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Robust stability with the smoothed spectral abscissa

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Problem setting

Stability optimization of continuous-time dynamic systems both difficult and relevant problem

- Periodic open-loop control in robotics
- Tunable parameters often stem from feedback control, e.g. using a linear low-order output feedback controller
- Especially hard when number of free variables much smaller than system's dimension, closed-loop system matrix $A + BKC$ with B en C^T tall matrices

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Can essentially be transformed into an optimization problem

$$\min_x \ \Phi_{\mathbf{stab}}(A(x)), \quad \text{s.t.} \quad g(x) = 0,\ h(x) \leq 0,$$

where

- $A(x)$ system matrix depending smoothly on the parameters x
- the function $\Phi_{\mathbf{stab}}(\cdot)$ expresses the desire to optimize stability

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Problem setting

What do we mean by optimal stability?
What are good choices as objective function $\Phi_{\mathbf{stab}}(A(x))$?

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The closed-loop system

1) must at least be asymptotically stable
   - $\Leftrightarrow$ all system trajectories eventually die out in time
   - $\Leftrightarrow$ the integral $\int_0^\infty ||e^{At}||^2 dt$ is finite for any matrix norm $||\cdot||$
   - $\Leftrightarrow$ the spectrum of the system matrix A resides completely in the left half complex plane, i.e. $\forall \lambda \in \Lambda(A) : \Re\{\lambda\} < 0$
   - $\Leftrightarrow$ the spectral abscissa $\alpha(A)$ ($=$ the real part of the rightmost eigenvalue) is strictly negative

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   - $\Leftrightarrow$ the spectral abscissa $\alpha(A)$ ($=$ the real part of the rightmost eigenvalue) is strictly negative
2) must have optimal robustness properties
   - $\Leftrightarrow$ The system should be "as far way" as possible from instability
   - $\Leftrightarrow$ maximize smallest perturbation that *may* destabilize system
   - $\Leftrightarrow$ $\mathcal{H}_2$-norm and/or $\mathcal{H}_\infty$-norm from input to output are minimal

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Stabilization via optimization

Some optimization approaches for stabilization/robustification

- Nonsmooth optimization using the gradient sampling algorithm
  - minimizing the spectral abscissa for pure stabilization
  - stabilizing parameters x attained as soon as $\alpha(A(x)) < 0$
  - further minimization leads to maximal exponential decay of the trajectories, but also to poor behaviour in terms of robustness
  - $\Rightarrow$ optimize stability radius or H_∞-norm afterwards
  - integrated in the matlab package HIFOO

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  - $\Rightarrow$ optimize stability radius or H_∞-norm afterwards
  - integrated in the matlab package HIFOO
- Nonsmooth mathematical programming for mixed H_2/H_∞ design
  - requires a stabilizing controller as initialisation
  - uses a progress function F with matching optimality conditions with the mixed H_2/H_∞ program
  - solve a tangent problem to compute descent directions for F
  - two phase behaviour: 1. minimization of the H_∞ constraint 2. minimization of H_2 while remaining feasible

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Stabilization via optimization

Some optimization approaches for stabilization/robustification

- Minimization of the pseudo-spectral abscissa
  - The pseudo-spectral abscissa α_ϵ is the maximal real part of the ϵ-*pseudo*spectrum $\Lambda_\epsilon(A) = \{\lambda \in \Lambda(X),\ \|X - A\| \leq \epsilon\}$

$$\alpha_\epsilon(A) = \sup\{\Re(z) \mid z \in \Lambda_\epsilon(A)\},$$

  - α_ϵ still requires specialized nonsmooth optimization methods
  - Interesting robustness property: $\mathcal{H}_\infty$ is bounded by ϵ^{-1} for negative $\alpha_\epsilon(A)$

$$\alpha_\epsilon < 0 \quad \Leftrightarrow \quad \|\mathbf{H}\|_{\mathcal{H}_\infty} < \frac{1}{\epsilon}$$

  - Minimizing α_ϵ stabilizes and robustifies at the same time
  - If $\min \alpha_\epsilon = 0$, equivalent to a H_∞ minimization, or likewise a maximization of the distance to instability

