Multistable inflatable origami structures at the meter-scale

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From stadium covers to solar sails, we rely on deployability for the design of large-scale structures that can quickly compress to a fraction of their size (1–4). Historically, two main strategies have been pursued to design deployable systems. The first and most common approach involves mechanisms comprising interconnected bar elements, which can synchronously expand and retract (5–7), occasionally locking in place through bistable elements (8, 9). The second strategy instead, makes use of inflatable membranes that morph into target shapes by means of a single pressure input (10–12). Neither strategy however, can be readily used to provide an enclosed domain able to lock in place after deployment: the integration of protective covering in linkage-based constructions is challenging and pneumatic systems require a constant applied pressure to keep their expanded shape (13–15). Here, we draw inspiration from origami, the Japanese art of paper folding, to design rigid-walled deployable structures that are multistable and inflatable. Guided by geometric analyses and experiments, we create a library of bistable origami shapes that can be deployed through a single fluidic pressure input. We then combine these units to build functional structures at the meter-scale, such as arches and emergency shelters, providing a direct pathway for a new generation of large-scale inflatable systems that lock in place after deployment and offer a robust enclosure through their stiff faces.

Origami | Multistability | Inflatable structures | Deployable structures

Large, deployable structures should ideally (i) occupy the minimum possible volume when folded; (ii) be autonomous when deploying; (iii) lock in place after deployment; and (iv) provide a structurally robust shell (if they are designed to define a closed environment). To satisfy all these requirements, we here present a novel approach with roots in the Japanese art of paper folding: origami. Extensively used in robotics (16–20), metamaterials (21–25) and structures (26–30), origami principles have potential to lead to efficient large-scale deployable structures as they offer (i) a versatile crease-based approach to shape design (31–33); (ii) an easy actuation through inflation, if enclosed (34–36); (iii) self-locking capabilities when designed to support multiple energy wells (37–44); and (iv) the possibility to create a protective environment through their faces. While previous origami systems have explored inflatability and multistability separately (34–44), here we show that these two properties can coexist, unlocking an unprecedented design space of meter-scale inflatable structures that harness multistability to maintain their deployed shape without the need for continuous actuation (see schematics in Fig. 1a)

Triangular facets as a platform for bistable and inflatable structures. To create inflatable and bistable origami structures, we start by considering a triangular building block ABC and denote with α and β the internal angles enclosed by the edges AB - AC and AB - BC , respectively (see Fig. 1b). The trian-

gle initially lies in the xy -plane and is subsequently deployed through a rotation around its edge BC . As shown in Fig. 1b, this deployment results in the displacement w_A of vertex A along the z -direction as well as in a volume V_{ABC} under the triangle

$$V_{ABC} = \frac{w_A \|AB\|^2}{6} \frac{\sin \alpha}{\sin(\alpha + \beta)} \sqrt{\sin^2 \beta - \frac{w_A^2}{\|AB\|^2}}, \quad [1]$$

where $\|AB\|$ indicates the length of AB . By focusing on the xy -plane, through simple geometrical considerations, one can see that if $\beta \in [\pi/4 - \alpha/2, \pi/2 - \alpha]$, the projection of vertex A during the deployment intersects the circle circumscribed to the initial configuration (see Fig. 1b) when

$$w_A = w_A^c = \|AB\| \sqrt{1 - \frac{\cos^2 \beta}{\sin^2(\alpha + \beta)}}. \quad [2]$$

It follows from the inscribed angle theorem (45) that for $w_A = w_A^c$ the angle α is recovered on the xy -plane (see the Supplementary Materials, Section S1 for details). As such, if triangles of this type are used as building blocks to form origami polyhedra, the assembled systems will have two distinct compatible configurations: one flat (identified by $w_A = 0$) and one expanded (identified by $w_A = w_A^c$). By contrast, any configuration with $0 < w_A < w_A^c$ will be geometrically frustrated, with incompatibility, Δ_{ABC} , that can be estimated as

$$\Delta_{ABC} = \|AC_{xy}\| \cdot \sin(\alpha_{xy} - \alpha), \quad [3]$$

where AC_{xy} and α_{xy} are the projection on the xy -plane of edge AC and angle α , respectively (note that $\alpha_{xy} = \alpha$ only for $w_A = 0$ and w_A^c — see the inset in Fig. 1c).

