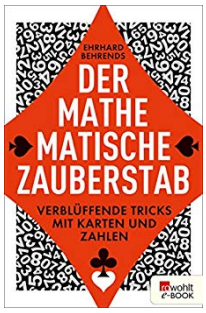
The Math behind the Magic, *Ehrhard Behrends*, AMS, 2019 (xx +208 p.) isbn: 978-1-4704-4866-0 (pbk).







Ehrhard Behrends

The AMS published under this title a translation of Behrends' *Der mathematische Zauberstab* (Rowohlt Taschenbuch, 2015). Behrends explains many number tricks that can be obtained, based on mathematical rules. Most of them are hidden into card tricks. It is known that many mathematicians are fond of games and magic. If you catch the attention of a youngster with magic and you want to sneak in some mathematics, then this book can be of great help for you.

There are several ways to use mathematics. Of course there are the mathematical properties of integers that can be used (chapter 1), also combinatorial and modulo calculus (chapter 2), some coding strategies (chapter 3), or just probability (chapter 4). Mostly the scenario is to explain the trick, how to prepare the cards (if needed) and then the explanation of why it works. Usually there are some variations or more complicated versions at the end. Let me give some examples.

It is not difficult to prove that a 4 digit number minus the number that is obtained by reversing the order of the digits, is a multiple of 9. So if you ask to reveal 3 of the 4 digits of the result (for example represented as cards), you can immediately say what the fourth is because the sum of the digits has to be 9. Other tricks can make use of prime numbers. Suppose p is prime and you have p cards, $K_0, \ldots, K_{p-1}$. If one cyclically reveals every mth card ($1 < m < p$) then the last card to be uncovered will be K_0.



riffle shuffle

From chapter 2, we recall the fact that if you have n pairs of cards (like two cards of the same colour and the same value) and you stack the $2n$ cards separating the elements of the pairs n places apart, then you can cut the stack of $2n$ cards as many times as you want, the pairs will remain n places apart. For several variants of Gilbreath tricks, one has to start from an arranged stack (for example red-black alternation) and one should master a riffle shuffle (mix two piles alternating between both). Symmetry can also be used. Most card faces are upside-down symmetric, but some are not (e.g. check 3,5,7,9). This can obviously be exploited in a trick.

In chapter 3, coding theory is used. Suppose you have 3 cards: A,B,C. Then there are 3! ways to order them, so that each ordering corresponds to a number from 1 to 6. With a deck of cards one can order by suits (for example ♥, ♦, ♣, ♠) and/or the values: $2, 3, \ldots, 9, 10, J, Q, K, A$. In this way the magician can hide a lot of information in the order of the cards that is unknown to the public. Or one could let the colours correspond to 0 and 1, so that cards laid out in some order represent a binary number. These are all ways in which an assistant can communicate with the magician.

The last chapter relies on probability, so the trick is not guaranteed to work, but the chance can be maximized. One is based on random walk. Just lay out a shuffled deck of cards face up in a row. Start near the beginning, read the value of the card, and move forward that number of cards and continue in that way till you reach the end. Provided there are enough cards, the chance is high that the end card, does not depend on the start card. Another trick relies on the non-transitive ordering of scissors-paper-rock. There is a winning strategy with high probability. Disguised as a card game you can have, with 3 cards, 8 different colour combinations. If the participant chooses e.g. RBB, then the magician inverts one of the colours that appeared twice and keeps the other. Thus he chooses RRB. The cards form the stack are turned one by one. It the stack is finished, it is shuffled and one continues with the shuffled tack. The one whose sequence of colours appears wins a point. The first one who wins five points wins the game. The strategy of the magician always has a greater chance of winning.

There are many more tricks that can be explored than the very brief sample that is given above. They are all great fun to perform and to explore the mathematics that is behind.

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