Optimising the robust stability of time-delay systems via nonsmooth eigenvalue optimisation

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1 Introduction

The stabilisation of a system with a finite number of free parameters is a problem that typically arises in the design of fixed-order controllers. Most methods attempt to solve this stabilisation problem by trying to satisfy certain types of stabilisibility conditions. The drawback of such methods is that these conditions are in general sufficient, but not necessary. Therefore, the system may be stabilisible although the stabilisibility conditions cannot be satisfied. Our approach is to directly act on the spectrum of the system to optimise the corresponding stability. We will first consider a nonsmooth optimisation method that maximises the exponential decay of the system trajectories. Since this commonly yields unrobust solutions, we will also apply this method to the optimisation of the robustness of a system against perturbations.

2 Spectral abscissa minimisation

A straightforward method to solve the stabilisation problem is to minimise the largest real part of the eigenvalues, also called spectral abscissa α, in function of the optimisation parameters. Asymptotic stability is then achieved as soon as this objective function becomes negative, because the trajectories are bounded by an exponential decay that lies below e^{α}. However, the spectral abscissa is in general a nonsmooth and nonconvex function, precluding the use of standard optimisation methods. Instead, we use a recently developed bundle gradient optimisation algorithm, namely the gradient sampling algorithm [1], which has already been successfully applied to fixed-order controller design problems for systems of ordinary differential equations.

We apply this approach to the stabilisation of systems of time-delay type. As the linearisation of such systems has in general an infinite number of eigenvalues, a direct pole placement is indeed not possible. In dealing with these time-delay systems, we thus extend the use of the gradient sampling algorithm to infinite-dimensional systems. This is realised by combining the optimisation method with advanced numerical algorithms that are able to efficiently and accurately compute the rightmost eigenvalues of time-delay systems with the use of linear multistep techniques.

A problem with the minimisation of the spectral abscissa is that it commonly results in a spectrum for which the eigenvalues are of higher multiplicity. Such eigenvalues are very sensitive with respect to small changes of the parameters or of the system's definition. This means that a tiny error in the system model may in practice lead to a huge increase of the spectral abscissa or may even result in the loss of stability. Thus, although it provides an optimal stability, the solution of a spectral abscissa minimisation is often accompanied by a very bad robustness w.r.t. stability against perturbations.

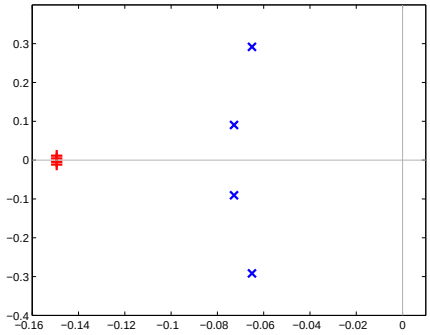

Figure 1: The spectrum of the optimised system w.r.t. exponential decay (+) and w.r.t. robust stability (x)

3 Robust stability

In [2], it is shown that the robustness of a system can be optimised by maximising the complex stability radius, measuring the smallest perturbation which would lead to instability. Since a point for which $\alpha < 0$ suffices to guarantee stability, we can use this point as a starting value for the robustification process. Fig. 1 depicts the spectra as a result of a spectral abscissa minimisation as compared to a complex stability radius maximisation, showing that the robustness is indeed improved, at the cost of a lower spectral abscissa.

References

[1] J.V. Burke, A.S. Lewis, and M.L. Overton, *A robust gradient sampling algorithm for nonsmooth, nonconvex optimization*, SIAM J. Optimization, 15 (2005), pp. 751-779.

[2] W. Michiels, D. Roose, *An eigenvalue based approach for the robust stabilization of linear time-delay systems*, Int. J. Control, 76 (2003), pp. 678–686.