Model Reduction by Balanced Truncation of Linear Systems with a Quadratic Output

Roel Van Beeumen

Katholieke Universiteit Leuven
Celestijnenlaan 200A, B-3001 Herverlee
Belgium
roel.vanbeeumen@student.kuleuven.be

Joint work with: K. Meerbergen

Balanced truncation is a widely used and appreciated projection-based model reduction technique for linear systems

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}.$$

This technique has the following two important properties: approximations by balanced truncation preserves the stability and the $\mathcal{H}_\infty$-norm (the maximum of the frequency response) of the error system is bounded above by twice the sum of the neglected singular values.

This paper tries to extend the framework of linear balanced truncation to systems with a quadratic output

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= x(t)^T Sx(t) \end{cases}.$$

For such systems, the controllability Gramian remains the same, but it is not possible any more to calculate the observability Gramian. Three different approaches are introduced to overcome this problem.

We also give numerical examples for large-scale systems. In this case the Gramians are computed by a block form of the Arnoldi-Lyapunov method.

Keywords: Numerical linear algebra, Model order reduction.