Journal Pre-proof

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PII: S0022-460X(20)30303-5

DOI: <https://doi.org/10.1016/j.jsv.2020.115471>

Reference: YJSVI 115471

To appear in: *Journal of Sound and Vibration*

Received Date: 28 September 2019

Revised Date: 19 May 2020

Accepted Date: 21 May 2020

Please cite this article as: K. Koo, B. Pluymers, W. Desmet, Acoustic impedance sensitivity approach for defining absorbing material layout using the wave based method, *Journal of Sound and Vibration* (2020), doi: <https://doi.org/10.1016/j.jsv.2020.115471>.

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Kunmo Koo: Conceptualization, Methodology, Formal analysis, Software, Writing-Review & Editing

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Acoustic impedance sensitivity approach for defining absorbing material layout using the wave based method

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Submitted to **Journal of Sound and Vibration**
18th May, 2020

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Abstract

This study used the wave based method (WBM) to investigate the optimum layout of an absorbing material on a vibrating surface. The structural dynamic behavior was examined using the finite element model, and the acoustic response was computed using the WBM. The thickness of the absorbing material on each structural element was considered as the design variable. The sensitivity of the acoustic response to the acoustic impedance was derived using wave based system matrices, and a polynomial function was used to describe the impedance variation with respect to the thickness of the absorbing material. Thereafter, the final design sensitivity of a vibro-acoustic model was derived with respect to the thickness of the damping material using the chain rule. The design sensitivity results were found to be in good agreement with the results of the central finite difference methods. The design optimization of the absorbing material layout was applied to a vibrating surface using the design sensitivity results under various excitations, and the results of the optimization showed a layout similar to the active intensity plot of the vibrating surface. The proposed method can easily be applied to reduce the noise produced by a vibrating structure.

Keywords: Acoustic impedance sensitivity; design optimization; wave based method; absorbing material layout.

1. Introduction

Noise reduction of vibrating structures is an essential requirement throughout the industry. Along with reinforcing materials, sound absorbing materials are also applied onto the vibrating surface for reducing noise. However, the excessive use of absorbing materials leads to an increase in the weight and a higher manufacturing cost. Moreover, complete treatment of the absorbing material does not help achieve a better damping when compared to a partial treatment [1]. Therefore, an optimum layout of sound-absorbing material on the vibrating surface is to be determined, which is the objective of this study.

Topology optimization (TO) is widely employed in the design of the absorbing material layout (AML) in an acoustic model. It is useful to create a conceptual design at the beginning of the design stage. Maria et al. [2] obtained the optimum AML along the walls of an acoustic cavity to reduce the sound pressure at field points. Lee et al. [3] proposed the design of poroelastic foam using Biot's theory [4], and Yoon [5] applied an empirical material formulation for optimizing the fibrous material layout in a cavity. TO adopts the finite element method (FEM) for designing the AML. It is classified into two approaches according to the mathematical model of topology description. The first approach is performed by applying the material distribution concept. It represents the element-wise or mesh-independent point-wise material distribution in a design domain, which allows for free refinement of

the analysis mesh in a density field [6]. The other approach is performed via geometry boundary description, such as the level set based TO. It uses a level set function defined on a set of grid points as an implicit description of the material distribution; hence, it is good at handling complex topological changes and describing the evolution of moving boundaries [7, 8]. Most TO problems are non-convex; therefore, a global optimum design cannot be guaranteed. However, TO derives a meaningful solution for guiding the optimum engineering design under the given constraints [9]; thereby making it applicable to various types of engineering problems. Another numerical approach for designing the absorbing material is by using the boundary element method (BEM) that considers the acoustic impedance at the boundaries as design variables. Zhang et al. [10] developed a formulation of the acoustic impedance sensitivity analysis based on the indirect BEM. Totaro et al. [11] applied the patch transfer function method to find the efficient positioning of absorbing material at the boundary of a cavity, and Besset et al. [12] adopted the simplified energy method for optimizing the acoustic absorption coefficients in the mid–high frequency range.

Absorbing materials are also used for reducing the noise that radiates from vibrating structures. Several numerical methods have been developed for predicting this noise. One of them is to define the acoustic impedance on the boundary of the radiating structure. Fundamental research on the impedance modeling of the absorbent panel on a moving boundary wall was established by Dowel et al. [13], and the formulation was applied to the vibro-acoustic model comprising complex boundary conditions by Suzuki et al. [14]. Yamamoto et al. [15] designed the optimum thickness of a multilayered structure by considering the vibro-acoustic effects using a one-dimensional transfer matrix. The TO of the damping layer using structural FEM and acoustic BEM was suggested by Xu [16] and Zhang [17] to reduce the radiated sound power and sound pressure at a field point, respectively.