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Minimal shift interpretation

Recall the property

$$\alpha_\epsilon(A) < 0 \quad \Leftrightarrow \quad \|(j\omega I - A)^{-1}\|_{\mathcal{H}_\infty} < \epsilon^{-1}$$

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Minimal shift interpretation

Recall the property

$$\alpha_\epsilon(A - sI) < 0 \quad \Leftrightarrow \quad \|(j\omega I - (A - sI))^{-1}\|_{\mathcal{H}_\infty} < \epsilon^{-1}$$

Introducing a shift-to-the-left s

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Introducing a shift-to-the-left s, we get

$$\alpha_\epsilon^{\mathrm{ps}}(A) < s \quad \Leftrightarrow \quad \|\mathbf{H}_s\|_{\mathcal{H}_\infty} < \epsilon^{-1}$$

with $\mathbf{H}_s := (j\omega I - (A - sI))^{-1}$ denoting the shifted transfer function.

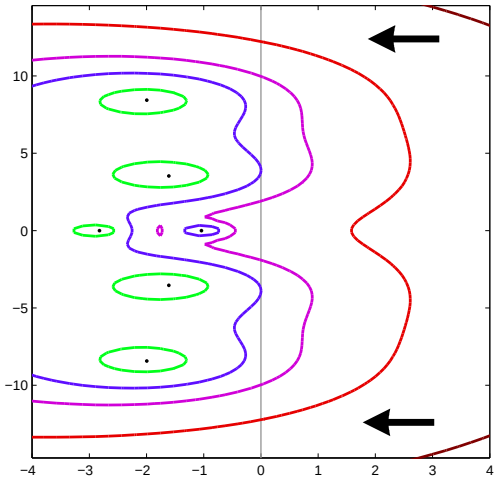
Minimal shift interpretation of the pseudo-spectral abscissa

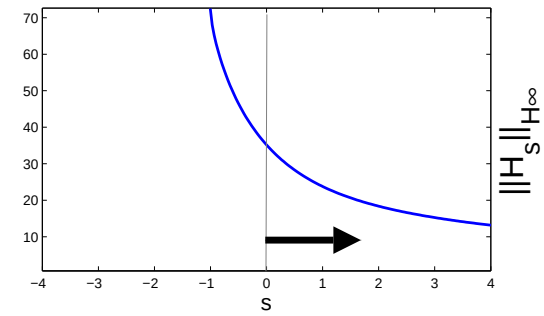
α_ϵ is the minimal shift-to-the-left s for which holds that

- the shifted system is stable

and

- its $\mathcal{H}_\infty$-norm is smaller than $\frac{1}{\epsilon}$





shifted $\mathcal{H}_\infty$-norm i.f.o. the shift s

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The smoothed spectral abscissa

Can we make a similar regularization for the $\mathcal{H}_2$-norm?

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The smoothed spectral abscissa

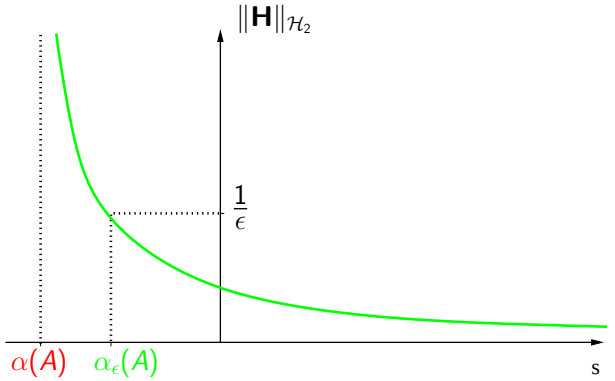
Can we make a similar regularization for the $\mathcal{H}_2$-norm?

