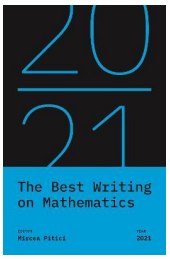
The best writings on mathematics 2021 Mircea Pitici (ed.). Princeton University Press, 2022 (320 p.), isbn: 978-0691225708 (pbk).





Mircea Pitici

Although somewhat hindered by the pandemic situation in 2021 (not all papers are available online, and not all libraries could be visited), Pitici has again collected a fine anthology of 26 papers published in 2021 dealing with mathematical topics from an outside perspective.

It is very appropriate that after a lock-down period, the opening article is about mathematicians that have been very creative during a period of isolation. Newton is of course the the best known example who created his best work when he left Cambridge because of the plague. There are however several other examples of mathematicians who, during a period of war or imprisonment, made some good work, which was sometimes triggered exactly because of the limitations of the situation. Examples are Ibn al-Haytham, Poncelet (geometry), Jakow Trachtenberg (mental calculation), and Jean Leray (PDE and algebraic topology). The next text is about another kind of viral spreading: what exactly is the design behind the creation of Bitcoin and a block chain. It is clearly illustrated that it is not a matter of just cryptography.

It is impossible to discuss each and every paper in this volume, so let me group them according to their main topic. Under the general topic *geometry* we can classify a paper on modular design in architecture where it can happen that when two identical modules meet in a corner, there can be logical accidents. Another one shows how the complex exponential transforms a rectangular grid into circular bands as in Cosmatesque design (named after the Roman Cosmati family). On a somewhat more playful note is the paper where an alphabet is designed using algebraic curves. Each letter consists of a set of curves, so that by simple translations, a text can be written as long algebraic equation in two variables. Professor Maria Trnkova has gathered some fame for her crocheting of hyperbolic geometrical objects. In an equilateral triangulation of the plane there will always be 6 triangles meeting in a corner. When less than 6 meet, the curvature is positive, when there are more than 6, we get negative curvature. So when at a corner more than 6 triangles (crocheted with yarn) are meeting, we get some curvy hyperbolic flowers.



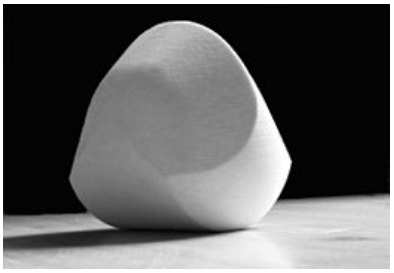
Santo Spirito Florence



Cosmati design



hyperbolic flower



gömböc

More geometry appears in a play from the 17th century where the characters are abstract geometrical figures, like in Abbott's Flatland. A paper describing the 3D printing of chaotic attractors as produced by dynamical systems brings us back to moden times. Next is the story of Gábor Domokos, inventor of the mono-monostatic Gömböc, who teamed up with Douglas Jerolmack to show their theory that in geology things break up on average in roughly cubic shapes.

Computer science is presented with a number of papers drawing parallels between quantum algorithms and the physical world, complexity, dark data and erroneous data.

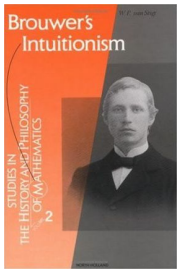
Games, puzzles and paradoxes are also standard topics in these anthologies. A hat-guessing game and the verification of a bicycle problem originally posed by V. Arnold. A coloring problem in a hexagonal pyramid turns out to be an excellent exercise in solving linear systems in $\mathbb{F}_3$.

One paper has particularly triggered my interest. It is related to the discussion of the very old question whether mathematics is invented or discovered. The discussion is about whether or not our universe is blocked or not. Some people are convinced that irrational numbers with an infinite number of non-repeating digits are inventions that do not exist in our universe. Each additional digit is extra information, and information requires energy. If we believe that, given enough information, the universe is predictable, excluding free will, then this infinite amount of information should have been present at the Big Bang, which is impossible. In fact, new information is created all the time. Many multiverses are possible as long as we do not observe, but by observing, we truncate the infinite sequence of information that is not yet complete. Observation destroys possible futures, it destroys information. Thus what we experience is a quantum universe where time is not continuous, and where "now" is an nonexisting vanishing instant of time. There are many related questions and problems. It resolves the philosophical question of free will, a blocked universe allows (at least in theory) wormholes, time travel, etc. This blocked universe excludes determinism and makes physics inherently uncertain. Physicist Nicolas Gisin recently picked up this theory again which follows the intuitionist views of Brouwer. It is a controversy that has not been settled yet.

The penultimate contribution of the book somehow relates to this: can one own a theorem? What about results about cryptography that the NSA wants to control or other research obtained by an employee of a company that wants to keep it secret? If mathematics is discovered, then nobody owns it, but of course you can keep your discovery secret if you want to.



Nicolas Gisin



Brouwer's intuitionism



Sophie Germain



dark data

This leads directly to the *history of mathematics*. Should we continue building mathematics starting form the latest layer added, or should we learn from history and study the old masters to progress? Another contribution explains the history of the *Notices of the AMS*, which started as a newsletter bringing info about the AMS and some letters bringing a political opinion about situations affecting mathematicians worldwide. Another paper highlights the importance of Sophie Germain as a mathematician.

And there is also the usual section on *education and teaching of mathematics*. One paper is a rightful plea not to learn our students only the rules to solve a problem or a class of problems, but to learn the reasoning that is behind it, which is not an obvious thing to do since there are different ways of reasoning (inductive, abductive, analogy, relational). Very relevant is a report that revealed that the achievement of pupils is not reflecting the amount of mathematical training of their teachers. Mathematical training for teaching is not the same as training for research. What are the skills, do's and don'ts, of someone supervising undergraduates? The last contribution is by Terence Tao who confesses that he didn't work too hard as a student since studying was too easy so that he only spent time on the courses he liked, but after nearly failing an exam, he pulled himself together and became the well respected professional mathematician he is now.

So this is again a very interesting set of papers made public in 2021, that are presented here in a uniform way. As usual there is also a long list (9 pages) with more, similar, highly interesting, reading stuff.

Adhemar Bultheel