The most common methods used for predicting the acoustic pressure from absorbent patches mounted on vibrating structures are the element based methods such as FEM and BEM. These methods are beneficial in model construction regardless of their geometrical complexity. However, their applications are limited to lower frequencies owing to the need for mesh refinement at higher frequencies, which further leads to a higher computational cost. To overcome this limitation, the wave based method (WBM), which is a numerical method used for predicting the vibro-acoustic response at mid–low frequencies, has been developed [18, 19]. The WBM offers the benefits of a higher convergence rate and accuracy even at a lower computational cost because its basis function is constructed to satisfy the governing differential equation, and a small number of partitioned subdomains are sufficient to describe the dynamic behavior within the concerned domain. Unlike the element based numerical approach, the WBM does not require mesh refinement at higher frequencies. It only requires multiple convex subdomains for its convergence, which are independent of the frequencies. As a result, the WBM yields a small number of subdomains and exhibits a higher

computational efficiency. Several publications have highlighted the efficiency of the WBM at higher frequencies for various applications [18, 19]. In this study, a vibrating hollow cube built of plates was modeled using the FEM, and the acoustic domain outside the structure was modeled using the WBM. Acoustic impedance is defined on the boundary to describe the dynamic behavior of the absorbent patch on the vibrating surface, and harmonic loads are applied on the surface as an excitation source. To determine the design sensitivity accurately, an analytic design sensitivity formulation of acoustic pressure was derived with respect to the acoustic boundary impedance from the system matrices of the WBM [20, 21]. Thereafter, the derivative of the normal impedance with respect to the thickness of each absorbing material patch was obtained. To this end, a polynomial function was used to describe the relation of the impedance value with respect to the thickness of the absorbing material based on the full Biot model [22]. Finally, the design sensitivity of the radiated sound pressure from the vibrating surface with respect to the thickness of the absorbent patch was applied to the optimization of the AML. The optimum AML was compared with the acoustic intensity plot of the vibrating surface, and it showed a similar configuration of active acoustic intensity on the surface.

2. Numerical model of the vibrating structure with absorbing material

This section presents the numerical model of a vibrating structure with an absorbing material. The structural FEM and acoustic WBM are introduced along with the detailed procedure of the acoustic impedance modeling on the vibrating structure that can be applied to the WBM.

2.1 Structural response

A linear structural dynamic system of the viscous damped model is defined as

$$\mathbf{M}\mathbf{z}_{,tt}(t) + \mathbf{C}\mathbf{z}_{,t}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{F}(t), \quad t > 0, \quad (1)$$

where $\mathbf{z}(t)$ is the displacement; $(\cdot)_{,t}$ denotes a derivative with respect to time; and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the structural mass, damping, and stiffness matrices, respectively. The damping loss factor of structures built of a metal beam or plate remains constant within a certain frequency range and mainly impacts its corresponding natural frequencies. Hence, the damping model can be described using linear hysteretic damping, which is known as structural damping. By applying the hysteretic damping ratio η to the equation of motion, the dynamic system equation can be expressed as

$$\mathbf{M}\mathbf{z}_{,tt}(t) + (1 + j\eta)\mathbf{K}\mathbf{z}(t) = \mathbf{F}(t). \quad (2)$$

When the harmonic force $\mathbf{F}(t) = \mathbf{f} e^{j\omega t}$ is applied to a structure, the steady-state response of the system can be described using the complex displacement $\mathbf{z}$:

$$[-\omega^2 \mathbf{M} + (1 + j\eta) \mathbf{K}] \mathbf{z}(\omega) = \mathbf{f}(\omega). \quad (3)$$

To obtain the boundary conditions of the vibro-acoustic system in a straightforward manner, the displacement vector expressed by Eq. (3) is changed into velocity using the relationship between displacement and velocity, $\mathbf{v} = j\omega \mathbf{z}$. The dynamic equation is represented as

$$(j\omega \mathbf{M} + \kappa \mathbf{K}) \mathbf{v}(\omega) = \mathbf{f}(\omega), \quad (4)$$

where $\kappa = (1 + j\eta) / j\omega$.

2.2 Acoustic WBM

Assuming that the mass load of the absorbing material on the vibrating plate is small compared with the base plate and the acoustic-induced vibrations are negligible, the vibro-acoustic behavior of an absorbent panel on the vibrating plate can be regarded as a one-way coupled WB model under an impedance boundary condition. For constructing the acoustic WB model of an unbounded problem, the entire domain is divided into bounded and unbounded domains by the truncation surface. Thereafter, the bounded domain is partitioned into several subdomains satisfying its geometrical convexity. Fig. 1 shows the domain partitioning for constructing the wave function of a vibrating cube. The wave functions are defined separately for each subdomain. In this section, the definition of the wave functions and wave model constructions are presented by considering the acoustic damping material on the vibrating panel.

Fig. 1. Domain partitioning of 3D exterior acoustic problem.

2.2.1. Three-dimensional bounded wave function

The governing differential equation for time-harmonic linear acoustics is presented as a Helmholtz equation:

$$\nabla^2 p(\mathbf{r}) + k^2 p(\mathbf{r}) = 0, \quad \mathbf{r} \in \Omega. \quad (5)$$

Here, p is the acoustic pressure, $k (= \omega / c)$ is the wave number, ω is the angular frequency, c is the speed of sound, $\mathbf{r}$ is the position of a field point in the acoustic domain Ω , and ∇^2 is the Laplace operator. The steady-state response of the field point within a subdomain is defined as a linear combination of the wave function and its contributing factors:

$$\begin{aligned} \hat{p}^{(\alpha)}(\mathbf{r}) &= \sum_{w=1}^{n_w^{(\alpha)}} p_w^{(\alpha)} \Phi_w^{(\alpha)}(\mathbf{r}) \\ &= \sum_{w_r=1}^{n_{w_r}^{(\alpha)}} p_{w_r}^{(\alpha)} \Phi_{w_r}^{(\alpha)}(\mathbf{r}) + \sum_{w_s=1}^{n_{w_s}^{(\alpha)}} p_{w_s}^{(\alpha)} \Phi_{w_s}^{(\alpha)}(\mathbf{r}) + \sum_{w_t=1}^{n_{w_t}^{(\alpha)}} p_{w_t}^{(\alpha)} \Phi_{w_t}^{(\alpha)}(\mathbf{r}) \\ &= \Phi^{(\alpha)}(\mathbf{r}) \mathbf{p}_w^{(\alpha)}. \end{aligned} \quad (6)$$