Yes! The $\mathcal{H}_2$-norm, being the square root of the integral

$$\int_0^\infty ||e^{At}||_{\mathrm{F}}^2 dt \quad \text{with} \quad \|\cdot\|_{\mathrm{F}} = \text{the Frobenius norm},$$

behaves exactly the same as the $\mathcal{H}_\infty$ norm w.r.t. a shift-to-the-left s :

- finite when s is stabilizing ($s > \alpha(A)$)
- decreases for growing s
- $\longrightarrow \infty$ when s becomes destabilizing



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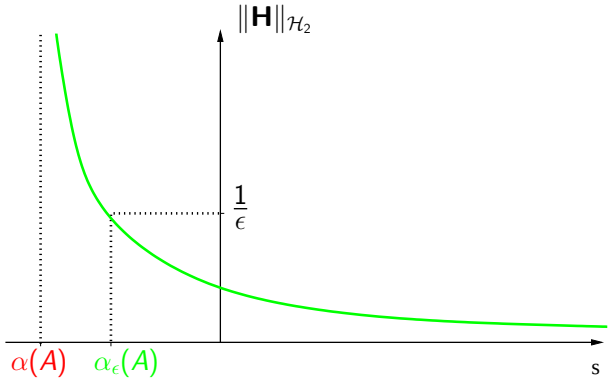

Definition

The smoothed spectral abscissa $\tilde{\alpha}_\epsilon$ is the minimal shift s for which the $\mathcal{H}_2$-norm of the shifted transfer function $\mathbf{H}_s$ stays below $\frac{1}{\epsilon}$

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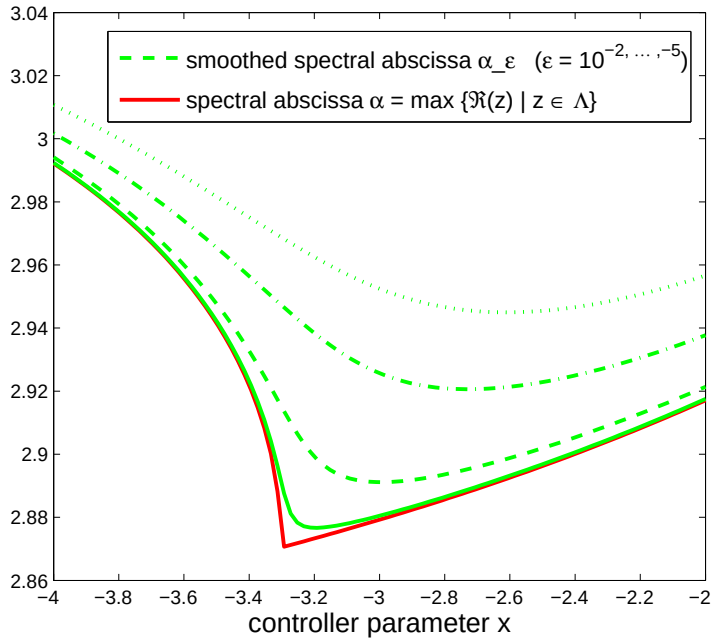

Some key properties :

- $\tilde{\alpha}_\epsilon(A)$ is completely **smooth** in A (and thus also in any controller parameters x upon which $A(x)$ might depend)

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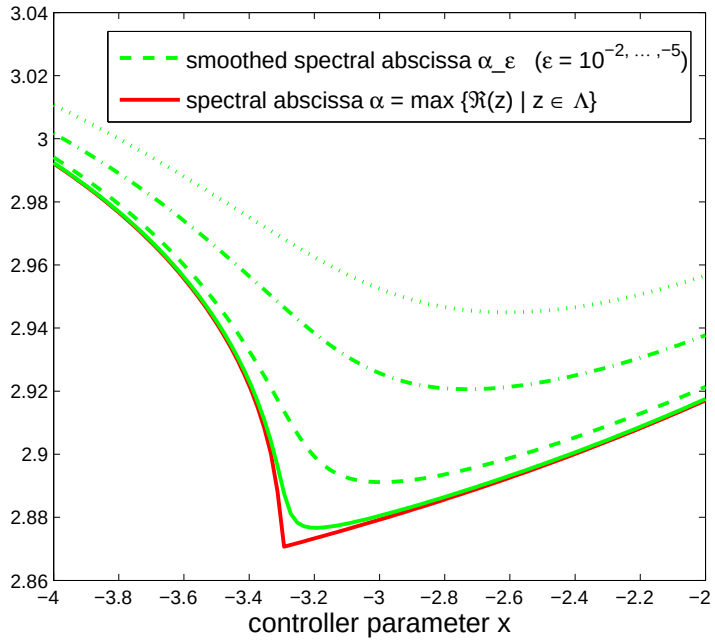

