# Optimal Mechanical Force-Velocity Profile for Sprint Acceleration Performance 

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#### Abstract

The aim was to determine the respective influences of sprinting maximal power output ( $P_{H} \max$ ) and mechanical Force-velocity ( $\mathrm{F}-\mathrm{v}$ ) profile (i.e. ratio between horizontal force production capacities at low and high velocities) on sprint acceleration performance. A macroscopic biomechanical model using an inverse dynamics approach applied to the athlete's centre of mass during running acceleration was developed to express the time to cover a given distance as a mathematical function of $P_{H} \max$ and F-v profile. Simulations showed that sprint acceleration performance depends mainly on $P_{H} \max$, but also on the F-v profile, with the existence of an individual optimal F-v profile corresponding, for a given $P_{H} \max$, to the best balance between force production capacities at low and high velocities. This individual optimal profile depends on $P_{H} \max$ and sprint distance: the lower the sprint distance, the more the optimal F-v profile is oriented to force capabilities and vice versa. When applying this model to the data of 231 athletes from very different sports, differences between optimal and actual Fv profile were observed and depend more on the variability in the optimal F-v profile between sprint distances than on the interindividual variability in F-v profiles. For a given sprint distance, acceleration performance ( $<30 \mathrm{~m}$ ) mainly depends on $P_{H} \max$ and slightly on the difference between optimal and actual F-v profile, the weight of each variable changing with sprint distance. Sprint acceleration performance is determined by both maximization of the horizontal power output capabilities and the optimization of the mechanical $\mathrm{F}-\mathrm{v}$ profile of sprint propulsion.


Key Words: horizontal force production; all-out running; maximal power output; biomechanics

## INTRODUCTION

Sprint running acceleration is a key performance determinant in many sports (e.g. track and field events or team sports) and refers to all-out efforts aiming at covering distance in the shortest time possible (or the largest distance in a given time of action). Forward acceleration capabilities have gained interest over the last decade in sport sciences, notably because individual top speed is rarely reached by athletes during games (e.g., in soccer or rugby). Therefore, examining the factors that determine all-out sprint acceleration performance may inform scientists and practitioners about the physical characteristics underlying sprinting performance.

The effect of force production qualities on sprint acceleration performance (sprint times from 10 to 100 m ) has been widely studied through lower limb strength output, notably in squat, leg press or jumps (e.g. ${ }^{1-3}$ ). Overall, sprint times were shown to be correlated to lower-limb maximal strength and power output, with an overall decreasing magnitude when sprint distance and subject level increase. However, in these studies, the strength indexes did not cover the entire spectrum of force-velocity qualities, notably the force production capacity at high velocity was not thoroughly assessed. Moreover, force production capabilities were not sprintspecific but only inferred from lower limb gym-based strength indexes. From basic principles of dynamics, the motion of one athlete's centre of mass (CoM) mainly depends on the ground reaction force applied on it, the latter resulting directly from the external force the athlete produced onto the ground ${ }^{4}$. Although running (with support and aerial phases) is possible only if net force is developed in the vertical direction, the forward acceleration of the CoM from one step to another is directly related to the net force developed by the athlete onto the ground in the horizontal, antero-posterior direction (backward so that the reaction is directed forward) ${ }^{4}$. All other things equal, the greater this net horizontal force component relative to body mass, the higher the acceleration of the body in the forward direction, ceteris paribus ${ }^{4}$. Force production during sprinting has been widely described in studies analysing only some steps during the acceleration phase or steady-speed (i.e., not accelerated) runs including top speed ${ }^{5-}$ ${ }^{7}$. Collectively, they showed (i) that early acceleration performance is related to high horizontal propulsive force ${ }^{4,5}$, and (ii) that the ability to maintain maximal running velocity is associated to high mass-specific vertical force applied over very short support time during the constant top speed phase ${ }^{6,7}$. Based on ground reaction force measurement over an entire acceleration phase, recent studies confirmed the importance of the net horizontal force component to explain inter-
athlete's differences in sprint acceleration performance (Rabita et al 2015, Morin et al 2012, Colyer et al 2018).

Consequently, sprint running acceleration performance depends on the athlete's capacity to produce net "horizontal force" onto the ground over each step. A macroscopic view of these capacities is given by the force-velocity ( $\mathrm{F}-\mathrm{v}$ ) and power-velocity ( $\mathrm{P}-\mathrm{v}$ ) relationships in sprinting ${ }^{8,9}$. Even if terminology is similar, sprinting F-v and P-v relationships are far from the original intrinsic muscle F-v relationships reported on isolated muscle by Hill and colleagues ${ }^{10}$ since they represent here the external horizontal force production capacities of the overall body during sprint acceleration. They integrate other neuromuscular and biomechanical mechanisms than those inherent to the muscle fibres only: basically mechanisms associated to the transmission efficiency between the muscle force and the external force ${ }^{11}$. These relationships describe the change in the athlete's maximal horizontal external force and the associated power production capabilities when running speed increases. As previously described for other multi-joint movements (pedalling, squat jump, leg press) ${ }^{12-17}$, sprinting Fv and $\mathrm{P}-\mathrm{v}$ relationships provide an objective quantification of force/power production abilities through the maximal power output an athlete can develop in the horizontal direction ( $P_{H} \max$, power capabilities), the theoretical maximal horizontal force an athlete can produce onto the ground ( $F_{H} 0$, force capabilities) and the theoretical maximal velocity until which the athlete is still able to produce positive net horizontal force ( $v_{H} 0$, velocity capabilities). Conceptually, $F_{H} 0$ and $v_{H} 0$ are independent and are associated to different physical and technical abilities related to producing high amount of horizontal force at low running velocities $\left(F_{H} 0\right)$ and horizontal force at high velocities $\left(v_{H} 0\right)$. These different key mechanical variables result from the complex integration of the different physiological, neural and biomechanical mechanisms involved in the total external force production and characterizing different athlete's abilities ${ }^{14-}$ ${ }^{17}$. Moreover, when focusing on sprint running movement, F-v and P-v relationships are specific to running acceleration propulsion and in turn also integrate the ability to apply the external force "effectively" (i.e. with a horizontal orientation in the antero-posterior direction) onto the ground ${ }^{8,18,19}$. These relationships thus refer to overall sprinting propulsion capacities rather than muscle properties only. The ratio between $F_{H} 0$ and $v_{H} 0$ corresponds to the athlete's mechanical F-v profile ( $S_{F v}$, slope of the F-v linear relationship) ${ }^{12,20}$. Interestingly, as for vertical jumping ${ }^{21}$, two athletes can present very different $F$-v profiles with the same maximal power capability ( $P_{H} \max$ ). Among these different force production capacities, scientists, coaches or athletes wonder which one is more important (if any) for sprint running acceleration
performance (mostly quantified through time to cover a given distance)? One of the main questions for sport practitioners is to determine where to place the training "cursor" and how to program training within the continuum between these two extreme force production capacities of the F-v profile: maximal horizontal force at low and high velocities.

