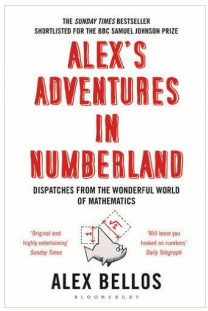
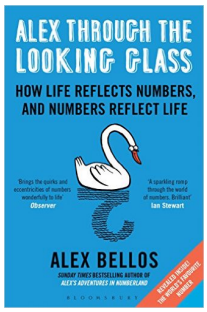
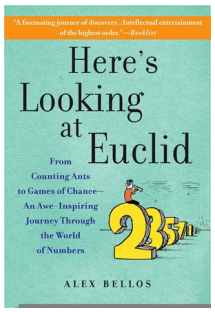
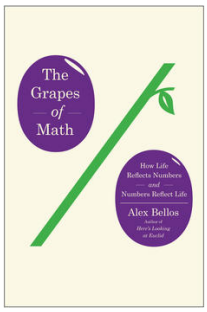
Two books by **Alex Bellos**. The UK editions are
Alex's Adventures in Numberland Bloomsbury, (2011) ISBN 978-1408809594 (pbk), 448 pp
Alex Through the Looking Glass Bloomsbury (2015) ISBN 978-1408845721 (pbk), 352 pp.
and their US counterparts are
Here's Looking at Euclid Free Press, (2011) ISBN 978-1408845721 (pbk), 336 pp.
The Grapes of Math Simon & Schuster (2014) ISBN 978-1451640113 (pbk), 352 pp.









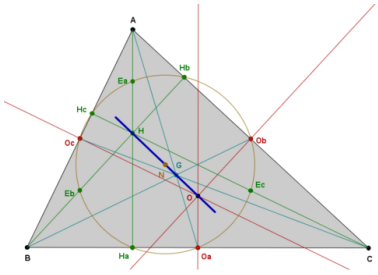

Alex Bellos

Alex Bellos wrote two successful books on popular mathematics. The first one is from 2010 and the most recent one from 2014. It just got a US paperback edition in 2015. The titles chosen may create some confusion because the UK and the US edition got different titles. Although the books are not that recent, they still deserve to be recommended for reading. Alex Bellos is a British mathematician who also got a degree in philosophy. At first he was South-American correspondent for the Guardian and wrote about Brazil. After his return to the homeland he produced these two marvelous books about mathematics.

I like the first one the most. Its subtitle is *From Counting Ants to Games of Chance - An Awe-Inspiring Journey Through the World of Numbers*. It was well received and it won several prizes. It brings an overview of many aspects of mathematics in a way that it is most entertaining for anyone, and you don't need to be a mathematician to appreciate it. It is clear that Bellos enjoys the subject and he doesn't get his information from dull books, but he actually interviewed several people and met them in person, which makes it a lively account. The historical facts and anecdotes do not diminish the liveliness of his story.



French decimal clock



Euler line

The book starts with the origin on how and why we human started counting. Bellos considers this a prequel to the actual story and hence calls it chapter zero. What is the origin of numbers, and how do humans perceive numbers (we catch small amounts without counting, but larger numbers need grouping or counting), and how much of the number concept can a chimp acquire?

The real stuff starts in chapter 1 where we meet the different number systems, how to count on your hands or your whole body. Why we have a decimal system for almost everything but why attempts to introduce the decimal clock were not successful. We get a manual of how to compute with an abacus or its Japanese counterpart, the soroban.

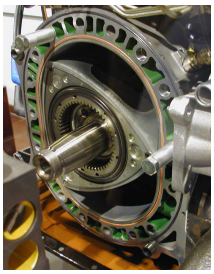
In chapter two we meet numerology. Pythagoreans detected many patterns in number arrangements and geometric patterns: square numbers, triangular numbers, and we get many different geometrical proofs of the theorem of Pythagoras. We also meet the Platonic solids and the Euler line containing the four possible centers of an arbitrary triangle. Tessellations of the plane and Penrose tiles, the Menger sponge, and origami.