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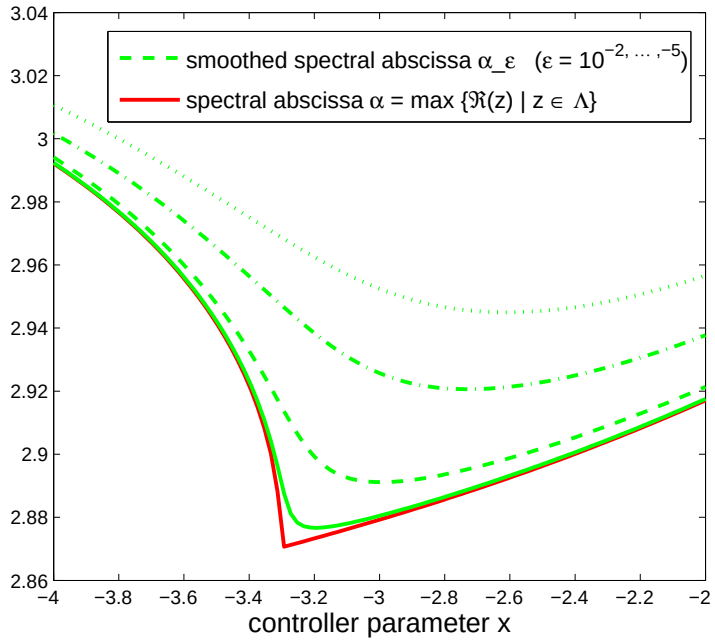

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- $\tilde{\alpha}_\epsilon(A)$ becomes also **smoother** with growing ϵ, and thus presumably more robust

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Relation with the pseudo-spectral abscissa

- By $\|\mathbf{H}\|_{\mathcal{H}_\infty} < 2\|\mathbf{H}\|^2_{\mathcal{H}_2}$ ([*Balakrishnan & Vandenberghe*, 2002]), we can derive

$$\alpha_{\epsilon/2}(A) \;<\; \tilde{\alpha}_\epsilon(A).$$

  $\Rightarrow$ The smoothed spectral abscissa forms an upper bound to the pseudo-spectral abscissa with an ϵ half as large.

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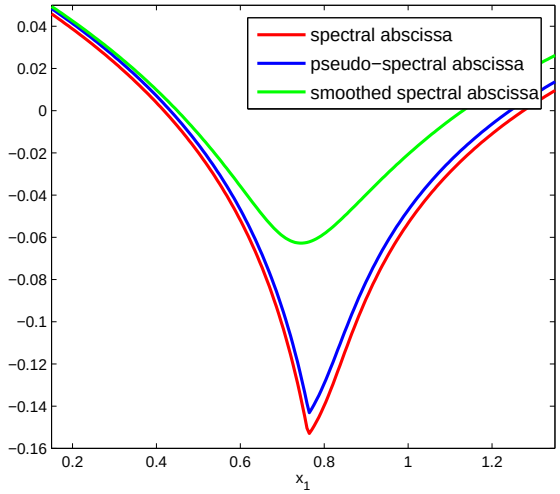
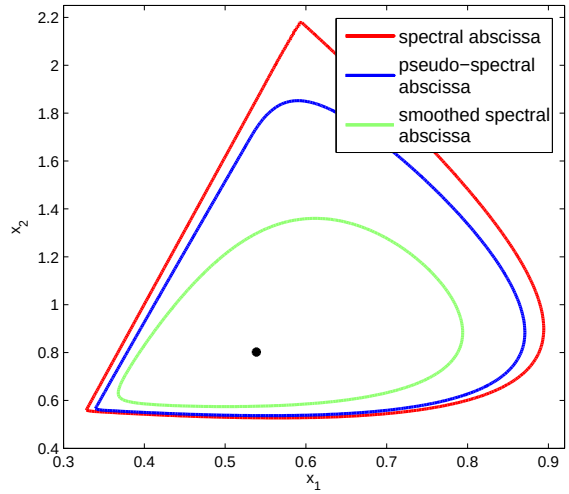
- Example:



1D parameter space



2D parameter space

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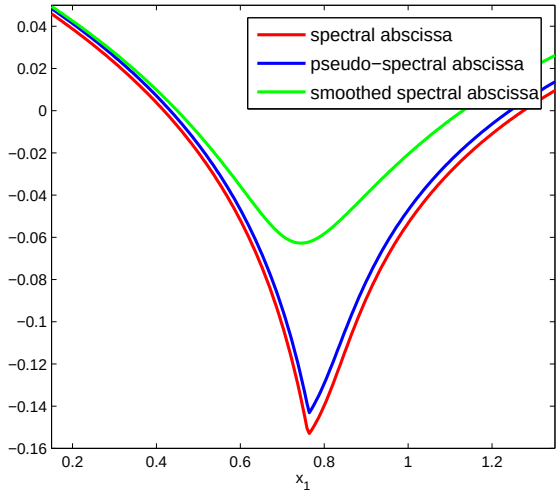

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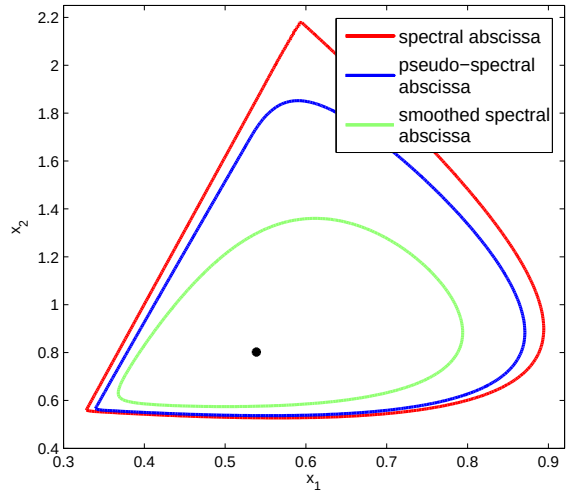

2D parameter space

- We can guarantee a desired distance to instability $\beta(A) = \frac{\epsilon}{2}$ as soon as we have that $\tilde{\alpha}_\epsilon(A) < 0$

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The smoothed spectral abscissa

- $\tilde{\alpha}_\epsilon(A)$ is the solution for s of the nonlinear equation

$$\int_0^\infty \| e^{(A-sI)t} \|_{\mathrm{F}}^2 \, dt = \epsilon^{-1},$$

but can be computed using matrix equation calculus.

The smoothed spectral abscissa

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$$\int_0^\infty \| e^{(A-sI)t} \|_{\mathrm{F}}^2 \, dt = \epsilon^{-1},$$

but can be computed using matrix equation calculus.

- Indeed, let P and Q be the symmetric solutions of the primal-dual Lyapunov equation pair

$$\begin{aligned} (A-sI)P + P(A-sI)^{\mathrm{T}} + I &= 0, \\ (A-sI)^{\mathrm{T}}Q + Q(A-sI) + I &= 0, \end{aligned}$$

then $\tilde{\alpha}_\epsilon(A)$ and its derivatives w.r.t. A are simply

$$\begin{aligned} s = \tilde{\alpha}_\epsilon(A) \quad &\Leftrightarrow \quad \mathrm{trace}(P) = \mathrm{trace}(Q) = \epsilon^{-1}, \\ \frac{\partial \tilde{\alpha}_\epsilon(A)}{\partial A} \quad &= \quad \frac{QP}{\mathrm{trace}(QP)}, \end{aligned}$$

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- This allows for an easy and efficient integration in a derivative based optimisation framework (BFGS, SQP).