In the last decade, several studies explored the mechanical determinants of sprint running acceleration through sprinting power-force-velocity relationships ${ }^{8,18,19,22}$. The main findings of these studies were that sprint performance $(40-\mathrm{m}$ or $100-\mathrm{m}$ times, maximal speed or $4-\mathrm{s}$ distance) depends on the mean horizontal power and force produced over the acceleration phase $8,18,19,23$. This has been shown to be related to a high athlete's $P_{H} \max$ associated to a high $v_{H} 0$, notably due to a high mechanical effectiveness, in recreational sportsmen ${ }^{18}$, old trained sprinters ${ }^{24}$, high level ${ }^{8,19,23}$ to world class men and women sprinters ${ }^{22}$. Consequently, $P_{H}$ max seems to be the main determinant of sprint acceleration performance, notably due to a high ability to produce horizontal force at high velocities rather than a high maximal horizontal force production at low velocities. However, these studies only focus on relatively long sprint accelerations ( $40-\mathrm{m}$ to $100-\mathrm{m}$ times, $4-\mathrm{s}$ distance) and did not assess the relative importance of each extremum of the horizontal force production capacities ( $F_{H} 0$ versus $v_{H} 0$ ), i.e. of mechanical F-v profile ( $S_{F v}$ ), according to the sprint distance and independently from the effect of $P_{H} \max$. Does the increase in performance depend only on an overall shift of the F-v relationship upwards and to the right (i.e. an increase in $P_{H} \max$ )? Or could a change in its slope (i.e. an increase or decrease in $S_{F v}$ ) independently from its overall position also contribute to performance improvement? And if so, to what extent for each sprint distance?

The question of the effect of the mechanical F-v profile on performance, independently of the power capacities, has been studied for ballistic push-off exercises such as jumping ${ }^{12,25}$. The maximal take-off velocity was shown to depend on both lower limb maximal power output (relative to body mass) and F-v profile, with the existence of an individual optimal F-v profile representing, for a given maximal power output, the best balance between force and velocity capacities maximizing the performance. The higher the maximal power and the lower the difference between actual and optimal $\mathrm{F}-\mathrm{v}$ profile, the higher the jumping performance ${ }^{12,21,25}$. Given the differences in movement modalities between acyclic single push-off jumping and cyclic multiple steps running movements, this concept, based on a specific jumping biomechanical model, could not be directly applied to sprint acceleration. The existence of such an effect of F-v profile, and notably the existence of an optimal profile in sprint running, is still unknown. This could be of great interest for scientists to better understand the mechanical
determinants of running acceleration, but also for sport practitioners to help improving sprint performance.

The overarching aim of this study was to determine the respective influences of sprinting maximal power output and mechanical F-v profile on sprint acceleration performance. Since the importance of horizontal force production capacities at low or high velocities can be expected to depend on the sprint acceleration distance, the secondary aim of this study was to investigate to what extent the effects of maximal power output and F-v profile (if any) depend on the sprint acceleration distance (until to 30 m ). To address these aims, a macroscopic biomechanical model was developed, simulated, and then applied on experimental data to quantify the respective contributions of maximal power output and mechanical F-v profile on acceleration performance of athletes from different sports and over different distances.

## THEORETICAL BACKGROUND

This section, associated to the first aim, is an analysis of kinematics and kinetics of the runner's body CoM during a linear sprinting acceleration starting from null velocity using a macroscopic inverse dynamics approach aiming to be the simplest possible and only focusing on the net stepaveraged horizontal component of the external force (and associated power output) ${ }^{20,26}$. All variables presented in this section are modelled over time, without considering intra-step changes, and thus correspond to step-averaged values (over contact plus subsequent aerial times).

Net horizontal antero-posterior ground reaction force as a function of maximal running velocity and acceleration time constant

During an all-out sprint running acceleration, horizontal velocity ( $v_{H}$, in m.s. ${ }^{-1}$ )-time $(t)$ curve has long been shown to systematically follow a mono-exponential function for recreational to highly trained sprinters ${ }^{20,26,27}$ :
$v_{H}(t)=v_{H} \max .\left(1-\mathrm{e}^{-\mathrm{t} / \tau}\right)$
with $v_{H} \max$ the maximal velocity reached at the end of the acceleration (in m. $\mathrm{s}^{-1}$ ) and $\tau$ the acceleration time constant (in s). The horizontal position ( $x_{H}$, in m) and acceleration ( $a_{H}$, in
$\mathrm{m} . \mathrm{s}^{-2}$ ) of the body CoM as a function of time during the acceleration phase can be expressed after integration and derivation of $v_{H}(t)$ over time, respectively, as follows ${ }^{20}$ :
$x(t)=v_{H} \max .\left(\mathrm{t}+\tau . \mathrm{e}^{-\frac{t}{\tau}}\right)-v_{H} \max . \tau$
$a_{H}(t)=\left(\frac{v_{H} \max }{\tau}\right) \cdot \mathrm{e}^{-\frac{\mathrm{t}}{\tau}}$
Applying the fundamental laws of dynamics in the horizontal direction, the net horizontal antero-posterior ground reaction force relative to body mass (BM) and applied to the body CoM ( $F_{H}$, in $\mathrm{N} . \mathrm{kg}^{-1}$ ) can be modelled over time as:
$F_{H}(t)=a_{H}(t)+F_{\text {aero }}(t)$
with $F_{\text {aero }}(\mathrm{t})$ the BM-relative aerodynamic drag (in $\mathrm{N}_{\mathrm{kg}} \mathrm{kg}^{-1}$ ) to overcome during sprint running which is proportional to the square of the velocity of air relative to the runner:
$F_{\text {aero }}(t)=k \cdot v_{H}(t)^{2}$
with $k$ the runner's aerodynamic friction coefficient ${ }^{20,28}$, expressed relative to BM , considering a situation without any wind.

From equation (1),
$v_{H} \max . \mathrm{e}^{-\mathrm{t} / \tau}=v_{H} \max -v_{H}(t)$
So, from equations [4], [5], [6], $F_{H}$ can be modelled as:
$F_{H}(t)=\frac{v_{H} \max }{\tau}-\frac{1}{\tau} v_{H}(t)+k \cdot v_{H}(t)^{2}$

## Maximal running velocity and acceleration time constant as a function of force production abilities

The linear sprinting acceleration performance can be represented by the distance covered within a given time (equation (2)), which depends on two parameters characterizing two different (but not independent) parts of the performance: $v_{H} \max$ and $\tau$. This section first aims at expressing these two parameters as a function of force production abilities $F_{H} 0$ and $v_{H} 0$, and then as a function of $P_{H} \max$ and $S_{F v}$.

From equation (7), the theoretical maximum $F_{H}$ than can be developed at null $v_{H}\left(F_{H} 0\right.$, in $\mathrm{N} . \mathrm{kg}^{-}$ ${ }^{1}$ ) can be expressed as:

$$
\begin{equation*}
F_{H} 0=\frac{v_{H} \max }{\tau} \tag{8}
\end{equation*}
$$

And so $v_{H} \max$ can be expressed by:

$$
\begin{equation*}
v_{H} \max =\tau . F_{H} 0 \tag{9}
\end{equation*}
$$

The theoretical maximal velocity until which $F_{H}$ can be developed ( $v_{H} 0$ ) can be obtained solving $F_{H}(t)=0$ (equation (7)) with $v_{H}(t)=v_{H} 0$. This gives:
$v_{H} 0=\frac{1}{2 k \tau}\left(1-\sqrt{1-4 k . \tau \cdot v_{H} \max }\right)$

From equations (9) and (10), and isolating $v_{H_{\max }}$ gives:
$v_{H} \max =\frac{F_{H} 0 \cdot v_{H} 0}{k \cdot v_{H} 0^{2}+F_{H} 0}$

From equations (9) and (11), and isolating $\tau$ gives:
$\tau=\frac{v_{H} 0}{k \cdot v_{H} 0^{2}+F_{H} 0}$

Despite equation (7) showed that F-v relationship is mathematically described by a polynomial function, the simplifying assumption of a linear F-v relationship was used here based on the previously reported experimental data on human multi-joint movements ${ }^{12,14}$ and sprinting ${ }^{8,29}$ (the induced-errors were further tested and discussed in the following sections, see Supplementary Material). In this case, the maximal horizontal power output ( $P_{H} \max$, in W. $\mathrm{kg}^{-}$ ${ }^{1}$ ) can be expressed as:
$P_{H} \max =\frac{F_{H} 0 \cdot v_{H} 0}{4}$

