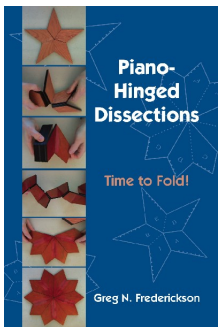
Piano-hinged Dissections: Time to Fold! *Greg N. Frederickson*, A K Peters, Wellesley, MA, 2006 (320 p.), Hard cover, ISBN 978-1-56881-299-1, 49.00 USD.



This is a sequel to the two previous books on dissections by the author [1-2] (both reviewed in this newsletter no. 44, September 2003). In the first book shapes (e.g. a square) were cut to separate (polygonal) pieces and puzzled together again to form another shape (e.g. a triangle). In the second book, extra constraints were introduced. Some of the vertices of the pieces were connected so that they form one or possibly more chains. Two connected pieces can only rotate in the plane around their joined vertex. The present book is a logical sequel. Now the pieces are connected along edges. Although the shapes are two-dimensional, one needs a third dimension to flip the pieces along the connecting hinges. Take for example a rectangular piece of paper that is twice as wide as it is high. Folding it along a vertical line in the middle, one obtains a double layered square; folding it along a horizontal line in the middle gives a double layered rectangle that is four times as wide as high. See the left of Fig. 2.

This is an example of how it is possible to unfold a double layered shape (a square) and refold it into another double layered shape (a rectangle). This is a trivial example of a piano-hinged dissection. Of course much more complicated examples are possible. Another simple example is shown in Fig. 1.

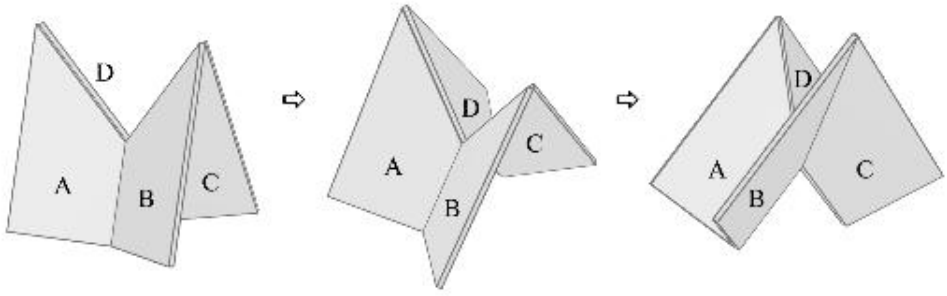


Fig.1: A piano-hinged dissection. From a mitre to a gnomon.
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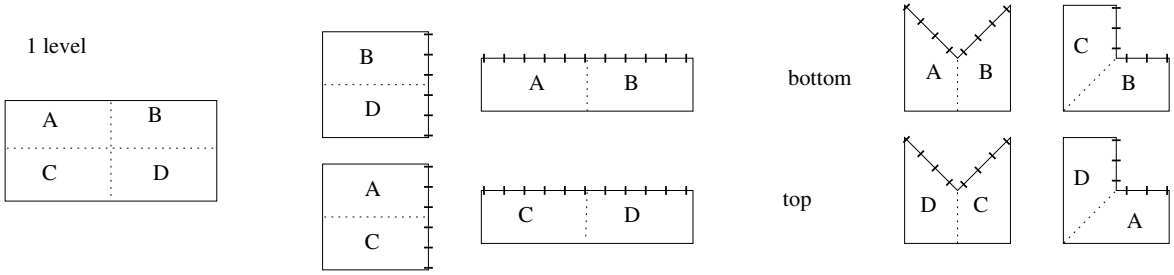

Fig.2: Notation for a very simple piano-hinged dissection (left) and for the mitre to gnomon (right).

The representation in Fig. 1 is a nice but rather unpractical way to represent in general all the cuts and hinges that are needed as well as the final arangements of the two shapes. This was solved in the following way. The previous (trivial) example of reshaping the square into a rectangle is represented on the left of Fig. 2. The mitre to gnomon example is on the right.

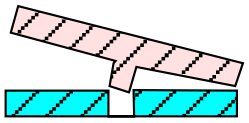

Fig.3: A tap into a hole needs rounding when the plates are hinged.

The dotted lines indicate hinges on the same level, the crossed lines indicate hinges between top and bottom level. Full lines are cuts (for the visible layer). The letters refer to the different pieces.