$\hat{p}^{(\alpha)}(\mathbf{r})$ is the predicted sound pressure at the α^{th} subdomain, $n_w^{(\alpha)} = n_{w_r}^{(\alpha)} + n_{w_s}^{(\alpha)} + n_{w_t}^{(\alpha)}$ is the total number of wave functions, and $p_w^{(\alpha)}$ is the contribution factor for each wave function $\Phi_w^{(\alpha)}(\mathbf{r})$. The row vector $\Phi^{(\alpha)}(\mathbf{r})$ of size $1 \times n_w^{(\alpha)}$ contains all the wave functions, and the column vector $\mathbf{p}_w^{(\alpha)}$ of size $n_w^{(\alpha)} \times 1$ is the contribution factor set. Three wave function sets are defined for the bounded subdomains [23] as

$$\Phi_w^{(\alpha)}(\mathbf{r}(x, y, z)) = \begin{cases} \Phi_{w_r}^{(\alpha)}(x, y, z) = \cos(k_{xw_r}^{(\alpha)} x) \cos(k_{yw_r}^{(\alpha)} y) e^{-jk_{zw_r}^{(\alpha)} z} \\ \Phi_{w_s}^{(\alpha)}(x, y, z) = \cos(k_{xw_s}^{(\alpha)} x) e^{-jk_{yw_s}^{(\alpha)} y} \cos(k_{zw_s}^{(\alpha)} z) \\ \Phi_{w_t}^{(\alpha)}(x, y, z) = e^{-jk_{xw_t}^{(\alpha)} x} \cos(k_{yw_t}^{(\alpha)} y) \cos(k_{zw_t}^{(\alpha)} z), \end{cases} \quad (7)$$

and the wave numbers of each wave function set are:

$$(k_{xw_r}^{(\alpha)}, k_{yw_r}^{(\alpha)}, k_{zw_r}^{(\alpha)}) = \left(\frac{w_1^{(\alpha)} \pi}{L_x^{(\alpha)}}, \frac{w_2^{(\alpha)} \pi}{L_y^{(\alpha)}}, \pm \sqrt{k^2 - (k_{xw_r}^{(\alpha)})^2 - (k_{yw_r}^{(\alpha)})^2} \right), \quad (8)$$

$$(k_{xw_s}^{(\alpha)}, k_{yw_s}^{(\alpha)}, k_{zw_s}^{(\alpha)}) = \left(\frac{w_3^{(\alpha)} \pi}{L_x^{(\alpha)}}, \pm \sqrt{k^2 - (k_{xw_s}^{(\alpha)})^2 - (k_{zw_s}^{(\alpha)})^2}, \frac{w_4^{(\alpha)} \pi}{L_z^{(\alpha)}} \right), \quad (9)$$

$$(k_{xw_t}^{(\alpha)}, k_{yw_t}^{(\alpha)}, k_{zw_t}^{(\alpha)}) = \left(\pm \sqrt{k^2 - (k_{yw_t}^{(\alpha)})^2 - (k_{zw_t}^{(\alpha)})^2}, \frac{w_5^{(\alpha)} \pi}{L_y^{(\alpha)}}, \frac{w_6^{(\alpha)} \pi}{L_z^{(\alpha)}} \right), \quad (10)$$

where $w_i^{(\alpha)} = 0, 1, 2, \dots; i = 1, L, 6; L_x^{(\alpha)}, L_y^{(\alpha)}$, and $L_z^{(\alpha)}$ represent the dimensions of the smallest rectangular box enclosing the considered subdomain.

2.2.2. Three-dimensional unbounded wave function

The wave functions for the unbounded domain are defined from the solutions of the Helmholtz equation in spherical coordinates. The sound propagation toward the unbounded domain can be approximated as [24]

$$\hat{p}(r, \theta, \phi) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l p_{lm} h_l^{(2)}(kr) P_l^{lm}(\cos \theta) e^{jm\phi} = \Phi(\mathbf{r}) \mathbf{p}_w, \quad (11)$$

where $l_{\max}$ is the truncation factor for the sound pressure expansion, $h_l^{(2)}$ is the second kind of spherical Hankel function, P_l^{lm} is the normalized associated Legendre function, and p_{lm} is the contribution factor of the functions. From Eq. (11), the wave function set for the 3D unbounded domain is constituted as

$$\Phi = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l h_l^{(2)}(kr) P_l^{lm}(\cos \theta) e^{jm\phi}. \quad (12)$$

The wave functions are basis functions that span the sound propagation within each subdomain. To complete the model construction for the WBM, the contribution factor must be determined to satisfy the boundary and continuity conditions for the boundaries and interfaces, respectively.

2.2.3. Model construction of boundary conditions

When the absorbing material is mounted on a vibrating structure, the normal structural velocity is different from the normal fluid velocity on the absorbing material. To describe these assembled boundary conditions, the relative velocity between the two sides of the absorbing panel is considered along with the normal impedance of the panel [14]. Fig. 2 shows an example of the boundary condition of an absorbent panel on a vibrating structure. The structural vibration energy is transmitted into the acoustic domain through the impedance of the absorbent panel. The dynamic behavior can be simplified using a mathematical model such as that depicted in Fig. 2(b), and the sound pressure on

the surface of the fluid domain is solely defined by the acoustic impedance of the absorbent panel and its relative velocity. Here, the inertial effect of the absorbing material is ignored by assuming that the exciting frequency on the panel is much lower than its natural frequency [13].