And the mechanical F-v profile $\left(S_{F v}\right)$ as :
$S_{F v}=-\frac{F_{H} 0}{v_{H} 0}$

From equations (11) to (14): $v_{H} \max$ and $\tau$ can be expressed as functions of $P_{H} \max$ and $S_{F v}$ :

$$
\begin{equation*}
v_{H} \max =\frac{2 \sqrt{-P_{H} \max \cdot S_{F v}}}{2 . k \cdot \sqrt{\frac{P_{H} \max }{-S_{F v}}}-S_{F v}} \tag{15}
\end{equation*}
$$

$\tau=\frac{1}{2 . k \cdot \sqrt{\frac{P_{H} \max }{-S_{F v}}}-S_{F v}}$

## Sprinting acceleration performance as a function of horizontal force production capacities

Linear sprinting acceleration performance can be represented by the distance covered within a given time (equation (2)), but also, and with a higher practical sense, by the time $\left(T_{X}\right)$ spent to cover a given distance (X). This can be obtained isolating $t$ in equation (2):
$T_{X}=\tau \cdot W_{0}\left(-e^{-\frac{X+\tau \cdot v_{H} \max }{\tau \cdot v_{H} \max }}\right)+\frac{X+\tau \cdot v_{H} \max }{v_{H} \max }$
with $\mathrm{W}_{0}$ the main branch of the Lambert W function defined on real values and respecting $\mathrm{W}_{0}(0)=0$ and $\mathrm{W}_{0}(-1 / \mathrm{e})=-1$.

From equations (15) to (17), $T_{X}$ can be expressed as function of $P_{H} \max$ and $S_{F v}$ :
$T_{X}=-\frac{W_{0}\left(-e^{-\frac{\sigma_{2}^{2} \sigma_{1}}{2 \sqrt{-P_{H} \text { max. } S_{F v}}}}\right)}{\sigma_{2}}-\frac{\sigma_{2} \sigma_{1}}{2 \sqrt{-P_{H} \max \cdot S_{F v}}}$
where
$\sigma_{1}=x+\frac{2 \sqrt{-P_{H} \operatorname{max.S} S_{F v}}}{\sigma_{2}{ }^{2}}$
$\sigma_{2}=S_{F v}-2 k \sqrt{-\frac{P_{H} \max }{S_{F v}}}$

## METHODS USED FOR MODEL SIMULATION

## F-v relationship regression model

Sprinting F-v relationship has been hitherto experimentally described by a linear regression 8,9,20 . Equation (7) shows here that, when velocity-time curve during a sprint acceleration is described by a mono-exponential function (equation (1)) ${ }^{20,27}$, the $\mathrm{F}-\mathrm{v}$ relationships follows a $2^{\text {nd }}$ order polynomial function, with a viscosity component associated to aerodynamic resistance. The Root Mean Square Error (RMSError) in $F_{H}$, as well as the differences in $F_{H} 0$ and $v_{H} 0$, between values obtained by the $2^{\text {nd }}$ order polynomial function (equation (7), (8) and (10)) and values obtained by a linear regression fitting of the values obtained by this polynomial function, were computed on different simulated sprints characterizing individuals with different $k$ (from 0.0025 to $0.0044 \mathrm{~N} . \mathrm{s}^{2} \cdot \mathrm{~m}^{-2} \cdot \mathrm{~kg}^{-1}$, increment step of 0.0001 ), $v_{H} \max$ (from 5 to $12 \mathrm{~m} . \mathrm{s}^{-1}$, increment step of 1 ) and $\tau$ (from 0.8 to 1.5 s , increment step of 0.1 ) values.

Effect of $\boldsymbol{P}_{\boldsymbol{H}} \max$ and $\boldsymbol{S}_{\boldsymbol{F v}}$ on sprint acceleration performance
The relative influences of $P_{H} \max$ and $S_{F v}$ on sprint acceleration performance ( $T_{X}$ ) were analysed via simulation of equation (18) for different sprint distances (X from 5 to 30 m , increment step of 5). For that, $T_{X}$ changes with $S_{F v}$ were determined for different $P_{H}$ max values and for different X values. The range of $P_{H} \max$ and $S_{F v}$ values used in the simulations correspond to those previously reported for humans : $P_{H} \max$ from 10 to $30 \mathrm{~W} . \mathrm{kg}^{-1}$ (increment step of 2) and $S_{F v}$ from -1.5 to -0.038 N.s.m ${ }^{-1} . \mathrm{kg}^{-1}$ (increment step of 0.006 ) ${ }^{20,21,30}$. In case of a curvilinear change in $T_{X}$ with $S_{F v}$ at a given $P_{H} \max$ and for a given sprint distance X (as observed during ballistic push-off, ${ }^{12}$ ), the $S_{F v}$ values associated to the minimum $T_{X}$ value (and so the best acceleration performance), corresponding to an "optimal" sprinting F-v profile $\left(S_{F v} O P T\right)$, were determined for different sprinting distances $(\mathrm{X})$ and $P_{H} \max$ values. The respective effects of $P_{H} \max$ and $X$ on $S_{F v} O P T$ were then studied.

## METHODS USED FOR MODEL APPLICATION TO EXPERIMENTAL DATA

The relative influences of maximal power output and mechanical F-v profile on sprint acceleration performance ( $T_{X}$ ) theoretically assessed by the model simulation were then tested on experimental data to quantify their respective contributions to explain inter-individual differences in acceleration performance over different distances of athletes from different sports.

## Subjects, experimental protocol and measurements

After giving their written informed consent, 231 athletes ( 144 men and 87 women, their mass, stature and body mass are presented in Table 1) from various sport disciplines volunteered to participated in this study, which was approved by the local ethical committee of the Catholic University of San Antonio (Murcia) in agreement with the Declaration of Helsinki (more details in ${ }^{30}$. The sport disciplines were chosen to potentially cover a large spectrum of different horizontal force production capacities regarding the importance and type of sprint acceleration within each discipline: track-and-field sprinters ( $\sim 30-50 \mathrm{~m}$ sprint accelerations), soccer players ( $5-30 \mathrm{~m}$ sprint accelerations without important strength training habits), rugby players ( $5-30 \mathrm{~m}$ sprint accelerations with strength training habits), basketball players ( $5-10 \mathrm{~m}$ sprint accelerations and ballistic actions) and weightlifters (no sprint acceleration but high strength training habits). After a complete warm-up (jogging and joint mobility exercises followed by three progressive sprints of $30-40 \mathrm{~m}$ at increasing running velocities), athletes performed two
or three all-out $40-\mathrm{m}$ sprints (separated by $>10 \mathrm{~min}$ ) from a crouched position (staggeredstance), the fastest trial being considered for further analyses. During each trial, athlete's instantaneous velocity was measured at a sampling rate of 46.875 Hz with a radar system (Stalker ATS System, Radar Sales, Minneapolis MN, USA) placed on a tripod 10 m behind the subjects at a height of 1 m . All data were collected using STATS software (Model: Stalker ATS II Version 5.0.2.1, Applied Concepts, Dallas, TX, USA) provided by the radar device manufacturer.

