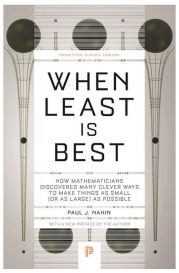
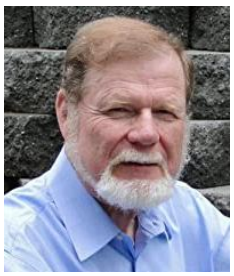
When least is best by *Paul J. Nahin*, Princeton University Press, 2020 (392 p.), isbn: 9780691218762.




Paul J. Nahin

This is an accessible discussion of the mathematics that are used throughout history to solve all kinds of optimization problems. In 2007 a paperback edition appeared introducing some extra optimization problem in the preface. Selecting the book as a volume in the wonderful Princeton Science Library series is most welcome. In the preface for this new edition, Nahin again adds a new problem that he always regretted not to have included in the original version (2004).

This problem is illustrative for some of the problems discussed in this book. Given are two points (2 cities) A and B in a plane at a positive distance from a line R (a river) and a point P (pump station) on R. Find the (the position of P giving a) minimal sum of the lengths AP + PB (the length of the pipelines needed). It is in fact the same as what is in chapter one called the Huygens problem. What is new is that the minimal length can be obtained without knowing the position of P. The latter can only be obtained by numerical approximation.

The problem is easy to understand, requires only basic mathematical techniques, its analytical solution is not trivial, but becomes much simpler when using a clever trick, and it is not completely detached form reality. Relation to a real problem is not unimportant because historically new methods and even new mathematics were invented when previous knowledge did not suffice to solve a new realistic problem.

That first chapter serves as an introduction, illustrating with several minimization problems how stationary points connect to vanishing derivatives, but that there are also non-differentiable problems. From chapter two on, the historical evolution is followed, starting with antiquity. The Dido problem (find the largest area enclosed in a closed curve of given length) and related problems (think of fractals where a finite area has an infinite perimeter) are discussed.

The Middle Ages are represented by several problems among which the Regiomontanus problem (what is the optimal distance to look at a painting on the wall to maximize the viewing angle). Then the 'war' between Fermat and Descartes over Snell's law in optics is explained. It illustrates that these algebraists were very close to describe what became later calculus when Kepler, Newton and Leibniz studied gravity and motion. So far, all these problems were considered and solved (at least historically) without calculus. Obviously, with calculus available, derivatives became an important tool to solved them. Galileo studied gravity and the Bernoulli brothers investigated the brachistochrone problem (find the curve of fastest descent between two given points), which is considered to be the first variational problem like the catenary problem (hanging chain) and minimal area surfaces (that generalizes the Dido problem).

In the final chapter, all types of optimization problems are introduced inspired by practical problems from our time like finding the optimal location of a distribution centre, the best configuration of railway tracks, the travelling salesman problem, etc. These require new solution methods like linear and dynamic programming.

Besides the interesting historical framing of the problems and their interplay with the development of mathematics, Nahin takes his time to explain all the formulas and mathematics to solve the problems. It is remarkable that all of this needs little more than some basic calculus. The text has several inserts giving solution to extra problems, or historical quotes. There is no list of references, but the reader is regularly referred to papers that studied some of the historical solution.

This book can be classified as a popular science book, but unlike the effort of many author to ban all formulas, in order not to decimate the number of readers, Nahin includes all the witty mathematics to actually solve the problems, and because the problems are often formulated as one that triggers the puzzling instinct (how to make a perfect basketball shot, what is the best way to get out of the wood when lost,...), the genuinely interested reader will read this book almost as a page turner.

It is very fortunate and actually only a just decision that Princeton picked this book as one in their Princeton Science Library which can be seen as a true quality mark. It is indeed an inexpensive must-read for a broad professional and general audience written by a true master of the genre.

Adhemar Bultheel