Fig. 2. Boundary conditions of the absorbent panel on the vibrating structure: (a) example of velocity boundary conditions on absorbent panel, (b) lumped mathematical model, where k_a and c_a are the stiffness and damping of the absorbent panel, respectively [13].

The definition of the relative velocity and sound pressure at the acoustic domain is:

$$v_r = v_a - v_s, \quad (13)$$

$$p_a = Z_n v_r, \quad (14)$$

where v_r is the relative velocity, v_s and v_a are the structural and fluid velocities, respectively, and Z_n is the normal impedance of the absorbing material. By substituting Eq. (13) into Eq. (14) and converting the fluid velocity into acoustic pressure, Eq. (14) can be expressed as

$$\frac{\hat{p}^{(\alpha)}(\mathbf{r}_b)}{Z_n(\mathbf{r}_b)} = L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r}_b)) - v_n(\mathbf{r}_b), \quad \mathbf{r}_b \in \Gamma_{v,Z}^{(\alpha)}, \quad (15)$$

where $\mathbf{r}_b$ is the position vector at the boundary $\Gamma_{v,Z}$, $L_v^{(\alpha)}(\mathfrak{g}) = (j/\rho_0\omega)(\partial(\mathfrak{g})/\partial n^{(\alpha)})$ is the fluid velocity operator, and v_n is the normal component of v_s . For constructing the wave based system matrices, the method of weighted residuals such as the Galerkin method, is applied to every boundary of the acoustic model. The residual error functions for the velocity and impedance boundary conditions are defined as

$$\mathbf{r}_b \in \Gamma_v^{(\alpha)} : R_v^{(\alpha)}(\mathbf{r}_b) = L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r}_b)) - v_n(\mathbf{r}_b), \quad (16)$$

$$\mathbf{r}_b \in \Gamma_Z^{(\alpha)} : R_Z^{(\alpha)}(\mathbf{r}_b) = L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r}_b)) - \frac{\hat{p}^{(\alpha)}(\mathbf{r}_b)}{Z_n(\mathbf{r}_b)}. \quad (17)$$

By moving the term on the left-hand side of Eq. (15) to the right-hand side, the residual error functions of the absorbent panel on the vibrating plate can be defined as

$$\mathbf{r}_b \in \Gamma_{v,Z}^{(\alpha)} : R_{v,Z}^{(\alpha)}(\mathbf{r}_b) = (L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r}_b)) - v_n(\mathbf{r}_b)) - \frac{\hat{p}^{(\alpha)}(\mathbf{r}_b)}{Z_n(\mathbf{r}_b)}. \quad (18)$$

Eq. (18) is a combined form of the error functions for the velocity and impedance boundary conditions. For a large normal impedance value, the error function is determined by considering the velocity boundary condition, whereas for a small impedance value, the error function is determined by considering the impedance boundary condition. As such, the model of absorbent panel on the vibrating plate can be constructed.

2.2.4 Model construction of interface and system equation

For formulating the WBM, a continuity condition is required between adjacent subdomains. To obtain a more stable numerical model, the impedance continuity is applied for the coupling at the interface [19]. The equivalent velocity operators L_{eq+} and L_{eq-} are applied for defining the impedance coupling as

$$L_{eq+}^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r})) = L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r})) - \frac{1}{Z_{imp}} \hat{p}^{(\alpha)}(\mathbf{r}), \quad (19)$$

$$L_{eq-}^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r})) = -L_v^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r})) - \frac{1}{Z_{imp}} \hat{p}^{(\alpha)}(\mathbf{r}). \quad (20)$$

Here, Z_{imp} is a scaling factor that imposes a relative weighting on the linear combined pressure and velocity continuities and acts as an artificial damping at the interface. From Eqs. (19) and (20), the residual error function at the interface is defined as

$$\mathbf{r} \in \Gamma_I^{(\alpha,\beta)} : R_I^{(\alpha,\beta)} = L_{eq+}^{(\alpha)}(\hat{p}^{(\alpha)}(\mathbf{r})) - L_{eq-}^{(\beta)}(\hat{p}^{(\beta)}(\mathbf{r})). \quad (21)$$

By applying the Galerkin weighted formulation with algebraic manipulations of all the error functions defined on the boundaries and the interfaces, the wave based system equation is formulated as

$$\mathbf{A}\mathbf{p}_w = \mathbf{b}, \quad (22)$$

where $\mathbf{A}$ is the system matrix of the wave model, $\mathbf{p}_w$ is the set of contribution factors, and $\mathbf{b}$ is the force vector composed of the wave functions and input velocity on Γ_v . The required sound pressure at the field point can be obtained through a post-process using the wave functions and contribution factors as defined in Eqs. (6) and (11).

3. Design sensitivity analysis

Design sensitivity analysis is an essential part of the gradient-based optimization. To compute the design sensitivity of radiating sound from an absorbent panel with respect to the thickness, two steps of design derivatives are required. The first is the derivative of the acoustic impedance sensitivity, and the second is the derivative of the impedance with respect to the thickness of the absorbent panel.