Therefore, to accommodate geometric frustration and realize closed origami shapes (i.e. shapes forming a closed inflatable cavity) capable of switching between two compatible configurations, we connect stiff triangular building blocks to stretchable hinges. Importantly, whereas polyhedra comprised of rigid triangular faces connected by perfect rotational hinges are known to be either rigid (46) or volume-invariant during deployment (47, 48), we anticipate our closed origami with stiff facets and flexible hinges to be bistable. Indeed, for hinges with low enough bending stiffness, we expect the energy profile of the closed origami to exhibit two local minima in correspondence of the flat and expanded compatible states (where the energy in the system can only be attributed to hinge bending), separated by an energy barrier caused by the deformation of

Fig. 1. Triangular facets as building blocks for large-scale inflatable and bistable origami structures. (a). Schematics illustrating the deployment via inflation of a large-scale origami structure comprising triangular facets. (b). Deployment of two triangular building blocks with angles $(\alpha^{(1)}, \beta^{(1)})$ and $(\alpha^{(2)}, \beta^{(2)})$. (c). Projected view of the deployment. (d). Evolution of incompatibility, Δ_{ABC} , and underlying volume, V_{ABC} , as a function of the deployment height, w_A . (e). Evolution of incompatibility, Δ_{ABC} , as a function of the underlying volume, V_{ABC} . (f-h). Contour maps of the compatible deployment height, w_A^c , maximum incompatibility, Δ_{ABC}^{max} , and inflation constraint, h_{ABC} .

the facets and the hinges required to accommodate geometric incompatibility.

To gain more insights into the behavior of our building blocks, we focus on the deployment of two triangles with $(\alpha^{(1)}, \beta^{(1)}) = (30^\circ, 50^\circ)$ and $(\alpha^{(2)}, \beta^{(2)}) = (30^\circ, 33^\circ)$. In Fig. 1d, we report the evolution of the incompatibility, Δ_{ABC} , and the underlying volume, V_{ABC} , as a function of the deployment height, w_A , for both triangles. We find that the triangle with $\beta^{(1)} = 50^\circ$ is characterized by both larger w_A^c and maximum incompatibility, $\Delta_{ABC}^{max} = \max(\Delta_{ABC})$. However, for this triangle the expanded compatible state is located after the configuration corresponding to the maximum underlying volume ($w_A^{c(1)} > w_A^{V_{max}(1)}$) and, therefore, cannot be reached when V_{ABC} is controlled. As such, whereas we expect a closed origami structure realized using these triangles to have two stable states with very different internal volume, we cannot use inflation to switch between the two of them. By contrast, the triangle with $\beta^{(2)} = 33^\circ$ exhibits much smaller Δ_{ABC} and w_A^c , but can be deployed when controlling the volume since $w_A^{c(2)} < w_A^{V_{max}(2)}$. This suggests that closed origami realized using this triangle can be deployed using inflation, but have an expanded configuration very similar to the flat one. Further, we expect such structures to be only marginally bistable, as small perturbations are enough to overcome the energy barrier associated to the small $\Delta_{ABC}^{max(2)}$.