## Data analyses

Individual force- and power-velocity relationships in sprinting were assessed as described in previous studies (details in Samozino et al 2016, Morin et al 2019). Briefly, for each trial, velocity-time curve was fitting using equation 1 (with a time delay as described in Morin et al 2019) and least-square regression method to determine individual $v_{H} \max$ and $\tau$ values. From the latter and equations (3) to (5), $F_{H}$ and the associated power output in the horizontal direction were computed at each instant to determine individual force- and power-velocity relationships in sprinting, and associated $F_{H} 0, v_{H} 0, P_{H} \max$ and $S_{F v}$ values. Using equation (2), split times at $5,10,15,20$ and 30 m were computed. If the effect of $S_{F v}$ on sprint acceleration performance was observed as curvilinear, as supposed here from what occurs in jumping ${ }^{12}$, the optimal $S_{F v}$ ( $S_{F v} O P T$ ) was computed for each athlete and each sprint distance simulating equation (18) using individual $k$ and $P_{H} \max$ values. Then, the actual athlete's $S_{F v}$ value was expressed in percentage of $S_{F v} O P T$ of each sprint distance, and the F-v difference between $S_{F v}$ and $S_{F v} O P T$ ( $F v_{\text {diff }}$, in \%) was computed as:
$F v_{d i f f}=100 .\left|1-\frac{S_{F v}}{S_{F v} p t}\right|$

## Statistical analyses

All data are presented as mean $\pm$ standard deviation (SD). If $S_{F v} O P T$ existed, the respective contributions of $P_{H} \max$ and $F v_{\text {diff }}$ (independent variables) to explain inter-individual variability in acceleration performance (dependent variable) were assessed using stepwise multiple regression analyses (standardised $\beta, F, R^{2}$ change [per variable], and model $R^{2}$ [adjusted], $p$ ) performed separately for the different sprint distances (5, 10, 15, 20 and 30 m ) on all data pooled together. $P_{H} \max$ and $F v_{\text {diff }}$ at the different sprint distances originally violated the assumption of distribution normality and were then log- and square-root-transformed,
respectively. Note that if $S_{F v} O P T$ did not exist, $S_{F v}$ was used in the multiple regression analysis instead of $F v_{\text {diff }}$.

## RESULTS

## MODEL SIMULATION

For sprint acceleration performances simulated for individuals with different values of $k$ (from 0.2 to 0.35 ), $v_{H_{\max }}\left(\right.$ from 5 to $12 \mathrm{~m} . \mathrm{s}^{-1}$ ) and $\tau$ (from 0.8 to 1.5 s ), the RMSError in $F_{H}$ over the entire F-v relationship between values modelled by the $2^{\text {nd }}$ order polynomial function and values obtained by a linear regression were from 0.382 to 4.50 N (i.e. from 0.005 to $0.056 \mathrm{~N} . \mathrm{kg}^{-}$ ${ }^{1}$, figure $1 \mathrm{~A}, \mathrm{~B}$ ). The associated differences in $F_{H} 0$ and $v_{H} 0$ were from -10.05 to -0.848 N (i.e. from -0.123 to $-0.01 \mathrm{~N} . \mathrm{kg}^{-1}$ ), and from -0.206 and $-0.009 \mathrm{~m} . \mathrm{s}^{-1}$, respectively (Figure 1C, D).

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*** INSERT FIGURE 1 ABOUT HERE ***
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As expected, $P_{H} \max$ positively affects sprint acceleration performance, as illustrated in Figure 2B with a decrease in sprint acceleration time when $P_{H} \max$ increases, whatever the F -v profile. The main original result was the curvilinear changes in sprint acceleration time with F-v profile for a given $P_{H} \max$ and sprint distance (Figure 2A, B). Such variations support the existence of an optimal $S_{F v}\left(S_{F v} O P T\right)$ minimising the sprint time (and so maximising sprint acceleration performance) for given $P_{H} \max$ and sprint distance. Moreover, $S_{F v} O P T$ values change according to both $P_{H} \max$ and sprint distance values: $S_{F v} O P T$ values tend to velocity-oriented F-v profiles when sprint distance increases or when $P_{H} \max$ decreases (Figure 3).
*** INSERT FIGURES 2 and 3 ABOUT HERE ***

## MODEL APPLICATION TO EXPERIMENTAL DATA

As initially expected, athlete's $P_{H} \max$, F-v profile and sprint acceleration times at different sprint distances were very different across sport activities (Table 1). Figure 4 presents the individual F-v profiles expressed relatively to the optimal one for each sprint distance and each sport activity. Modelled and actual sprint acceleration performances are presented in Figure 5
(and associated videos presented in supplementary materials) to illustrate the respective effects of $P_{H} \max$ and F-v profile on acceleration performance over different sprint distances of male and female athletes from different sports. Figures 4 and 5 show that the variability in $S_{F v} O P T$ across sprint distances (whatever $P_{H} \max$ ) is larger than the inter-individual variability in F-v profile. For all sprint distances, multiple regression analyses showed that both $P_{H} \max$ and $F v_{\text {diff }}$ significantly contributed to sprint acceleration performances and explained their quasientire variance ( $R^{2>}>0.99$, Table 2). For the different sprint distances, the variances in performance (time to cover the distance) were mainly explained by $P_{H} \max$ ( $R^{2}$ change from 0.92 to 0.99 ) with a high sensitivity (standardised $\beta$ from -0.88 to 1.05 ). Yet significant and non-negligible, the part of the explained variance of $F v_{\text {diff }}$ in sprint performance ( $R^{2}$ change from 0.004 to 0.063 ), as well as its weight to predict sprint performance (standardised $\beta$ from 0.06 to 0.26 ), are lower.
*** INSERT TABLES 1 and 2 ABOUT HERE ${ }^{* * *}$
*** INSERT FIGURES 4 and 5 ABOUT HERE ${ }^{* * *}$

## DISCUSSION

The main finding of this study is that sprint acceleration performance over short distances ( $<30$ m ) depends on both maximal horizontal power output ( $P_{H} \max$ ) and individual F-v profile, with the existence of an individual optimal F-v profile corresponding to the best balance between horizontal force and velocity capacities. This optimal profile, which can be accurately determined for each individual by numerical simulation, depends on maximal power output and sprint distance. The validity of the macroscopic biomechanical model of sprint acceleration performance on which these findings were based was supported by very low errors between modelled and measured values. When applying this model to the data of 231 male and female athletes from very different sports, differences between optimal and actual F-v profile ( $F v_{\text {diff }}$ ) were observed and depend more on sprint distance than on individual F -v profile. For a given sprint distance, acceleration performance ( $<30 \mathrm{~m}$ ) mainly depends on $P_{H} \max$ and slightly on $F v_{\text {diff }}$, the weight of each variable changing regarding sprint distance.