3.1 Acoustic impedance sensitivity

The impedance design sensitivity is derived by differentiating both sides of Eq. (6) with respect to the normal impedance as

$$\frac{d\hat{p}^{(\alpha)}(\mathbf{r})}{dZ_n} = \frac{\partial\Phi^{(\alpha)}(\mathbf{r})}{\partial Z_n} \mathbf{p}_w^{(\alpha)} + \Phi^{(\alpha)}(\mathbf{r}) \frac{\partial\mathbf{p}_w^{(\alpha)}}{\partial Z_n}. \quad (23)$$

Here, the wave function set Φ is defined by the wave number and the dimensions of its corresponding subdomain; hence, the derivative of the wave function with respect to the impedance vanishes. For evaluating $\partial\mathbf{p}_w^{(\alpha)} / \partial Z_n$, the acoustic WB system equation defined in Eq. (20) is differentiated:

$$\frac{\partial\mathbf{A}}{\partial Z_n} \mathbf{p}_w + \mathbf{A} \frac{\partial\mathbf{p}_w}{\partial Z_n} = \frac{\partial\mathbf{b}}{\partial Z_n}. \quad (24)$$

The force vector $\mathbf{b}$ comprises the velocity boundary conditions on the surface, and is independent of the impedance similar to the wave function. Therefore, the differentiation of the contribution vector with respect to the impedance can be expressed as

$$\frac{\partial \mathbf{p}_w}{\partial Z_n} = -\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial Z_n} \mathbf{p}_w. \quad (25)$$

The system matrix $\mathbf{A}$ is constructed using the weighted residual formulation from all boundary and continuity conditions of the acoustic model. Here, the components of $\mathbf{A}$ that are associated with the impedance boundary condition are only used for formulating the impedance sensitivity. The system matrix derived from the impedance boundary conditions are computed from Eq. (18) as

$$\mathbf{A}_z^{(\alpha)} = \frac{j}{\rho\omega} \Phi^{(\alpha)}(\mathbf{r}_b)^T \frac{\partial}{\partial n} (\Phi^{(\alpha)}(\mathbf{r}_b)) - \frac{1}{Z_n} \Phi^{(\alpha)}(\mathbf{r}_b)^T \Phi^{(\alpha)}(\mathbf{r}_b). \quad (26)$$

As shown in Eq. (26), $\mathbf{A}_z^{(\alpha)}$ is an explicit function of impedance; hence, the impedance sensitivity is derived using the following simple derivative formulation:

$$\frac{\partial \mathbf{A}_z^{(\alpha)}}{\partial Z_n} = \frac{1}{Z_n^2} \Phi^{(\alpha)}(\mathbf{r}_b)^T \Phi^{(\alpha)}(\mathbf{r}_b). \quad (27)$$

By substituting Eqs. (25) and (27) into Eq. (23), the impedance sensitivity can be formulated as

$$\frac{d \hat{p}^{(\alpha)}(\mathbf{r})}{d Z_n} = -\frac{1}{Z_n^2} \Phi^{(\alpha)}(\mathbf{r}) \mathbf{A}^{-1(\alpha, \alpha)} \left(\Phi^{(\alpha)}(\mathbf{r}_b)^T \Phi^{(\alpha)}(\mathbf{r}_b) \right) \mathbf{p}_w^{(\alpha)}, \quad (28)$$

where $\mathbf{A}^{-1(\alpha, \alpha)}$ of size $n_w^{(\alpha)} \times n_w^{(\alpha)}$ is the submatrix of $\mathbf{A}^{-1}$ corresponding to the α^{th} subdomain.

3.2 Thickness sensitivity of the absorbing material

Once the impedance sensitivity is obtained, the acoustic pressure sensitivity with respect to the thickness of the absorbent panel can be derived using the chain rule:

$$\frac{d|p|}{d\mathbf{u}} = \frac{\partial|p|}{\partial Z_n} \frac{dZ_n}{d\mathbf{u}} = \frac{\partial|p|}{\partial R_n} \frac{dR_n}{d\mathbf{u}} + \frac{\partial|p|}{\partial X_n} \frac{dX_n}{d\mathbf{u}}, \quad (29)$$

where $|p| = \sqrt{p_r^2 + p_i^2}$ is the amplitude of the sound pressure, $\mathbf{u}$ is the thickness of the absorbing material, and R_n and X_n are the real and imaginary parts of the impedance $Z_n = R_n + jX_n$,

respectively. The normal impedance of the absorbing material is a function of the excitation frequency and its thickness. For the curve fitting of the empirical impedance model with respect to the thickness, a sixth-order polynomial function is employed. The polynomial function has a simple formulation and is useful for describing a monotonically increasing or decreasing curve with a small fluctuation over a wide frequency range.

Fig. 3. Impedance curves of polyurethane: (a) with a layer thickness of 50 mm with respect to the frequency, (b) at 400 Hz with respect to the thickness, and (c) at 500 Hz with respect to the thickness.

Fig. 3 shows the impedance curve of polyurethane defined using Biot's theory. Fig. 3(a) depicts the impedance curve of a 50 mm layer of polyurethane with respect to the frequency, and Figs. 3(b) and 3(c) depict the impedance curves of polyurethane with respect to the thickness at 400 and 500 Hz, respectively. The blue and red solid lines are the real and imaginary values of the empirical impedance, and the curve fittings are represented using dashed lines. The polynomial function for the curve fitting at 400 Hz is defined as

$$R_n = 16.607 \times 10^{-7} t^6 - 35.261 \times 10^{-5} t^5 + 29.94 \times 10^{-3} t^4 - 1.301 t^3 + 30.755 t^2 - 387.38 t + 2417, \quad (30)$$

$$X_n = -12.66 \times 10^{-6} t^6 + 27.82 \times 10^{-4} t^5 - 0.2456 t^4 + 11.147 t^3 - 276.53 t^2 + 3679.6 t - 23777, \quad (31)$$

where t is the thickness of the absorbent panel. The approximated curves using the polynomial functions are in good agreement with the empirical impedance curves of polyurethane. Through

polynomial fitting, the acoustic pressure sensitivity with respect to the thickness of an absorbent panel is easily implemented, and the intermediate design obtained during the optimization still retains the physical characteristics of polyurethane, thereby resulting in an accurate model description with a simple formulation.