The most important number, the zero in only introduced in chapter three. There we also meet the different ways to compute with numbers. The mythical Vedic mathematics turn out not to be so much different from finger counting that was used by shepherds in France and Russia. The Swami Bharati Krishna Tirtha that Bellos interviewed was not really convincing. The Hindu gave us the zero, but for them the zero is certainly not a synonym of nothing. *Nothing is an existing entity; nothing is everything. Zero was invented by a culture that accepted the void as the essence of the universe.*

Chapter 4 is called 'Life of Pi'. Remarkably it starts with some stories about lightning calculators, the

human computers on steroids. Probably a dying species since the introduction of the electronic computer. Japanese pupils are still drilled and there is a worldwide hype where mental calculators challenge one another in international competitions. Japanese sometimes use a mental abacus to do the computations since you see their hands shifting the beads like mad. Reciting digits of π as far as possible is one of the possible competitions and this is an incentive to explain several ways to compute π. Of course π is directly connected with the circle, but the chapter ends with a discussion of other curves of constant width like the Reuleaux triangle (used in the Wankel motor and a drill of this form can produce square holes) and the heptagonal circumference of a 50p coin.


Reuleaux triangle

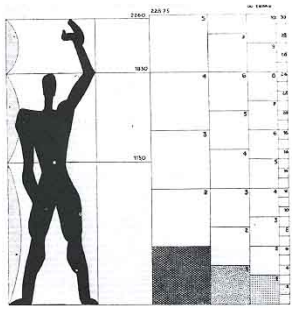

Wankel motor


supereggs

The history of algebraic equations is told in the next chapter. It also tells about the instruments that were used to help doing the arithmetic. The first ones were the slide rules based on the principle of logarithms. Then there were instruments like Fuller's calculator or the curta (a miniature pocket calculator), but then also the first electronic pocket calculators that were extremely expensive but on which you could produce words with the digits when the instrument was turned upside down. The chapter concludes with the introduction of quadratic equations, Descartes analytic geometry and the superellipse and his superegg with equations like $(x/a)^n + (y/b)^n = 1$ that Piet Hein commercialized in design objects.

What we know as a 9 digit magical square is known in China as lo shu, symbolizing harmony in the universe, but magic squares and other number patterns have been used in all religions. For example the improper 4×4 square on Gaudi's Sagrada Familia cathedral in Barcelona modified to add up to 33, the age of Christ when he died. And then there is the birth of another 'magic square', the Sudoku, as a worldwide phenomenon. Other puzzles include the Chinese tangrams, the 15 puzzle, the Rubik cube and the rearrangement geometric puzzles rearranging a cut-up triangle into a square, or the missing square puzzles and more recently the hinged dissections.

Next are the most exotic sequences in Sloan's On-Line Encyclopedia. The most exotic ones can be found there. Some have produced conjectures as yet unproved. Goldbach's conjecture for example, but also the powertrain by J.H. Conway: take any number e.g. 3462 and transform it into $3^4 6^2 = 2916$ and repeat to get $2^9 2^6 = 512 \to 5^1 2 = 10 \to 1^0 = 1$. Almost all numbers will end up in 1, but Conway found an exception 2592 that ends up in a cycle. Sloan found a second one 24547284284665600000000000. Are these the only ones? Probably not, but there is no proof. And there are, besides the prime numbers, many other 'strange sequences' with unanswered questions like the Récaman sequence etc.


le Corbusier Modular Man

Chapter 8 deals with the golden ratio and the Fibonacci numbers and spirals that keep appearing so often in nature and in many man-made objects. The human legend created by Adolf Zeising in the late 19th century that the golden ratio is a spiritual ideal proportion and incarnates the key to beauty seems to hold only approximately, even though Le Corbusier's Modular Man is based on it.