## The biomechanical model

These results are based on a macroscopic biomechanical model using an inverse dynamics approach applied to the athlete's body CoM during linear sprint running acceleration. Based on
the commonly used mono-exponential model of the CoM kinematics during an all-out running acceleration, this approach models the net step-averaged external force the runner develops onto the ground in the horizontal antero-posterior direction, as well as the associated power output. Note that this power output (named 'horizontal power output' for simplicity ${ }^{31}$ ) corresponds here to the rate of the mechanical work only associated to the net step-averaged horizontal component of the external force, i.e. associated to i) the step-to-step change in mechanical kinetic energy of the CoM in the horizontal antero-posterior direction and ii) to the work performed against air friction. Because it aims at studying step-to-step athlete's acceleration capacities, this model focuses on step-averaged external horizontal force production and not on within-step changes in external force nor on the total external power generated by muscles including the internal power to accelerate the segments with respect to $\mathrm{CoM}^{32}$. Moreover, this approach does not focus on the several underpinning mechanisms, such as muscle architecture, neuromuscular function, movement pattern, running kinematics (contact time, step rate/length) or other motor behaviours involved in sprint performance. The latter are not neglected within this macroscopic model, but they are encompassed by the different parameters associated to the model, provided they affect external horizontal force production. The main other simplifying assumptions of this model are those inherent to the application of fundamental laws of dynamics to the whole human body considered as a system represented by its CoM ${ }^{4,12,20}$, and the estimation of the horizontal aerodynamic drag from only stature, body mass and a fixed drag coefficient ${ }^{28}$. The high concurrent validity and reliability of such a model to estimate stepaveraged external horizontal force and power output values have been recently supported through comparisons to reference force plate measurements during overground sprinting ${ }^{9,20}$. Although F-v relationships in sprinting have been experimentally described as strongly linear ${ }^{8,9,13,20,29}$, the present biomechanical model showed that, when the velocity-time curve is modelled by a mono-exponential regression during an all-out sprint acceleration ${ }^{20,27}$, the horizontal force developed onto the ground changes with velocity following a $2^{\text {nd }}$ order polynomial function due to aerodynamic friction force. However, model simulations, covering the entire range of $k, v_{H_{\max }}$ and $\tau$ values characterizing typical human sprint accelerations, showed very low and negligible differences in $F_{H}, v_{H} 0$ and $F_{H} 0$ between linear and polynomial models. Note that the extreme, yet very low, error values reported here were obtained for a sprint simulated for an individual gathering extreme characteristics: $2-\mathrm{m}$ tall, $110-\mathrm{kg}$ individual who reaches $12 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ with a time constant of 1.5 s . These results support the practical validity and relevance of using linear regressions to describe F-v relationship in human sprint running acceleration, as well as the simplifying assumption made for equation (13) and following ones.

That said, it cannot be ruled out that the F-v relationship is actually linear, which would challenge the mono-exponential model used here to describe velocity-time curve during sprint acceleration. In both cases, the differences between models are largely lower than the measurement noise for human performances.
The validity of the proposed biomechanical model was tested comparing the acceleration times measured using a laser device to times estimated using equation (18) with, as input data, individual $P_{H} \max$ and F-v profiles measured by force plates (see Supplementary Material). The results showed no differences between predicted and measured values, associated to a low systematic ( $<0.4 \%$ ) and random ( $<3.5 \%$ ) errors. These low differences were within the range of measurement noise previously reported for different sprint time or force-velocity relationship variables ${ }^{20,33,34}$, and showed that the errors induced by the above-mentioned simplifying assumptions are very low. Therefore, sprint acceleration times ( $<30 \mathrm{~m}$ ) can be accurately predicted using the proposed model from individual $P_{H} \max$, F-v profiles and $k$ values.

## Horizontal force production capacities and sprint acceleration performance

Sprint acceleration performance over short distances ( $<30 \mathrm{~m}$ ), considered here as the time required to cover a given distance, was shown to depend on both the maximal power output developed in the horizontal direction $\left(P_{H} \max \right)$ and the F-v profile $\left(S_{F v}\right)$, as previously reported for acyclic ballistic push-off ${ }^{12}$. This was supported here by numerical simulation of the model, but also from regression analyses on more than 230 athletes from different sports. Even if their respective magnitude of influence on sprint acceleration performance changes with the value of one another and with the sprint distance, simulations of the model (figure 2) showed that $P_{H} \max$ has overall the greatest weight. From a purely mechanical point of view and from previous experimental studies ${ }^{8,18,19}$, sprint acceleration performance (whatever the distance) is directly related to the average horizontal power output produced over the entire targeted distance. As expected, the model simulation results confirmed that this largely depends on $P_{H} \max$, which is a macroscopic index informing on horizontal force production capacities over the entire spectrum of velocities (i.e. related to a shift of the F-v relationship upwards and to the right). When investigating in more details the horizontal force production abilities, sprint acceleration performance, and in turn average power output produced over the entire acceleration distance, also depends on F-v profile, that is the ratio between horizontal force production capacities at low $\left(F_{H} 0\right)$ and high $\left(v_{H} 0\right)$ velocities (i.e., the slope of the $\mathrm{F}-\mathrm{v}$ relationship). For each individual, characterized by both $P_{H} \max$ and $k$ values, and for each sprint distance, a sprint optimal F-v profile ( $S_{F v} O P T$ ) exists that represents the best balance
between $F_{H} 0$ and $v_{H} 0$ (i.e. the best $\mathrm{F}-\mathrm{v}$ relationship slope) maximizing acceleration performance (and so minimizing sprint times). This optimal F-v profile allows one athlete to stay the closest as possible of his/her optimal velocity over the entire acceleration phase, corresponding to horizontal power output within the upper part of the parabolic P-v relationship (Figure 6). The average horizontal power output produced over the entire acceleration phase is then maximised and the acceleration time minimized. Another athlete, with the same $P_{H} \max$, but presenting a non-optimal F -v profile too much oriented towards $F_{H} 0$ and not enough towards $v_{H} 0$ (i.e. too 'steep' $\mathrm{F}-\mathrm{v}$ relationship) would produce power output mostly on the descending part of his $\mathrm{P}-\mathrm{v}$ relationship (Figure 6), and vice versa for an athlete presenting a non-optimal F-v profile not enough oriented towards $F_{H} 0$ (i.e. too 'flat' F -v relationship). For the latter cases, the average horizontal power output produced over the acceleration phase would be reduced, and so would performance. Note that in the typical example presented in Figure 6, the performance difference between the two athletes at 20 m is $\sim 0.1 \mathrm{~s}$ (or $\sim 80 \mathrm{~cm}$ ), which represents a meaningful advantage in team sports.

Obviously, besides being influenced by $P_{H} \max$ values, the optimal F-v profile value mostly changes with sprint distance: the lower the sprint distance, the more $S_{F v} O P T$ oriented towards force capabilities $\left(F_{H} 0\right)$ and vice versa. The present approach brings insights about the optimal F-v profile values between these two extreme sprints situations and to what extent they may affect sprint acceleration performance. Note that the effect of sprint distance on $S_{F v} O P T$ is important for short sprint distances (until $\sim 15 \mathrm{~m}$, Figure 3), which represent the most common sprint acceleration distances in many sport activities, except in track-and-field sprinting events. Concerning these short sprint accelerations ( $<15 \mathrm{~m}$ ), although the effect of F -v profile on sprint performance may seem to be low in absolute values (Figure 2A), this effect is quite important when considering relative changes (from $\sim 10$ to $20 \%$ ). Moreover, for short distance accelerations in numerous sport activities, the aim of sprint acceleration is not to largely outdistance the opponent, but only to take advantage over the very first seconds. For longer sprint accelerations ( $>15 \mathrm{~m}$ ), $S_{F v} O P T$ values correspond to the upper part of the range of $S_{F v}$ human values oriented towards velocity capacities. This explains why sprint acceleration performance, mostly studied on relatively long sprint accelerations ( $40-\mathrm{m}$ to $100-\mathrm{m}$ times, $4-\mathrm{s}$ distance), was previously only related to $P_{H} \max$ and $v_{H} 0^{8,18,19,22}$. For shorter distance accelerations ( 5 to 15 m ), $F_{H} 0$ presents more importance through notably, as shown here, an optimal balance between $F_{H} 0$ and $v_{H} 0$.