4. Numerical examples

4.1 Symmetric excitation

Fig. 4. Model information: (a) geometry and boundary conditions, (b) positions of harmonic excitation on the flexible plate at $y = -0.2$ m.

Fig. 4 shows the numerical model of a hollow cube built of plates. The dimensions of the model are $L_x = L_y = L_z = 0.4$ m and its origin $(0, 0, 0)$ is at the center of the cavity. Fixed boundary conditions are imposed at the walls of the model except on the flexible panel at $y = -0.2$ m, where the harmonic point loads are excited symmetrically as depicted in Fig. 4(b). The FEM is used for structural dynamic analysis, and the acoustic WBM is used for predicting the radiating noise from the structure. Each panel is meshed by 100 shell elements with the following properties: thickness of the steel plate = 2 mm, Young's modulus = 200 GPa, Poisson's ratio = 0.33, mass density = 7850 kg/m^3 , and structural damping ratio = 0.01. The acoustic medium is air with a density of 1.225 kg/m^3 , which can transmit sound at a speed of 340 m/s.

Fig. 5 shows the structural-acoustic response of the cube model. One-way coupling is employed in this problem by assuming that the acoustic pressure does not impact the structural response. The frequency response at the field point is depicted in Fig. 5(a), where a sine sweep of 1 Hz resolution from 50 to 500 Hz is applied to the panel as the input excitation. The sound propagation plot at 400 Hz, which corresponds to one of the local peaks of the frequency response, is depicted in Fig. 5(b), where the red dashed lines indicate the interfaces between the neighboring subdomains of the WB

model. The results indicate that the acoustic wave propagates symmetrically over all the response points and is continuous along the interfaces.

Fig. 5. Structural-acoustic response: (a) frequency response at the field point, (b) acoustic wave propagation at 400 Hz.

4.2 Asymmetric excitation

Fig. 6. Model information: (a) geometry and boundary conditions, (b) positions of harmonic excitation on the flexible plate at $y = -0.2$ m.

Fig. 6 shows another numerical model with the same geometry as the previous one. However, in this model, the excitation point shifted to an arbitrary point on the vibrating plate to consider more practical excitation cases. Fig. 7 depicts the structural–acoustic response of the model shown in Fig. 6. Owing to the difference in the excitation, the sound propagation from the vibrating panel and the frequency response function at the field point are different from those in the symmetric excitation case. In this study, the design sensitivity analysis and optimization of the absorbent panel were applied to

the two abovementioned excitation cases and were validated through numerical examples.

Fig. 7. Structural-acoustic response: (a) frequency response at the field point, (b) acoustic wave propagation at 400 Hz.

4.3 Design sensitivity analysis

Table 1 Comparison of design sensitivity results obtained using analytic method and the finite difference method (FDM) at 400 Hz.

DV	$\Psi(\mathbf{u})$ (Pa)	$\Psi(\mathbf{u} + \Delta\mathbf{u})$ (Pa)	$\Psi(\mathbf{u} - \Delta\mathbf{u})$ (Pa)	FDM (Pa/mm)	Analytic (Pa/mm)	Analytic/FDM (%)
1		7.542349	7.542355	-2.539E-05	-2.550E-05	100.4
2		7.542345	7.542360	-7.514E-05	-7.550E-05	100.5
3	7.542353	7.542341	7.542364	-1.167E-04	-1.171E-04	100.3
23		7.542268	7.542436	-8.367E-04	-8.365E-04	100.0
45		7.542408	7.542298	5.475E-03	5.470E-04	99.9

Table 1 lists the design sensitivity results of the numerical model under the symmetric excitation at 400 Hz. The objective function Ψ is the amplitude of the sound pressure level at the field point, and the design variable u is the thickness of the polyurethane layer on the vibrating shell element. A 5 mm thickness of the polyurethane layer is considered as the initial design variable $u_0 = 5$ because the polynomial fitting of the impedance curve was defined within the range of at least 5 mm and at most 50 mm. To validate the design sensitivity, the finite difference method (FDM) is employed as a reference value. The central FDM with a perturbation of $\Delta u = 0.1$ mm is used for computing the sensitivity. Five design variables are selected for validating the design sensitivity including those with the highest and lowest sensitivity. The analytic sensitivities of the design variables are in good

agreement with those obtained using the FDM. The analytic sensitivity formulation requires only matrix multiplication of the predefined system matrix to obtain the design sensitivity whereas the FDM requires iterative numerical solutions of the structural and acoustic responses of the perturbed design. Hence, the analytic sensitivity formulation has the advantage of computational accuracy and efficiency compared to the FDM.

Fig. 8. Design sensitivity results with symmetric excitation at 400 Hz: (a) design sensitivity plot, (b) design sensitivity graph.