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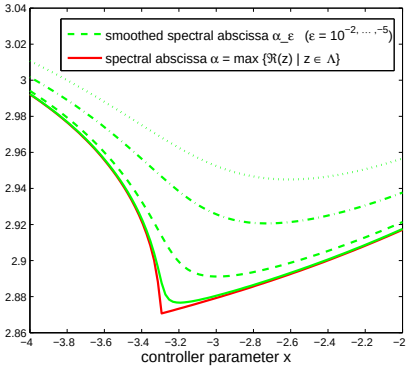
Optimisation problems involving the smoothed spectral abscissa

- Two possible optimization formulations
  - **1st optimization problem:**
    Choose a fixed ϵ, and try to minimize $\alpha_\epsilon^{\mathrm{sm}}(A)$

$$\min_x \ \tilde{\alpha}_\epsilon(A(x))$$

$\longrightarrow$ If this problem has a negative the solution, system is stabilized.
$\longrightarrow$ If it hasn't, decrease ϵ!



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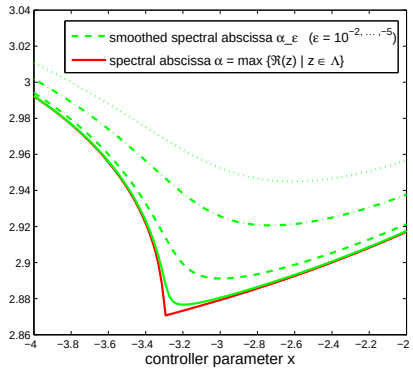

$\longrightarrow$ If this problem has a negative the solution, system is stabilized.
$\longrightarrow$ If it hasn't, decrease ϵ!

  - **2nd optimization problem:**
    Since the choice of ϵ is a bit arbitrary in the previous problem, find the largest ϵ for which the stability certificate still holds.

$$\max_{x,\epsilon} \ \epsilon \qquad \text{s.t.} \qquad \tilde{\alpha}_\epsilon(A(x)) \leq 0$$

$\longrightarrow$ the constraint $\tilde{\alpha}_\epsilon(A(x)) \leq 0$ will be active in the optimum!