When considering actual human F-v profile values presented here and previously reported 8,18,19,22,30 ranging from $\sim-1.6$ to -0.4 N.s.m ${ }^{-1} \cdot \mathrm{~kg}^{-1}$, differences between actual and optimal F-v profiles ( $F v_{d i f f}$ ) occur, whatever the sprint distance (Table 1, Figures 4 and 5). Multiple regression analyses performed here on $\sim 230$ athletes showed that sprint acceleration performance depends on both $P_{H} \max$ and $F v_{\text {diff }}$, with contribution weight changing as a function of sprint distance. Overall, for acceleration up to 30 m , sprint performance was largely explained (variance and standardized weight) by $P_{H} \max$ and to a lesser extent by $F v_{d i f f}$ (i.e. by the fact that the $\mathrm{F}-\mathrm{v}$ profile is not optimal). For acceleration distances between 10 and 20 m , the weight of $F v_{\text {diff }}$ is very low (explained variance $<1.5 \%$, standardized beta $<0.13$ ). For shorter ( $<10 \mathrm{~m}$ ) or longer ( $>20 \mathrm{~m}$ ) accelerations, the contribution of $F v_{\text {diff }}$ to performance increases with optimal F-v profiles oriented towards $F_{H} 0$ or $v_{H} 0$, respectively.

One of the main findings highlighted by the application of the model to experimental data of athletes from very different sports (sprinting, team sports, weightlifting) was that $F v_{\text {diff }}$ depends more on the variability in the optimal F-v profile between sprint distances than on the interindividual variability in F-v profiles, as well illustrated in Figures 4 and 5. For short sprint accelerations ( $<\sim 10 \mathrm{~m}$ ), all the athletes (from sprinters to weightlifters) present a F-v profile not oriented enough towards $F_{H} 0$ compared to the optimal F-v profile (i.e. too 'flat' F -v relationship, or force deficit). For long sprint accelerations ( $>\sim 20 \mathrm{~m}$ ), all the athletes tested here (and likely all humans) present a F -v profile not oriented enough towards $v_{H} 0$ compared to the optimal F-v profile (i.e. too 'steep' F-v relationship, or velocity deficit). Concerning intermediate sprint distances ( $\sim 10$ to 20 m ), F-v profiles are distributed on both sides of the optimal F-v profile, but the weight of $F v_{\text {diff }}$ in acceleration performance over such distances is very low. The larger influence of acceleration distance (compared to the influence of individual F -v profiles) on $F v_{\text {diff }}$ is mainly due to large differences in the optimal F-v profile between different sprint distances which is larger than the substantial interindividual variability in F-v profile rather observed within each sport than between disciplines. This is well illustrated in figure 4 and in line with Haugen et al.'s results obtained on more than $\sim 650$ elite athletes from 23 different sports ${ }^{35}$. Note that the interindividual variability in F-v profile observed here (and associated $F v_{\text {diff }}$ values) is quite related to the sample of athletes tested and may be slightly different with other athletes, notably elite ones.

## Force-velocity profile and sprint performance optimization.

When a training program is designed to improve sprint acceleration performance, assessing Fv profiles in addition to $P_{H} \max$ may help more finely and individually orient training modalities ${ }^{13,21}$. Note that both of these variables can be obtained in field conditions using a recently validated method requiring only anthropometric (body mass and stature) and spatiotemporal (split times or instantaneous velocity) parameters ${ }^{9,20}$. Values of $S_{F v}$ make the comparison among athletes possible independently from their power capabilities and their sprint performances (split times at given distances) ${ }^{21}$, and thus to know whether an athlete, as compared to another one, is characterized by a "force" or a "velocity" sprint profile. The individualization of the training content could hitherto only be done from comparisons to others or to normative values. As previously proposed for ballistic movement ${ }^{12}$, expressing the individual value of $S_{F v}$ relatively to $S_{F v} O P T$ for a target sprint distance would allow to quantify to what extent an individual $\mathrm{F}-\mathrm{v}$ profile is not optimal to take the most advantage of $P_{H}$ max regarding the distance-specific sprint requirements. This can then be used to orient training modalities for a given athlete according to their own strengths and weaknesses, movement specificities and sport context. The present results showed that improving sprint acceleration performance may be achieved through increasing power capabilities (i.e. shifting F-v relationship upwards and to the right) and shifting the F-v profile as close to the optimal one as possible. In case of a 'force deficit' (i.e. F-v profile not oriented enough towards $F_{H} 0$ compared to the optimal one, $S_{F v}<100 \% S_{F v} O P T$ ), force production capacities at low velocity should be trained in priority to increase $P_{H} \max$, and vice versa in case of 'velocity deficit' $\left(S_{F v}>100 \%\right.$ $S_{F v} O P T$ ). The magnitude of priority can be given by the value of $F v_{\text {diff }}$ (i.e. the difference between $S_{F v}$ and $S_{F v} O P T$ ): the higher $F v_{\text {diff }}$ (i.e. $S_{F v}$ largely lower or higher than $100 \%$ $S_{F v} O P T$ ), the higher the interest to both optimize the F-v profile and increase $P_{H} \max$. Since the variability in $S_{F v} O P T$ across acceleration distances is higher than the inter-individual variability in F-v profile, sprint acceleration training should be individualized rather by considering the target distance over which the acceleration performance should be maximized than considering the individual F-v profile, as previously suggested and shown for jumping ${ }^{21,36}$. Even if in some sports or codes (e.g. team sports) it can be quite complex to define only one training-targeted acceleration distance, the present findings can give some overall insights about how (and with what level of priority) to orient sprint acceleration training content regarding acceleration distances mainly occurring for a given athlete: increasing $P_{H}$ max through $F_{H} 0$ improvement if short sprint accelerations are targeted $(<\sim 10 \mathrm{~m})$, through $v_{H} 0$
improvement for long accelerations ( $>\sim 20 \mathrm{~m}$ ) or through training horizontal force production capacities over the entire velocity spectrum for acceleration distances between 10 and 20 m . Moreover, the biomechanical model used here makes possible, for a given athlete, to estimate the potential gains in acceleration performance associated to an improvement in power capabilities or to a change in F-v profile. This prediction could help coaches to prioritize training towards the one or the other of these two targets. Such changes in the sprint F-v relationship, notably in its slope, may be achieved by specific training focusing rather on $F_{H} 0$ or $v_{H} 0$. The latter are very different since they refer to opposite training modalities associated to different movement velocities, force to produce, body positions or segment configurations. For instance, heavy resisted sled training represents a specific means of providing overload to horizontal force production capacities to increase the training exposure to high force-low velocity conditions, which was clearly shown to increase $F_{H} 0$ and $P_{H} \max$ with trivial effect on $v_{H} 0{ }^{37,38}$. Contrastingly, although less studied, training horizontal force production specifically at very high velocities could be performed during maximal speed sprinting or over speed conditions, and should improve $v_{H} 0{ }^{39}$. It is worth noting that at high running speed, there is an interplay between horizontal and vertical force production capacities: the higher the running speed, the more the athlete have to produce high vertical force onto the ground to limit the contact duration and the associated breaking impulse ${ }^{6,23}$. This double target lower limb should face during the support phase in the late acceleration (high forces in both horizontal and vertical direction) could also contribute to the difficulty to keep producing net backward horizontal force at very high velocities, which was partly shown by the decrease in the ability to orient effectively the force produced onto the ground when velocity increases ${ }^{8,18,19,23}$.
In both $F_{H} 0$ - and $v_{H} 0$-oriented training based on previous considerations, it is likely that both $P_{H} \max$ will increase and $S_{F v}$ will be optimized (i.e. change towards $S_{F v} O P T$ ). These two changes would both result in a higher sprint acceleration performance, as recently shown for jumping ${ }^{36}$. Note that the initial level of horizontal force production properties influences the degree of mechanical response when training at different ends of the F-v spectrum, which should be also considered by practitioners to optimize the individual effectiveness of resisted and assisted sprint training ${ }^{39}$. Contrastingly, an improvement in $P_{H} \max$ associated to a F-v profile even less optimal could induce no gain, if not an alteration, in sprint acceleration performance. Among other experimental reasons, this could partly explain the results of Rakovic et al. ${ }^{40}$ showing that individualized sprint-training based on F-v profile was no more effective in improving sprint performance than a generalized sprint-training program. The training individualization was performed based on individual F-v profile compared to group
values. Since the $S_{F v}$ group mean was $-0.90 \pm 0.06 \mathrm{~N} . \mathrm{s}^{2} \mathrm{~m}^{-1} . \mathrm{kg}^{-1}$ and the target sprint distance was 30 m , one can reasonably consider, regarding the present results, that all the subjects presented a non-optimal F -v profile towards a $v_{H} 0$ deficit, and so individualized training interventions may have increased the difference with the optimal F-v profile for numerous subjects. Therefore, yet the weight of F-v profile in acceleration performance is lower than the one of maximal power, considering the optimal F-v profile associated to the target sprint distance could have helped to better individualize training and provided better sprinting performances.