While in Figs. 1b-d we focus on two geometries, we next consider all deployable triangles (i.e. triangles with $\pi/4 - \alpha/2 \leq \beta \leq \pi/2 - \alpha$) and seek for those that can potentially lead to deployable structures that are simultaneously bistable and inflatable. Towards this end, we use w_A^c to estimate the change in shape between the compatible states and Δ_{ABC}^{max} to evaluate bistability (i.e. to estimate the energy required to snap back from the expanded to the flat state). Further, we introduce an inflation constraint

$$h_{ABC} = \frac{\Gamma_{ABC}^{V_{max}}}{\Gamma^c}, \quad [4]$$

where $\Gamma_{ABC}^{V_{max}}$ and Γ^c are the arc lengths measured on the Δ_{ABC} - V_{ABC} curve between the flat stable state and the state of maximum volume and between the flat and expanded stable configurations, respectively (see Fig. 1e). It follows from Eq. (4) that only geometries with $\log h_{ABC} \geq 0$ can be deployed through fluidic actuation as those are the only ones for which the expanded compatible configuration is reached before the one with maximum volume during inflation (note that $\log h_{ABC} = -1.46$ and 0.322 for the two triangles considered in Figs. 1b-c).

In Figs. 1f-h, we report w_A^c , Δ_{ABC}^{max} , and h_{ABC} for all deployable triangles. We find that both w_A^c and Δ_{ABC}^{max} are maximized in the region close to the upper boundary of the domain (i.e. when $\beta \rightarrow \pi/2 - \alpha$). By contrast, the triangles deployable through inflation (for which $\log h_{ABC} \geq 0$) are all close to the lower boundary of the domain (i.e. when $\beta \rightarrow \pi/4 - \alpha/2$) and exhibit small values of w_A^c and Δ_{ABC}^{max} . As such, these results indicate that we cannot realize closed origami structures that are at the same time bistable and inflatable using a single triangle building block.

Extending the design space to enable deployment via inflation. In an attempt to realize inflatable closed origami structures with stable flat and expanded configurations, we turn our focus

Fig. 2. Bistable and inflatable origami shapes. (a-f). Examples of (a-b) Designs I, (c-d) Design II, and (e-f) Design III geometries in both the flat and deployed stable configurations along with the regions in the $\alpha - \beta$ space leading to inflatable structures. Note that the bright and dark shaded regions in the plots represent the triangles with angles $(\alpha^{(1)}, \beta^{(1)})$ and $(\alpha^{(2)}, \beta^{(2)})$, respectively. Further, the contour lines represent the maximum incompatibility of the triangular building blocks, Δ_{ABC}^{max} . (g). Deployment of a triangular building block that has been initially rotated around its edge BC to have a height w_A^i . (h). Contour map of angle β_{IV}^i required to obtain Designs IV that are both inflatable and bistable. (i). Maximum incompatibility, Δ_{ABC}^{max} , versus the inflation constraint, h , for 500,000 random geometries of Designs I-IV. (j). Pressure-volume curves recorded when testing our centimeter-scale prototypes. See Supplementary Information for rationale behind geometry and material selection.

to systems realized by assembling two different triangles with internal angles $(\alpha^{(1)}, \beta^{(1)})$ and $(\alpha^{(2)}, \beta^{(2)})$. To begin with, we arrange $2n$ triangles of each type to form two identical layers with n -fold symmetry and connect them at their outer boundaries (see Figs. 2a and c). The resulting star-like structures (reminiscent of an origami waterbomb base (38, 49)) define an internal volume $V = 2n(V_{ABC}^{(1)} + V_{ABC}^{(2)})$, exhibit geometric incompatibility $\Delta = 2n(\Delta_{ABC}^{(1)} + \Delta_{ABC}^{(2)})$ and are inflatable only if $\log h = \log(\Gamma^{V^{max}}/\Gamma^c) \geq 0$, where $\Gamma^{V^{max}}$ and Γ^c are the arc lengths measured on the Δ - V curve between the states with $V = 0$ and $V = V^{max} = \max(V)$ and between the two stable configurations, respectively. However, it is important to note that to realize these star-like structures the pair of tri-

angles cannot be arbitrarily chosen. This is because geometric compatibility is guaranteed only if the two triangles have (i) identical deployed compatible height (i.e. $w_A^{(1)} = w_A^{(2)}$); (ii) connecting deployed edges (either AB or AC) of equal length; and (iii) angles that satisfy $\alpha^{(1)} + \alpha^{(2)} = \pi/n$.

