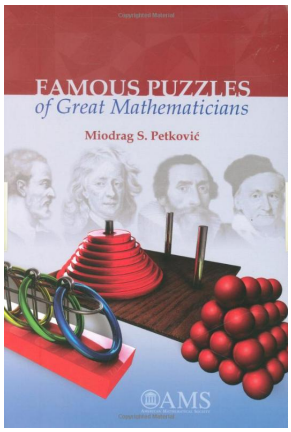
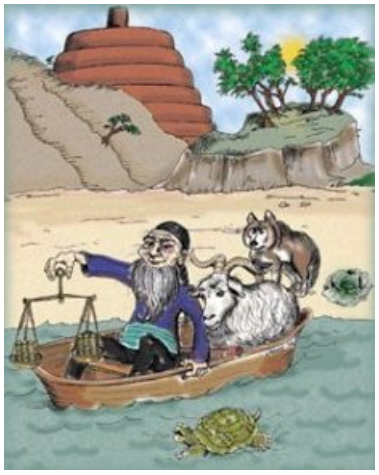
Famous puzzles of great mathematicians *Miodrag S. Petković*, American Mathematical Society, 2009 (xiii+325 p.), soft cover, ISBN 978-0-8218-4814-2, US$36.00.



There are already numerous books on mathematical puzzles and recreational mathematics. So what is so special about this one? M.S. Petković is a professor of mathematics at Niš (Serbia). Besides his work on interval arithmetic and polynomial root-finding, he has developed great interest in recreational mathematics. So he published a first book *Mathematics and Chess*, Dover Publications in 1997. In this book he collects a number of "classic" puzzles and connects them with their history. So what you can find is a collection of mostly well known puzzles, their solution, and some historical facts about the mathematicians that have "invented" these or at least were involved in finding a solution (but most well known biographical facts are skipped, just the witty bits are given) and some variations on these puzzles are included as brain teasers for the reader. The solution of these are only found at the end of each chapter.

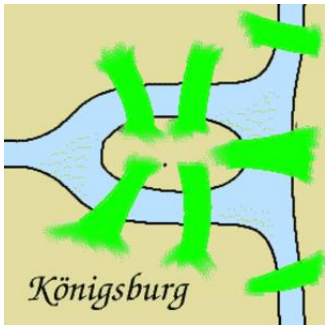
After an introductory chapter placing recreational mathematics in a broader mathematical and historical perspective (there is a lot of namedropping here for problems discussed in subsequent chapters), the book consists of eight chapters collecting the problems around mathematical disciplines (arithmetic, number theory, geometry, tiling and packing, physics, combinatorics, probability, and graphs) and a last chapter on chess problems, which seems to attract the author's special interest. Some miscellaneous teaser-problems are collected in a short chapter 11. The book ends with appendices giving some mathematical background and a list of short biographies of the important mathematicians featuring in the book.



River crossing problem. Illustration from *Introduction to the Design and Analysis of Algorithms* by Anany Levitin

The puzzles are almost all classical. For example, the river-crossing problem where a man has to bring a goat, a cabbage and a wolf across the river. This is attributed to *Alcuin of York* (735–804) and can be solved by finding a shortest path in a digraph. The proof by *Leonhard Euler* in 1736 that there was no solution to the problem of the Königsberg bridges was in fact the start of graph theory. This is one of the statements that Petković makes: what may have started as recreational mathematics, has turned later sometimes into a fully fledged mathematical discipline, so that they are more than just a gratuitous time-passing.

It will come at no surprise that probability found its origin in problems that were posed by gamblers. Antoine Gombaud, chevalier de Méré, who was a notorious gambler, asked his friend *Blaise Pascal* (1601–1665) to design a good strategy for his gambling. Pascal discussed the problem with *Pierre de Fermat* (1601–1665), which became the embryo of probability.

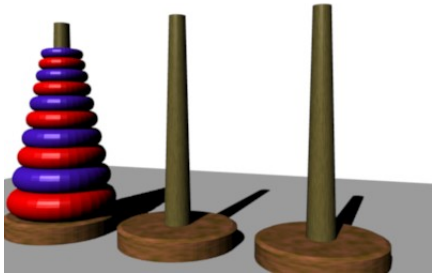


Bridges of Königsberg

Combinatorics is of course a rewarding subject to design puzzles. So this chapter is well stuffed. You may learn here for example that the formula for the combinatorial numbers $C_k^n = n!/(k!(n-k)!)$ is probably from the Indian mathematician *Mahāvira* (ca. 800– ca. 870). A very well known puzzle from this chapter is the "Tower of Hanoi" problem. This seems to originate from *Edouard Lucas* (1842–1891), who formulated it in 1883 under the alias M. Claus (an anagram of his name). Also the Chinese rings is a familiar toy that is sold even today. Its solution is similar to the solution of the Tower of Hanoi. Its origin is less certain. It could be that it was used by French countrymen as a lock for chests, or it could have been invented by a Chinese general *Hung Ming* (181–234) to amuse his wife while he was at war. In any case, *Geroldamo Cardano* (1501–1576) seems to be the first to describe it in 1550.

