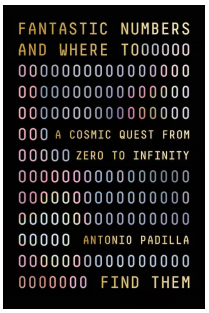
Fantastic numbers and where to find them, *Antonio Padilla*. Farrar, Strauss and Giroux, 2022 (352 p.), isbn: 978-0374600570. **Fabelachtige getallen en waar ze te vinden**, Ambo|Anthos, 2023 (392 p.), isbn: 978-9026353505.






Antonio Padilla

Antonio (Tony) Padilla is a cosmologist and professor of theoretical physics at the University of Nottingham and an active contributor to the popular Numberphile YouTube channel.

The title of the book may be a bit misleading because it's not a popularizing book about number theory. It is a combination of popularizing mathematics, cosmology and particle physics as there are books doing that for mathematics like for example several books by Ian Stewart, and there are similar books about theoretical physics like *A beautiful question* by F. Wilczek or *Theories of everything* By F. Close. It is more similar to G. Farmelo's *The universe speaks in numbers*.

Padilla takes the numbers as a hook to tell about theoretical physics. There are three parts: large numbers, small numbers, and infinity (infinity is not a number).

The large numbers are 1,000000000000000858, googol, googolplex, Graham's number, and TREE(3) while the small numbers are zero, 10^{-16}, and 10^{-120}.

The first of the 'large' numbers is not large at all, but when Usain Bolt on 16 August 2009 ran 100 m in 9.58 sec in Berlin, according to relativity theory he slowed down his clock by that factor. This is how Padilla starts his exposition about Einstein's special relativity theory and how gravity deforms the Minkowski space-time fabric and that energy and mass are essentially the same.

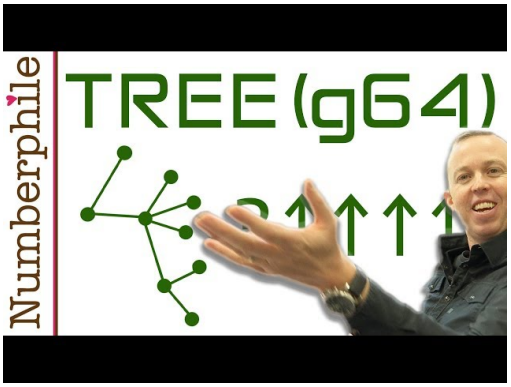
Googol (or 10^{100}, officially called ten sexdecilliard) is a name invented in 1920 by the 9-year old nephew of the American mathematician Edward Kasner. He also invented the name googolplex (10^{googol}) to indicate a fabulously larger number. Under the googol title Padilla introduces the reader to thermodynamics and entropy. When you pump more and more entropy, and thus energy, and thus mass, into a cubic meter, the gravity will create a black hole, which hides all the information it contains. All we can know is its mass, its electrical charge and its angular momentum. A black hole of a cubic meter would have an entropy of about 10^{69} which means that it can contain $10^{10^{68}}$ different microstates, a gigantic large number, but still finite. In the universe the number of microstates will be immensely larger, but still finite, meaning that there there can be at most one twin of yourself per googol but several per googolplex.

Googleplex, under this title we can read about quantum physics experimentally verified by Young's double slit experiment revealing that light behaves simultaneously a wave and as a particle. Schrödinger's probabilistic wave function puts an end to deterministic physics, at least on a microscopic scale and Heisenberg's uncertainty principle limited what can be measured.

To introduce the *Graham number*, some Ramsey theory needs to be introduced and Knuth's uparrow notation: $a \uparrow b = a^b$, $a \uparrow\uparrow b = a \uparrow (a \uparrow (\cdots \uparrow a))$ (b repetitions of $\uparrow$), $a \uparrow\uparrow\uparrow b = a \uparrow^3 b = a \uparrow\uparrow (a \uparrow\uparrow (\cdots \uparrow\uparrow a))$ (b repetitions of $\uparrow\uparrow$), etc. Then Graham's number is $2 \uparrow^{63} 3$. It is a perfectly computable number, but if you tried to remember all its digits, it would be so much information that even a black hole with the size of your head would not be large enough. Your head would simply explode. Even a black hole with the size of the de Sitter space (the largest space we could ever observe if we waited infinitely long) would not be large enough. This leads to a discussion of the entropy of a black hole.

And TREE(3) is even larger still. It is related to trees (from graph theory), hence the name TREE (although H. Friedman used the name TR). TREE(1) = 1, TREE(2) = 3, and then

suddenly TREE(3) explodes to a number that is so large that Graham's number is ridiculously small in comparison. Friedman estimated a lower bound for TREE(3) as $2 \uparrow^{187.195} 187.196$. Here Padilla takes a small excursion to Gödel and his incompleteness theorems. On a cosmological level, he leads us to the ultimate destiny of our universe and an introduction to the holographic principle and string theory. There is a curious observation made by Bekenstein and Hawking that the entropy of black holes is proportional to the area of its event horizon, and not to its volume. So G. 't Hooft and L. Susskind realized that all information must be stored at the surface and that this information should allow to recover the information of the inside like how a hologram is constructed. This gave rise to the theory of a high dimensional reality where some of the dimensions are curled up so small that we do not observe them and we have the illusion that we live in a 4-dimensional space-time. He concludes with a discussion of the correspondence between anti de Sitter (used in quantum gravity theory) vs. conformal field theory (like the Yang-Mills gauge theory used in particle physics).



In the second part, the stage is for the small numbers. Here Padilla starts with zero, which a long discussion about the origin of our positional number system and the acceptance of the zero as a number, and not only as a place holder as it was originally. Eventually this leads to the set-theoretical definition of the natural numbers. For him the zero stands for symmetry, and symmetry plays an important role in physics as Emmy Noether showed.

The next small number is 10^{-16}. When the Higgs boson was observed, its mass was only 10^{-16} times the expected mass. To explain this, there is a long chapter on particle physics. The last small number is 10^{-120}. This chapter is about the cosmological constant. Why does our universe not implode when there is so much vacuum? The answer is gravity. This means that there must be an enormous amount of energy stored in vacuum to prevent this, since on the contrary, it is expanding in an ever increasing speed. It is a bubbling soup with energy appearing but immediately disappearing. However, quantum theory calculations, confirmed by several experiments, shows that the cosmological constant is 10^{-120} times the expected value. This triggers a discussion of why this is true, which results in a discussion of the multiverse and of the anthropic principle: fundamental constants have their values because that is the only way that life can exist.

The final part is about infinity. This has again a larger mathematical part with Cantor as the main contributor: ordinal and cardinal numbers, countable sets, ω, $\aleph_0$, and higher forms of infinity, and the continuum hypothesis. In physics infinite values appear as singularities. However, when they cancel out in computations and give a finite result, they do no harm. The book ends with some ideas about string theory and the Theory of Everything, which is the ultimate goal.

Padilla tells his story in an entertaining way. There are very few formulas, but about every concept is explained using analogies. There are some graphical illustrations, but not many. The mixture of mathematics and mathematical physics and cosmology is present in a way that I have not met in other books. Most of the focus is on the latter, but on the other hand, there is also the extensive exposition about the history of our number system. What is also interesting is that there are also many scientists and mathematicians that have contributed to the theory as we know it today, and Padilla sometimes gives some surprising details on the lives or the character of these people. Because there are so many topics covered and so many people are mentioned, it would have been very useful to have an index of topics and/or people at the end of the book so that they could be looked up easily. Since most of the book consists of compact prose, it is not always easy to find a particular idea that one has met before.

Adhemar Bultheel