## PERSPECTIVES

Horizontal force-velocity relationship during sprinting has been shown to be insightful for training sprint propulsion abilities ${ }^{21}$. A simple field method, requiring only spatio-temporal and anthropometrical data, was proposed some years ago to assess horizontal force-velocity-power profile during sprinting with a high concurrent validity compared to force plate measurements ${ }^{9,20}$, which makes accessible this kind of testing to many athletes and coaches. Once this profile obtained, the remaining question many sport scientists and practitioners have is about how to train this profile in order to improve effectively sprint acceleration performance. The present study brings some answers showing the existence of an individual optimal F-v profile in sprinting which can be used as a training target, as it was shown for jumping ${ }^{12,21}$. Expressing the individual F-v profile relatively to the optimal one allows to identify the F-v quality to train in priority regarding the distance-specific sprint requirements. This can then be used to orient training modalities to improve the maximal power output while orienting the Fv profile closer to the optimal one by focusing training of horizontal force production at low or high sprinting velocities, or throughout the entire velocity spectrum ${ }^{21}$. Finally, the approach used here is based on macroscopic indices of step-averaged horizontal force production capacities during sprinting. The interactions between these variables and other force components (notably the vertical component), within-step changes in external force and internal mechanical power remain to be further explored to bring insights about the underpinning mechanical determinants of sprinting F-V-P relationship and better understand the transmission efficiency between muscle local function and external mechanical function ${ }^{11}$.

## CONCLUSION

Based simulations of a biomechanical model presenting a high concurrent validity compared to experimental values and on data measured on 230 athletes from different sports, sprint acceleration performance (over distances $<30 \mathrm{~m}$ ) was shown to mainly depends on maximal horizontal power capabilities, but also (even to a lesser extent) on F-v mechanical profile characterizing the ratio between maximal horizontal force production at low velocities and horizontal force production capacity at high velocities. For a given maximal horizontal power output, an individual optimal balance between these two capacities exists that maximizes sprint acceleration performance. This sprinting "optimal mechanical F-v profile" changes with the individual maximal horizontal power output, but also and mainly with the sprint distance: the shorter the sprint distance, the more the optimal F-v profile is oriented towards force profile, that is towards maximal horizontal force capacity. Consequently, differences between optimal and actual F-v profile are observed and depend more on sprint distance than on individual F-v profile. For a given sprint distance ( $<30 \mathrm{~m}$ ), the differences in acceleration performance between athletes mainly depends on differences in maximal power capacities and slightly in difference magnitude between actual and optimal F-v profile, the weight of each of them changing with sprint distances. These findings have direct practical applications for sport performance optimization to individualize sprint acceleration training regarding the sprint distance on which the performance has to be improved and the athlete's sprinting F-v profile.

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## Conflict of Interest

The authors declare that the research was conducted in the absence of any factors that could be construed as a potential conflict of interest.

## Availability of materials and data

The datasets generated by model simulations and analysed during the current study are available from the corresponding author on reasonable request

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Table 1: Mean $\pm$ SD of the body mass, maximal horizontal power output and $\mathrm{F}-\mathrm{v}$ profile, as well as the acceleration time for different sprint distances.

|  | n | Stature (m) | $\begin{gathered} \hline \text { Body mass } \\ (\mathrm{kg}) \end{gathered}$ | $\begin{gathered} P_{H} \max \\ \left(\mathrm{~W} \cdot \mathrm{~kg}^{-1}\right) \end{gathered}$ | F-v profile (N.s. $\mathrm{m}^{-1} \cdot \mathrm{~kg}^{-1}$ ) | 5 m | 10 m | $\begin{gathered} \hline \text { Sprint Time (s) } \\ 15 \mathrm{~m} \\ \hline \end{gathered}$ | 20 m | 30 m |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sprint | 28 | $1.72 \pm 0.08$ | $63 \pm 9.8$ | $18.4 \pm 3.05$ | $-0.79 \pm 0.08$ | $1.37 \pm 0.070$ | $2.09 \pm 0.109$ | $2.73 \pm 0.146$ | $3.32 \pm 0.182$ | $4.46 \pm 0.257$ |
| Men | 15 | $1.79 \pm 0.04$ | $70.1 \pm 7.6$ | $20.8 \pm 2.19$ | $-0.79 \pm 0.10$ | $1.32 \pm 0.056$ | $2.01 \pm 0.078$ | $2.61 \pm 0.094$ | $3.18 \pm 0.108$ | $4.25 \pm 0.134$ |
| Women | 13 | $1.65 \pm 0.05$ | $54.9 \pm 3.8$ | $15.8 \pm 0.91$ | $-0.79 \pm 0.06$ | $1.42 \pm 0.030$ | $2.19 \pm 0.043$ | $2.86 \pm 0.055$ | $3.49 \pm 0.069$ | $4.71 \pm 0.099$ |
| Soccer | 106 | $1.74 \pm 0.09$ | $69.6 \pm 11.6$ | $14.8 \pm 2.22$ | $-0.79 \pm 0.08$ | $1.46 \pm 0.067$ | $2.24 \pm 0.108$ | $2.93 \pm 0.150$ | $3.58 \pm 0.194$ | $4.84 \pm 0.287$ |
| Men | 72 | $1.79 \pm 0.06$ | $75.4 \pm 8.6$ | $15.8 \pm 1.78$ | $-0.78 \pm 0.07$ | $1.43 \pm 0.055$ | $2.19 \pm 0.082$ | $2.86 \pm 0.106$ | $3.48 \pm 0.131$ | $4.69 \pm 0.182$ |
| Women | 34 | $1.64 \pm 0.07$ | $57.4 \pm 6.1$ | $12.6 \pm 1.27$ | $-0.83 \pm 0.08$ | $1.52 \pm 0.047$ | $2.35 \pm 0.073$ | $3.09 \pm 0.100$ | $3.79 \pm 0.129$ | $5.16 \pm 0.193$ |
| Basket ball | 45 | $1.86 \pm 0.10$ | $77.7 \pm 12.9$ | $13.3 \pm 2.39$ | $-0.84 \pm 0.12$ | $1.5 \pm 0.094$ | $2.32 \pm 0.149$ | $3.05 \pm 0.204$ | $3.74 \pm 0.262$ | $5.09 \pm 0.388$ |
| Men | 28 | $1.92 \pm 0.08$ | $82.9 \pm 12.7$ | $14.5 \pm 1.78$ | $-0.81 \pm 0.13$ | $1.46 \pm 0.084$ | $2.25 \pm 0.112$ | $2.94 \pm 0.13$ | $3.60 \pm 0.144$ | $4.87 \pm 0.168$ |
| Women | 17 | $1.77 \pm 0.05$ | $69.3 \pm 7.9$ | $11.4 \pm 1.93$ | $-0.88 \pm 0.09$ | $1.56 \pm 0.078$ | $2.43 \pm 0.132$ | $3.22 \pm 0.188$ | $3.98 \pm 0.248$ | $5.45 \pm 0.375$ |
| Rugby | 35 | $1.77 \pm 0.1$ | $87.1 \pm 19.1$ | $16.9 \pm 3.19$ | $-0.97 \pm 0.11$ | $1.37 \pm 0.088$ | $2.13 \pm 0.134$ | $2.82 \pm 0.173$ | $3.47 \pm 0.211$ | $4.74 \pm 0.286$ |
| Men | 20 | $1.84 \pm 0.07$ | $102 \pm 9.8$ | $19.1 \pm 2.35$ | $-1.02 \pm 0.12$ | $1.31 \pm 0.061$ | $2.04 \pm 0.086$ | $2.69 \pm 0.105$ | $3.31 \pm 0.122$ | $4.52 \pm 0.155$ |
| Women | 15 | $1.68 \pm 0.05$ | $67.4 \pm 5.3$ | $13.9 \pm 1.04$ | $-0.91 \pm 0.05$ | $1.45 \pm 0.036$ | $2.26 \pm 0.054$ | $2.98 \pm 0.072$ | $3.67 \pm 0.09$ | $5.02 \pm 0.128$ |
| Weightlifting | 17 | $1.72 \pm 0.07$ | $76.4 \pm 11.2$ | $13.5 \pm 1.67$ | $-0.97 \pm 0.05$ | $1.46 \pm 0.055$ | $2.29 \pm 0.092$ | $3.03 \pm 0.129$ | $3.75 \pm 0.168$ | $5.16 \pm 0.248$ |
| Men | 9 | $1.76 \pm 0.06$ | $84.4 \pm 8.9$ | $14.4 \pm 1.75$ | $-0.98 \pm 0.06$ | $1.43 \pm 0.056$ | $2.24 \pm 0.093$ | $2.96 \pm 0.132$ | $3.66 \pm 0.171$ | $5.03 \pm 0.255$ |
| Women | 8 | $1.66 \pm 0.02$ | $67.4 \pm 4.4$ | $12.4 \pm 0.70$ | $-0.96 \pm 0.04$ | $1.5 \pm 0.025$ | $2.35 \pm 0.046$ | $3.11 \pm 0.070$ | $3.85 \pm 0.095$ | $5.3 \pm 0.148$ |