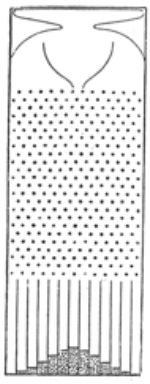
Probability is the next subject. That is based on chance, and it was also the origin since the foundations were made by Blaise Pascal when Chevalier de Méré asked him to compute his gambling chances in the casino. Pascal used it in his famous wager which states that if there is a 50% chance that God exists, then it is better to bet on it that he does exist. Bellos elaborates on chances you have on the roulette, and the enormous appeal the slot machines have on humans. This brings him to discuss randomness and random walks and there is the incredible story of Ed Thorp, a mathematician who showed that card counting could overcome the casino's advantage with blackjack. He later applied his technique to hedge funding on the financial market which brought him great fortune.

The bell-shaped normal distribution is the subject of chapter 10. We learn that Poincaré so exposed his baker as a fraud selling underweight bread to him. Even after the shop was warned by the economic inspection, the distribution was skew showing that the fraud continued but that they always give him

the heaviest one available. The Belgian Quételet was a professional collector of data, which illustrated the omnipresence of the normal distribution. This quote by Bellos is perhaps worth reproducing in this Newsletter: *Adolphe Quételet has good claim to being the world's most influential Belgian. (The fact that this is not a competitive field in no way diminishes his achievements.)*. The connotation 'normal' to the bell shape was however promoted by the English Galton who also designed the machine which he called the quincunx to illustrate the bell shape.

The last chapter is about hyperbolic geometry, including Poincaré's circle model, so beautifully illustrated by Escher, but also Daina Taimina earned some fame with her hyperbolic crochet works. Her idea is simple: start with a line of stitches and add more stitched per line as you add new lines. The topology of our universe brings along the concept of infinity and Cantor's $\aleph_0$ and the classification of the infinite.

The book is well illustrated and there are some appendices with some more technical content, and also per chapter many interesting references for further reading.

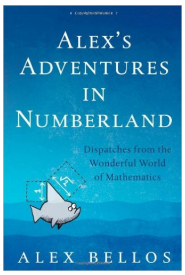


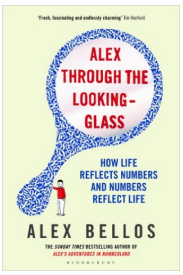
Galton board or quincunx

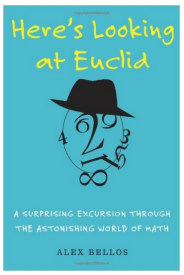


Adolphe Quételet @ Royal Academy Brussels

The sequel **Alex Through the Looking Glass** has the subtitle *How life reflects numbers and numbers reflect life* is a continuation with more of the same kind of material. Again some spotlight is put on numbers and their history, but also on the psychological aspects, how analysis emerged from counting and computing, the intuitive resistance by some against new concepts such as negative and imaginary numbers, the pitfalls of logic paradoxes, cell automata as models of biological systems.







The first chapter is about the feelings that are assigned to numbers like being lucky or unlucky, why 7 is most often picked as a random number between 1 and 10, how a number associated with a commercial product can make it a bestselling top or a flop, the well known pricing methods like 2.99 € instead of 3.00 € to mislead the costumer, etc. It is remarkable that if you ask people to select the numbers they like or think of as good, or interesting, heavy, favorable, etc., then it turns out that prime numbers are recognized in the patterns that result. This might have something to do with our familiarity with times tables.

The second chapter is about the well known Bedford's law. In many large sets of numbers, the lower digits appear more frequently as the first digit. In fact the frequency of the numbers 1-9 follow some exponentially decaying distribution. A similar distribution can be obtained for words in a long text like a book. These examples are used to introduce logarithmic plots for power laws, and the more general Zipf's law that appears for many types of data like Kleiber's law in biology or popularity laws in big networks.