1st optimization problem example

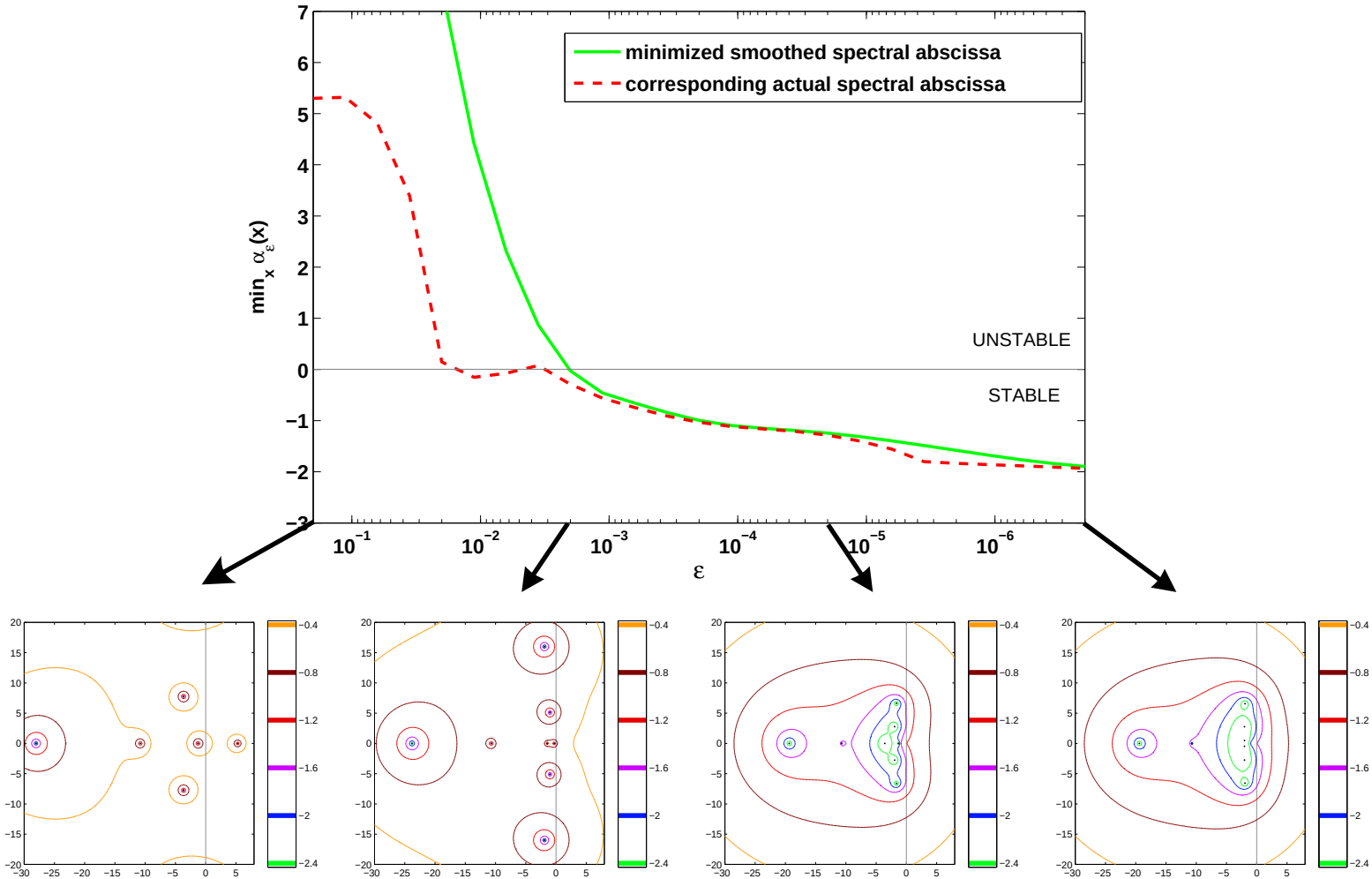

Figure: Pseudospectra of a turbo generator model in the solutions of $\min_x \tilde{\alpha}_\epsilon(A(x))$ for several values of ϵ $(2 \cdot 10^{-1,\ -3,\ -5,\ -7})$

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2nd optimization problem example

$$\max_{x,\epsilon} \ \epsilon \qquad \text{s.t.} \qquad \tilde{\alpha}_\epsilon(A(x)) \leq 0$$

Why maximize ϵ?

- It is easy to show that the minimizers of this problem (x^*, ϵ^*) also satisfy

$$\min_x \ \|\mathbf{H}\|_{\mathcal{H}_2} = \|\mathbf{H}^*\|_{\mathcal{H}_2} = \|j\omega - A(x^*)\|_{\mathcal{H}_2} = \frac{1}{\epsilon^*}$$

  $\Rightarrow$ Maximizing ϵ comes down to minimizing the $\mathcal{H}_2$-norm of the system over all feasible x

  $\Rightarrow$ The set of minimizers of $\tilde{\alpha}_\epsilon$ for $\epsilon \in [0, \epsilon^*]$ form a range of compromises between maximal asymptotic decay rate and maximal robustness (in the sense of minimal $\mathcal{H}_2$-norm)

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  $\Rightarrow$ The set of minimizers of $\tilde{\alpha}_\epsilon$ for $\epsilon \in [0, \epsilon^*]$ form a range of compromises between maximal asymptotic decay rate and maximal robustness (in the sense of minimal $\mathcal{H}_2$-norm)

- Recall that for $\tilde{\alpha}_\epsilon(A) < 0$, $\beta(A)$ is guaranteed to be $> \frac{\epsilon}{2}$

  $\Rightarrow$ By maximizing ϵ we also maximize a lower bound on the distance to instability