Table 2: Stepwise multiple regression analysis results assessing the importance of the maximal horizontal power output ( $P_{H} \max$ ) and force-velocity profile (via the difference between actual and optimal F-v profile, $F v_{d i f f}$ ) to predict sprint acceleration time at $5,10,15,20$ and 30 m .

|  | stand. $\beta$ | $\mathrm{R}^{2}$ change | $P$-value | F | df | model $\mathbf{R}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| time@5m |  |  |  | 14727 | 2, 230 | 0.992 |
| Prmax | -1.044 | 0.940 | <. 001 |  |  |  |
| $F V_{\text {diff }}$ | 0.239 | 0.052 | <. 001 |  |  |  |
| time@10m |  |  |  | 21790 | 2, 230 | 0.995 |
| Phmax | -1.010 | 0.989 | <. 001 |  |  |  |
| $F v_{\text {diff }}$ | 0.078 | 0.006 | <. 001 |  |  |  |
| time@15m |  |  |  | 30906 | 2, 230 | 0.996 |
| Prmax | -0.985 | 0.992 | <. 001 |  |  |  |
| $F V_{\text {diff }}$ | 0.060 | 0.004 | <. 001 |  |  |  |
| time@20m |  |  |  | 12646 | 2, 230 | 0.991 |
| Phmax | -0.948 | 0.976 | <. 001 |  |  |  |
| $F V_{\text {diff }}$ | 0.120 | 0.015 | <. 001 |  |  |  |
| time@30m |  |  |  | 11279 | 2, 230 | 0.990 |
| $P_{\text {Hmax }}$ | -0.889 | 0.927 | <. 001 |  |  |  |
| $F V_{\text {diff }}$ | 0.261 | 0.063 | <. 001 |  |  |  |

stand. $B$, beta-weight in standardized units; $F, F$ value; $d f$, degrees of freedom

## FIGURE LEGENDS

Figure 1: Linear (grey dash line) and $2^{\text {nd }}$ order polynomial (black line) model to describe Forcevelocity relationship of a typical individual: aerodynamic friction coefficient $(k)=0.3$, maximal velocity $\left(v_{H} \max \right)=10 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ and time constant $(\tau)=1.2 \mathrm{~s}$ (Panel A). Root Mean Square Error (RMSError) values in horizontal force (B), as well as the differences in maximal theoretical horizontal force $\left(F_{H} 0, \mathrm{C}\right)$ and velocity $\left(v_{H} 0, \mathrm{D}\right)$ between values obtained by the $2^{\text {nd }}$ order polynomial function and values obtained by the linear regression fitting the values obtained by this polynomial function, for different simulated sprints characterizing individuals with different $k$ (from 0.2 to 0.35 ), $v_{H} \max$ (from 5 to $12 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ ) and $\tau$ (from 0.8 to 1.5 s ).

Figure 2: Changes in sprint acceleration time as function of Force-velocity profile ( $S_{F v}$ ) for different sprint distances $(5,10,15,20,25$ and 30 m$)$ and a given maximal horizontal power output $\left(P_{H} \max =20 \mathrm{~W} . \mathrm{kg}^{-1}\right.$, panel A) and different $P_{H} \max \left(10,14,18,22,26\right.$ and $\left.30 \mathrm{~W} . \mathrm{kg}^{-1}\right)$ and a given sprint distance ( 15 m , panel B). The grey area represents commonly sprinting F-v profile values previously reported in sport and sciences. The white dots represent the best performances reached at the optimal force-velocity profile for each simulated condition.

Figure 3: Optimal Force-velocity profile according to the sprint acceleration distance for different maximal horizontal power output ( $P_{H} \max , 10,15,20,25$ and $30 \mathrm{~W} . \mathrm{kg}^{-1}$ ). The grey area represents commonly sprinting F-v profile values previously reported in sport and sciences.

Figure 4: Force-velocity (F-v) profiles expressed relatively to the optimal profile for each sprint distance and each sport activity. Individual data are presented as jittered dots. The summary of the data is shown as a violin plot reflecting the data distribution, a vertical black bar indicating the median of the data and a horizontal bar indicating the $95 \%$ confidence interval determined by bootstrapping.

Figure 5: Modelled sprint acceleration time as a function of F-v profile and maximal horizontal power output ( $P_{H} \max$ ) for different sprint distances. The markers represent experimental acceleration times measured on male and female athletes from different sport activities.

Figure 6: Illustration of horizontal force- and power-velocity relationships of two athletes (panel A) and their horizontal power production and velocity over time during a $20-\mathrm{m}$ sprint
acceleration (Panel B). The two athletes present the same maximal horizontal power output with different F-v profile: Athlete 1 (black lines) present an optimal F-v profile ( $S_{F v} O P T$ ) maximising a $20-\mathrm{m}$ acceleration and Athlete 2 (grey lines) present 'velocity deficit' with a velocity deficit for a $20-\mathrm{m}$ acceleration (i.e. F-v profile not oriented enough towards force production capacities at high velocities). These data are obtained from model simulation (with $k=0.0031$ ).