Triangles are the theme of the next chapter. From Thales's method to measure the height of the pyramids or Eratosthenes to measure the circumference of the Earth, to Pythagoras's theorem, to the triangulations of Gemma Frisius used in the 17th and 18th centuries to measure the meridian and find an answer to the dispute about the Earth being flattened at its poles or not.

Next focus is on conic sections and the finding of Galileo and Kepler about our solar system where the ellipses are featuring. In falling objects or in spotlights or in capturing systems, parabolas are the main object while hyperbolas appear in architecture. More general curves were used by the military in nomograms until mid 20th century. The circle is of course the one that is immediately connected to π. Bellos is obviously a strong promoter of the τ instead. Not the ratio of the circumference over the diameter, but the ratio of the circumference over the radius, which is $\tau = 2\pi$, is the proper constant that deserves all our attention. No more pi-day for him.

Other important curves are introduced from a physical point of view: cycloids, tautochrone, brachistochrone, different epicycloids, periodic waves and Fourier techniques. To illustrate what kind of anecdotes that can be found in the book, let me give this example. Bellos tells the story of Johan Bernoulli who challenged his peers to find the brachistochrone curve. Since after some time, he only got an answer from Leibniz, he challenged Newton by renewing the challenge and adding "those who boast that trough special

methods ... they have not only penetrated the deepest secrets of geometry but also extended the boundaries in marvelous fashion" and sent a copy to Newton. Newton was at the time not in Cambridge anymore but was in charge of the Royal Mint. After receiving the letter he worked on the problem till 4 in the morning and submitted his answer anonymously whereupon Bernoulli said "ex ungue leonem" (I recognize the lion by its claws).

After π, the number e is the next one to be in the spotlight. It starts by a convincing attempt to illustrate to the reader how incredibly fast exponential growth is. The growth of cities, of bacteria, of the world's population. The exponentials can be combined to form the cosh, which is the form of the catenary curve, used by Gaudi in its upside-down version in the construction of the Sagrada Familia church but it appears in several other constructions as well. While Jakob Bernouilli introduced e as the limit of $(1 + 1/n)^n$, it was Euler who wrote it as the series summing the reverse factorials. It also shows up as $1/e \approx 36.8$ in the Marriage problem which should be solved according to the rule: skip the first 36.8% of the candidates and take then the first one that is better than all the previous ones.



cycloid on Bernoulli's collected work
supra invidiam - above envy - for the dog


Gaudi's chain models

The negative numbers and the imaginary numbers did not come overnight. A huge psychological resistance had to be overcome and it required a serious leap of the mind to accept them as 'numbers'. Once complex numbers are represented in the plane and connected with radius and argument, nice Mandelbrot sets come into reach.

Further development of mathematics came with Newton and Leibniz to introduce calculus. Differential equations govern most of the physical phenomena but curvature is also important for the design of road or railway curving and for the design of roller coasters on fairgrounds. The clothoid is the curve that solves the latter problem. Several of the fairground roller coasters had to be shut down because it caused neck injuries. Werner Stengel looked into the problem and designed the *Great American Revolution* for Six Flags Magic Mountain in 1976, the first loop-the-loop that worked properly because of the characteristic inverted teardrop of the clothoid. The flamboyant character of Cédric Villani also gets an extensive description here.


Six Flags loop-the-loop (1976)


Cloverleaf exits on highway

The crisis in logic and set theory in the early 20th century was related to continuity of the real numbers, and the axiom of choice, leading to the ZFC axiomatic system, the start of the Bourbaki group in France, and the acceptance of proofs by computer.

The last chapter is about cell automata, the game of life of John Conway with its many particular configurations like gliders, guns, beehives, traffic lights, etc. Gurus of the game were anxiously designing patterns to obtain special effects.

As this survey illustrates, as much as possible, the mathematics are related to our daily life. The mathematics are accessible for anyone. Some details are moved to an appendix. There you can also find a glossary of terms used, and more details for further reading, and if you want to look up some name or concept, you can use the excellent name and subject index. Both books are a pleasant read that is warmly recommended.

Adhemar Bultheel