Fig. 8 represents the colored map of the design sensitivity with its graph under the symmetric excitation at 400 Hz. Design variable number 23 shows the lowest sensitivity, which implies that the sound pressure level is reduced the most when the absorbing material is attached at this location. On the contrary, the design variable number 45 shows a positive sensitivity. A positive sensitivity value implies that applying the material at such a location will have a negative effect and will lead to an increase in pressure. Based on the sensitivity results, the additional absorbing material should be applied on the whole panel except at the red-colored locations to reduce the sound pressure level.

Table 2 Comparison of design sensitivity results obtained using the analytic method and FDM at 500 Hz.

DV	$\Psi(\mathbf{u})$ (Pa)	$\Psi(\mathbf{u}+\Delta\mathbf{u})$ (Pa)	$\Psi(\mathbf{u}-\Delta\mathbf{u})$ (Pa)	FDM (Pa/mm)	Analytic (Pa/mm)	Analytic/FDM (%)
1		0.5272963	0.5272964	-5.47E-07	-5.21E-07	95.3
2		0.5272964	0.5272964	4.42E-08	4.96E-08	112.1
3	0.5272964	0.5272960	0.5272968	-4.18E-06	-4.12E-06	98.6
23		0.5272914	0.5273013	-5.00E-05	-5.00E-05	100.1
45		0.5272956	0.5272972	-8.04E-06	-7.98E-06	99.2

Fig. 9. Design sensitivity results with symmetric excitation at 500 Hz: (a) design sensitivity plot, (b) design sensitivity graph.

Table 2 lists the results of the design sensitivity analysis of the symmetrically excited model at 500 Hz. The sound pressure level and design sensitivities are significantly reduced compared to the results obtained at 400 Hz. Fig. 9 shows the color map of the design sensitivity and its graph. As most design variables represent negative sensitivity, applying the absorbing material throughout the panel is effective in reducing the sound pressure. However, design variable 2 has a small positive value close to zero. This means that the objective function is not impacted by the absorbing material attached to that location. The sensitivities of variables 1 and 2 obtained using the analytic method and the FDM show some differences. These are caused by the numerical error in the FDM owing to the low sensitivity of the design variables.

Fig. 10. Design sensitivity results with asymmetric excitation at 400 Hz: (a) design sensitivity plot, (b) design sensitivity graph.

Figs. 10 and 11 show the design sensitivity results obtained with asymmetric excitation on the flexible panel. Unlike the symmetric excitation case, the positive and negative sensitivities are distributed uniformly over the entire plate. As described in the sensitivity results with symmetric excitation at 400 Hz, the application of absorbing material at the location with positive design sensitivity increases the sound pressure, which further indicates that addition of absorbing material at such a location is not appropriate for reducing the sound pressure. It can be expected that the optimal absorbing material layouts in the asymmetric excitation case are completely different from those in the symmetric excitation case.

Fig. 11. Design sensitivity results with asymmetric excitation at 500 Hz: (a) design sensitivity plot, (b) design sensitivity graph.

4.4 Design Optimization

$$\min_{\mathbf{u}} \psi = |p(\mathbf{r}, \mathbf{u}(\mathbf{r}_b))| \quad \text{Objective function,} \quad (32)$$

Subject to

$$\frac{\int_{\Gamma_{v,Z}} \mathbf{u}(\mathbf{r}_b) d\Gamma_{v,Z}}{\int_{\Gamma_{v,Z}} \mathbf{u}_0(\mathbf{r}_b) d\Gamma_{v,Z}} - \beta \leq 0 \quad \text{Volume constraint,} \quad (33)$$

$$5 \leq \mathbf{u}(\mathbf{r}_b) \leq 50 \quad \text{Side constraint.} \quad (34)$$

In this section, the results of the design sensitivity analysis described in Section 4.3 are applied to the optimization for identifying the optimum sound absorption arrangement for noise reduction. The numerical formulation of the objective function and the design constraint in the optimization are defined in Eqs. (32)–(34). The constraint for the total volume of the absorbing material and the side

constraint for each design variable are defined in Eqs. (33) and (34), respectively. The control parameter β in Eq. (33) determines the amount of absorbing material used in the optimization process, $\beta = 10$ indicates the unrestricted use of the absorbing material within the side constraint. The optimization problem is solved using sequential linear programming in the design optimization tool (DOT).

Fig. 12. Iteration history with symmetric excitation at 400 Hz for (a) $\beta = 6$, (b) $\beta = 10$.

Fig. 13. Optimum layout with symmetric excitation at 400 Hz for (a) $\beta = 6$, (b) $\beta = 10$.

The iteration histories of the objective function and volume constraint with symmetric excitation at 400 Hz are depicted in Fig. 12. Here, the optimization history obtained using the control parameters $\beta = 6$ and $\beta = 10$ were compared with each other. The optimization process converges in a few iterations owing to the use of the analytic design sensitivity. Fig. 13 shows the optimization results of the symmetric excitation model at 400 Hz. The red box indicates the 50 mm-thick patch and the dark blue box indicates an initial thickness of 5 mm. When $\beta = 6$, the absorbing material is located as shown in Fig. 13(a), and its sound pressure is 6.04 Pa. Fig. 13 (b) shows the optimization results when $\beta = 10$. In this case, the absorbent patch covers the entire plate except the central region, and the

sound pressure level is reduced to 5.90 Pa. The optimum layout is similar to the array of design variables with negative sensitivity as shown in Fig. 8(a). For verifying the optimization results, the sound pressure level from the vibrating panel entirely covered by a 50 mm layer of polyurethane was evaluated. This level was 6.02 Pa, which is higher than that of the optimum model when $\beta = 10$ and similar to the case when $\beta = 6$. From these results, it can be expected that optimum design can lead to effective noise reduction with cost savings on the absorbent material.

