41, No. 1, Special Issue on Stochastic and Dynamic Models in Transportation, pp. 192-202

Small, K.A. (1982), ‘The Scheduling of Consumer Activities: Work Trips’, American Economic Review, 72(3), 467–479. Reprinted in The Economics of Transport, Herbert Mohring (ed.), in Series of The International Library of Critical Writings in Economics, Edward Elgar, 1994, Cheltenham, UK, 363–75.

Van Berkum, E.C. and P.H.J. van der Mede (1992), ‘The impact of traffic information: Dynamics in route and departure time choice’. Delft University of Technology. PhD Thesis.

Viti F. and Tampère C.M.J. (2010). Dynamic Traffic Assignment: Recent Advances and New Theories towards Real Time Applications and Realistic Travel Behaviour (Editorial). Book chapter of Tampère C.M.J., Viti F. and Immers L.H (Eds). New Developments in Transport Planning: Advances in Dynamic Traffic Assignment. Edward Elgar, Cheltenham UK.

Von Neumann, J. and O. Morgenstern (1944), ‘Theory of Games and Economic Behavior’, 1953 edition, Princeton, NJ: Princeton University Press.

Wardrop, J.G. (1952), ‘Some Theoretical Aspects of Road Traffic Research’, Proceedings of the Institute of Civil Engineers II(1), 325–78.

Watling, D. and M.L. Hazelton (2003), ‘Dynamics and Equilibria of Day-to-day Assignment Models’, Network and Spatial Economics 3, 349–70.