Just folding and cutting paper might be relatively simple. It becomes much more involved if the shapes are realized with thin wooden plates (with

a thickness that can not be neglected) and physical piano hinges (that can be attached to one side of the plates only). This implies that you can fold two hinged plates in one direction, but not in the other. Moreover there are physical limitations like in Fig. 3, where it is impossible to swing a tap into a tightly fitting hole if the tap is attached to a plate that is fixed by the hinge on the right. So here some rounding is needed to make things work.

Because essentially all the moves discussed need a three-dimensional space, it is sometimes difficult to give a clear explanation of the operations to be performed. This gave Frederickson the idea of including a cdrom on which he demonstrates the folding, decomposing and recomposing the dissections. Some are really complicated, and even if you see him doing it, it is sometimes impossible to see how all the pieces fall into place. At least it is a visual proof that the method does indeed work. It is quite funny if you see him fighting with a wooden model of a hinged dissection that is a loop consisting of 30 triangles hinged together. See Fig. 4. With this dissection it is possible to form a double layered square but also all 12 double layered letters from the pentomino alphabet *en.wikipedia.org/wiki/Pentomino*.



Fig.4: A piano-hinged loop of 30 isosceles right triangles. The left edge should be hinged to the right edge.

And there are many other of these geometrical puzzles. Some of them are relatively simple and are e.g., related to the way we fold a paper large map into some handy pocked sized format. Others, in fact most of them, are quite a challenge and are beautiful in their complexity. Some strategies for solving these puzzles are proposed.

Throughout the book reference is made to a manuscript by Ernest Irving Freese. In fact, this is a remarkable story. Freese was a Los Angeles architect who did this kind of puzzles and finished a manuscript about it in 1957. So he mentioned in a letter to a friend. Unfortunately he died before it ever got published. His widow never responded to people who tried to get hold of the manuscript. Later his son Bill did not find it, and hope was given up that it would ever be found. Only after Bill died in 2002, a cousin was cleaning up the house and found a letter by Frederickson inquiring about the manuscript and eventually, several months later, she did find the manuscript and this is how Frederickson got hold of it. Several drawings are reproduced in this book.

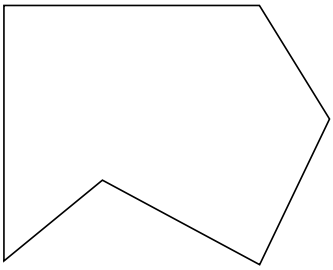

Fig.5: Double layered shape to be transformed into a square as a piano-hinged dissection.

More information, pictures, and additions can be found on the web page for the book which is *www.cs.purdue.edu/homes/gnf/book3.html*.

Finally, for those who have read this whole text through, here is a "simple" puzzle to be solved. The double layered shape in Fig. 5 has to be cut and equipped with piano-hinges such that it can be transformed in a double layered square. The solution can be found elsewhere in this Newsletter.

References

[1] G.N. Frederickson, Dissections: Plane & Fancy, Cambridge University Press, New York, 1997.

[2] G.N. Frederickson, Hinged Dissections: Swinging & Twisting, Cambridge University Press, New York, 2002.

[2] L. Pook, Flexagons inside out, Cambridge University Press, New York, 2003. (Revewed in this newsletter no. 46, January 2004.)

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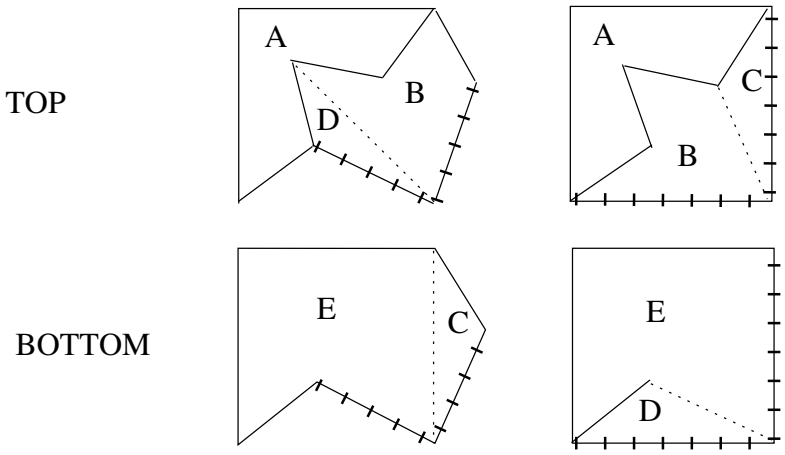

Solution of the piano-hinged dissection puzzle elsewhere in this newsletter.