Improved computation of characteristic roots of delay differential equation by pseudospectral differencing method

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We aim at the efficient computation of the rightmost, stability-determining characteristic roots of a system of delay diffenential equations. The approach which we use is based on the discretization of the infinitesimal generator of the solution operator by pseudospectral differencing method [1]. We present a procedure that can automatically select the number of discretization points N to compute all roots in a given right half-plane accurately.

Consider a system of linear delay diffenential equations (DDE) of the form

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{1}$$

where $x(t) \in \mathbb{R}$ is the state variable at time t, $A_i \in \mathbb{R}^{n\times n}$, $i = 0, 1, \ldots, m$, are real matrices, and $0 < \tau_1 < \tau_2 < \cdots < \tau_m$ represent the time-delays. The characteristic roots of (1) are defined as the roots λ of the characteristic equation

$$\det(\lambda I - A_0 - \sum_{i=1}^{m} A_i e^{-\lambda \tau_i}) = 0. \tag{2}$$

We follow the method in [1] to compute the rightmost characteristic roots of (2). Our contribution lies in an automatic determination of the number of discretization points N. Our proposed approach is based on connections between the pseudospectral differencing method and a rational approximation of exponential functions in (2). Roughly speaking, firstly, we estimate a region that contains all the desired characteristic roots by the method in [2]; secondly, we select the smallest N such that in this region the rational approximation of the exponential function is accurate.

References

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