The physics chapter has many problems about motions, which often result in summing infinite series. Everyone is probably familiar with Zeno's paradox on the race of Achilles and the tortoise. There is

also the problem of a girl and a pigeon both starting in the direction of a wall that is 500 feet away. The girl walks at 5 feet per second, the pigeon flies back and forth between the girl and the wall at 15 feet per second. The question is what distance the pigeon has travelled before the girl hits the wall. The hard way is to compute the subsequent distances between the girl and the wall when the pigeon reaches the girl and sum all these distances. However, the much simpler solution is to observe that the girl reaches the wall after 100 seconds and that the pigeon has travelled in that time 100 times 15 feet. The legend says that *John von Neumann* (1903–1957), immediately came with an answer, his teacher said "Ah, you noticed the trick" whereupon von Neumann answered "What trick? It was an easy series".



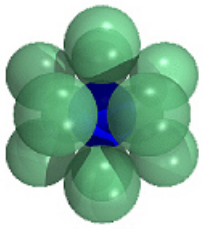
Tower of Hanoi

Problems related to tiling and packing have also been a recurrent subject of recreational mathematics. The drawings of M.C. Escher are wel known, but the names of mathematicians such as *Roger Penrose* (b. 1931) who discovered in 1974 nonperiodic tilings of the plane and also *John Horton Conway* (b. 1937) are immediately linked to this field. Conway was also the inventor of "the game of life" that gave rise to the study of cellular automata. Also *Donald Knuth* (b. 1938) participated in this and besides being the designer of TEX and METAFONT, and the author of the 4 volumes of *The art of computer programming*, he also invented many problems with polyominoes (any figure of n edge-connected unit squares). But some tiling and packing problems are much older. The problem of "kissing spheres" came as a result of a discussion between *David Gregory* (1659–1708) and *Isaac Newton* (1643–1727) about how many unit spheres can simultaneously touch a given unit sphere. According to Newton there were 12, and according to Gregory there were 13, but none of them had a proof. That proof was only given by K. Schüttle and B.L. van der Waerden in 1953: there are at most 12.



Chinese ring puzzle

There are numerous puzzles that relate to geometrical problems. For example the problem of finding the diameter of a sphere using only a ruler and compass is ascribed to *Tābit ibn Corra* (826–901), and there is the so called Dido problem: among all closed curves of a given length, which is the one enclosing the largest region. This naming refers to the the princess Dido, who negotiated to buy the land that could be enclosed by a bull's skin. She cut it in narrow strips and acquired the land on which later Carthage was founded.



Kissing spheres

From number theory, I quote the stamp problem of *James Joseph Sylvester* (1814–1897) and a similar problem formulated with coins by *Ferdinad Georg Frobenius* (1849–1917): what is the largest amount that can not be made up with un unlimited number of stamps of 5 and 17 cents? The answer is 63. Of course there are many generalizations of this problem.

A typical arithmetic problem as posed by *Isaac Newton* reads: In 4 weeks , 12 oxen will consume $3\frac{1}{3}$ acres of pasture. In 9 weeks 21 oxen will consume 10 acres of pasture. How many oxen will it take to consume 24 acres in 18 weeks? In his biographical note Petcović says that Newton was from Jewish origin, which is remarkable since all sources I could consult seem to deny this. A slip of the pen perhaps.

The most typical chess problems are the the "knight's circles" (a knight has to visit all squares exactly once) which has attracted the attention of many mathematicians, and the 'eight queens" problem (how many ways to place 8 queens on the chess board so that they do not attack one another).

There is thus a lot of petty facts, like funny quotes and oneliners by historical figures. I only showed a few sips from a bowl plentyful of facts. Thus, even for puzzle fanatics I think there is a lot of material in this carefully compiled book that will be new to them too. The name index at the end shows the long list of persons that appear in the text, but unfortunately it has only the names of persons. It would have been interesting to also have a subject index so that one could look up the history of some puzzle.

Adhemar Bultheel