2nd optimization problem example

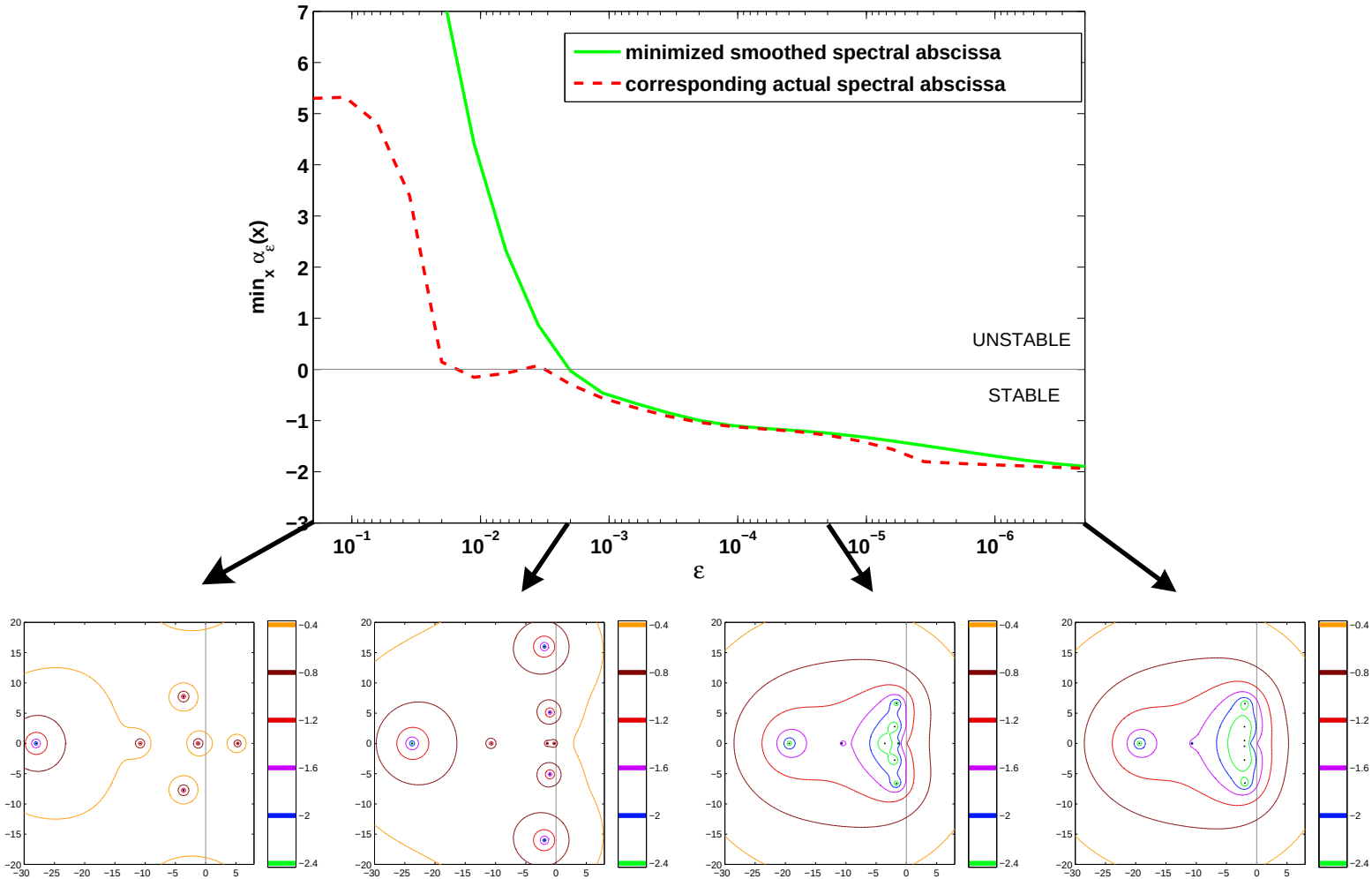

Figure: Pseudospectra of a turbo generator model in the solutions of $\min_x \tilde{\alpha}_\epsilon(A(x))$ for several values of ϵ $(2 \cdot 10^{-1,\ -3,\ -5,\ -7})$

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1. smooth and its derivatives can be computed cheaply
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- Solution: integrate fast solvers for Lyapunov equation (exploiting low rankness or sparsity) into the optimization procedures

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Thank you for your attention!