Les Pook *Flexagons inside out*, Cambridge University Press, 2003 (170p.) ISBN 0-521-52574-8.



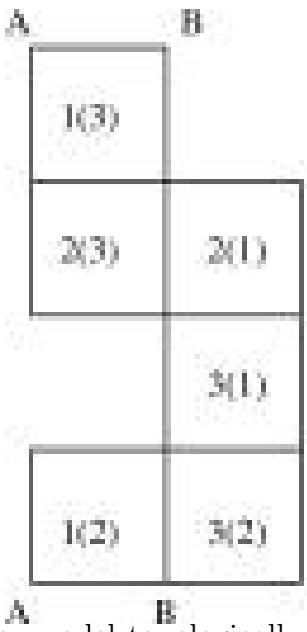
Flexagons were discovered in 1939 by Arthur H. Stone, a postgraduate student at Princeton University. He cut American size paper to fit it in an English binder. He folded the ribbons of paper that he had cut off, in a spiraling fashion and glued the ends together and that was the first flexagon.

It is difficult to explain to a reader what exactly a flexagon is if he has never actually made a paper flexagon or at least played with one that was produced by somebody else. A somewhat fuzzy definition from Eric Weinstein's encyclopedia of mathematics goes as follows.

An object created by folding a piece of paper along certain lines to form loops. The number of states possible in an n-flexagon is a Catalan number. By manipulating the folds, it is possible to hide and reveal different faces.

That is correct of course, but it will not help you much to imagine what this "object" actually looks like. Well I didn't have the faintest clue after reading and re-reading this definition a couple of times.

So, I think the first thing you have to do if you are interested in knowing what flexagons are and never actually played with one, is to produce one. So here is a little exercise that you can do yourself, or that you can ask the children to do for you if they are old enough to use scissors. I show in the accompanying figure a square grid which you should cut in paper. Fold along the lines in both directions and flatten again. The numbers without brackets are written on the top and the numbers in brackets are written on the back of the squares. Then fold along the lines such that (3) covers (3) and 3 covers 3. The top line AB and the bottom line AB will then match and these are glued together with a piece of tape. The result is a 2 by 2 square with on one side all 1's and on the other side all 2's. Fold along the horizontal line to let the 1's cover up each other and reopen to let the 3's reappear. Now the 2 by 2 square has all 3's on one side and all 2's on the other. This is the simplest possible square flexagon with only 2 possible states. If you look at this paper model topologically, you will see that it is actually a Moebius-like thing with only one side (after the gluing).



In general flexagons can have a polygonal outline when it is in one of its flexed states, and some folding/unfolding manipulation will transform it from one state into another.

A simple google search on the Internet will result in many weblinks related to flexagons. Some of them show how to make some flexagon, others show a movie in which you can see the flexing actually taking place. Others give nice decorations (the numbers can be replaced by colors or different decorations which are decomposed and reassembled by flexing).

Flexagons have been popular ever since they were discovered. A 'Flexagon Committee' was founded at Princeton with members R.P. Feynman (a physics student) and J.W. Tuckey (young mathematics lecturer). In fact already in 1940 they worked out a first theory on hexaflexagons (which have an hexagonal shape). Construction and a brilliant simulation can be found on www.drking.worldonline.co.uk/hexagons/flexagons/.

Martin Gardner has further popularized these "puzzles" is his *Scientific American* column in 1956 and 1957. It was only in 1957 that a serious mathematical paper on flexagons authored by Oakley and Wisner appeared in the *American Math. Monthly*. A key reference is an internal RIAS report by Conrad and Hartline that is now available at delta.cs.cinvestav.mx/~mcintoch/oldweb/new.html.

The mathematical theory is not simple, certainly not if one moves to all possible generalizations that reach beyond what is practically realizable, or when classical components like squares or triangles are replaced by other polygons. While flexagons are in their different states two-dimensional, but fold in three-dimensional space to switch between the different states, flexahedra are in their different states three-dimensional, while transformations are in the four-dimensional space.

Flexagons have always been in the realm of recreational mathematics, and the serious mathematical study has not known an overwhelming number of publications. The present book is also to be placed in the recreational sphere (the reader need not know anything about mathematics) but it still wants to stimulate the reader to think about some of the mathematical questions that could be asked in this context. However in the first place it is a collection of nets and instructions to construct numerous different flexagons. You just have to photocopy some pages and the rest is cutting and pasting. The reader who gets easily addicted to this kind of puzzles should refrain from reading this book, unless he/she is retired and has a lot of free time to spend on it. Fun is guaranteed.

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