Fig. 14. Iteration history with symmetric excitation at 500 Hz for (a) $\beta = 6$, (b) $\beta = 10$.

Fig. 15. Optimum layout with symmetric excitation at 500 Hz for (a) $\beta = 6$, (b) $\beta = 10$.

Fig. 14 (a) and (b) show the iteration histories with symmetric excitation at 500 Hz for $\beta = 6$ and $\beta = 10$, respectively. It can be observed that both cases converge to each of the optimum designs with a few iterations. Fig. 15 shows the optimum layout of the absorbing material under symmetric excitation at 500 Hz. When β is confined to 6, the absorbing material is attached to the elements with higher magnitudes of negative sensitivity, as shown in Fig. 9. The side and center of the plate are not covered by the absorbing material because of their low magnitude of design sensitivities. When β is increased to 10, the entire panel is covered by the absorbing material except for the design variable of positive sensitivity listed in Table 2. The sound pressure levels of the optimum layout using $\beta = 6$

and $\beta = 10$ are 0.39 Pa and 0.38 Pa, respectively. This implies that the objective function maintains a constant value regardless of the increase in the control parameters. When a 50 mm thick layer of polyurethane is applied on the entire vibrating plate, the sound pressure obtained is the same as that obtained with the optimization result using $\beta = 10$. As mentioned above, the objective function is not affected by the existence of the absorbing material on the elements having low design sensitivity. Thus, there is no significant change in the sound pressure level after applying the absorbing material on the elements.

Fig. 16. Iteration history with asymmetric excitation using $\beta = 10$ at (a) 400 Hz, (b) 500 Hz.

Fig. 17. Optimum layout with asymmetric excitation using $\beta = 10$ at (a) 400 Hz, (b) 500 Hz.

Fig. 16 (a) and (b) show the iteration histories of optimization for asymmetric excitation at 400 Hz and 500 Hz, respectively. Both optimized designs are obtained in 3 iterations, and these show good convergence histories. Fig. 17 shows the optimum layouts of the absorbing material under asymmetric excitation. These layouts are more complicated than those obtained with symmetric excitation. The sound pressure levels of the initial design are 0.44 Pa and 0.10 Pa at 400 Hz and 500 Hz, respectively. After optimizing with $\beta = 10$, the sound pressure levels at 400 and 500 Hz are reduced to 0.29 and 0.02 Pa, respectively. The absorbing material covers only the locations at which the sensitivity is

negative, as shown in Figs. 10 and 11. When the entire panel is covered by a 50 mm layer of the absorbing material, the sound pressure levels at 400 Hz and 500 Hz are 0.35 Pa and 0.07 Pa, respectively. The results are worse than those of the optimum design. Thus, it can be verified that the absorbing material is optimally distributed over the vibrating panel and a large amount of absorbing material does not result in a higher reduction of the sound pressure in the case of a vibrating structure.

4.5 Relationship with sound intensity

The sound intensity can be characterized as active and reactive parts, depending on the phase relationship between the sound pressure and velocity. The active intensity in the frequency band is defined as [25]

$$I_r = \frac{1}{2} \text{Re}\{pu_r^*\}, \quad (35)$$

where p is the sound pressure and u_r is the particle velocity.

Fig. 18 shows the active intensity plots of the symmetric excitation case at 400 Hz and 500 Hz. The intensity over the vibrating plate shows a similar color plot as the optimum layout obtained using $\beta = 6$ in Fig. 13 and 15. The absorbing material is applied in the region of higher active intensity.

Fig. 19 shows the active intensity plots when the plate is excited by the asymmetric point load at 400 Hz and 500 Hz, respectively. The intensity plots at 400 Hz and 500 Hz are exactly the same as those of the optimal layout shown in Fig. 17, where the absorbing material covers the locations of positive active intensity. From these results, it can be seen that the optimum AML on the vibrating plate is highly dependent on the active intensity plot, and the optimum layout is a reasonable result in terms of sound intensity for acoustic wave propagation.

Fig. 18. Active intensity plots of symmetric excitation case at (a) 400 Hz, (b) 500 Hz.

Fig. 19. Active intensity plots of asymmetric excitation case at (a) 400 Hz, (b) 500 Hz.

5. Conclusions

In this study, design optimization for the AML on a vibrating structure was investigated. The structural FEM and acoustic WBM were used to model the radiating noise from a vibrating structure. The residual error functions of the velocity and impedance boundary conditions were derived using the relative velocity of the absorbing material, and the acoustic impedance design sensitivity for the absorbing material was developed using the wave based acoustic model. Using the relation between the acoustic normal impedance and the thickness of the absorbing material, any intermediate design in the optimization process can easily be implemented, and its manufacturing feasibility can also be satisfied. The results show that the optimum layout of the absorbing material is related to the active intensity plot of the vibrating plate. Through the optimum layout approach suggested in this work for applying the absorbing material on a vibrating surface, noise reduction in the mid-low frequency range can be efficiently achieved at a lower cost with better performance.

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- Design sensitivity of acoustic impedance was derived using wave based method.
- The optimization of absorbent panel layout on a vibrating structure was conducted.
- The performance of the optimization was verified through numerical examples.
- The noise reduction can be achieved at a lower cost with better performance.

Declaration of interests

☐ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.