As shown in Fig. 2a, we first connect the two triangles via their longest edge ($\|AB^{(1)}\| = \|AB^{(2)}\|$) and refer to these structures as Designs I. We identify all possible geometries by enforcing requirements (i)-(iii) and find that designs deployable through inflation (for which $\log h \geq 0$) not only have a deployed shape almost indistinguishable from the initial flat one (see insets of structures in Fig. 2b), but are also made of two triangles with very low Δ_{ABC}^{max} (see area highlighted in ma-

genta in Fig. 2b). As such, the inflatable Designs I exhibit very low maximum incompatibility $\Delta^{max} = \max(\Delta)$ (see magenta markers in Fig. 2i for 500,000 randomly chosen geometries). To investigate the performance of these inflatable designs, we fabricate and test the geometry with $\log h \geq 0$ and the highest Δ^{max} (Design I-A with $(\alpha_{I-A}^{(1)}, \beta_{I-A}^{(1)}) = (22^\circ, 35^\circ)$ and $(\alpha_{I-A}^{(2)}, \beta_{I-A}^{(2)}) = (68^\circ, 14^\circ)$ for which $\Delta^{max}/||AB|| = 2.67 \times 10^{-2}$ —see magenta circular marker in Fig. 2i). A centimeter-scale prototype with $||AB^{(1)}|| = ||AB^{(2)}|| = 60$ mm is constructed by connecting 3D-printed stiff triangular facets with compliant hinges made of thin polyester sheets and an inflatable cavity is formed by coating it with a 0.5 mm-thick layer of silicone rubber (see insets in Fig. 2j and the Supplementary Materials, Section S2 and Video S1 for details). The sample is then deployed by supplying water at a constant rate of 10 mL/min with a syringe pump (Pump 33DS, Harvard Apparatus), while monitoring the pressure with a pressure-sensor (MPXV7025DP—see the Supplementary Materials, Section S3 for details). We find that the pressure, p , increases monotonically with V until the maximum volume for the cavity is reached (see Fig. 2j and Video S2). As such, our test reveals that the Δ^{max} of this design is not large enough to make the fabricated structure bistable.

Next, with the goal of increasing the geometric incompatibility of the inflatable designs, we investigate the response of star-like structures in which the longest edge of one triangle, $AB^{(1)}$, is connected to the shortest edge of the other one, $AC^{(2)}$ (we refer to these designs as Designs II—see Fig. 2c). Again, we impose requirements (i)–(iii) to identify all possible geometries and find that those deployable through inflation comprise two very distinct triangles: a first one with low Δ_{ABC}^{max} but $\log h_{ABC}^{(1)} > 0$ (see area highlighted in dark red in Fig. 2d) and a second one with substantially larger Δ_{ABC}^{max} but $\log h_{ABC}^{(2)} < 0$ (see area highlighted in bright red in Fig. 2d). Remarkably, we find that the combination of these different triangles results in inflatable designs with higher maximum incompatibility compared to Designs I (see red markers in Fig. 2i for 500,000 randomly chosen geometries). As a result, when we fabricate and test the inflatable geometry that maximizes Δ^{max} (Design II-B with $(\alpha_{II-B}^{(1)}, \beta_{II-B}^{(1)}) = (43.6^\circ, 25.2^\circ)$ and $(\alpha_{II-B}^{(2)}, \beta_{II-B}^{(2)}) = (46.4^\circ, 33.5^\circ)$ for which $\Delta^{max}/||AB|| = 8.58 \times 10^{-2}$ —see red marker in Fig. 2i) we observe a negative pressure region (see red curve in Fig. 2j and Video S2). This confirms the presence of an expanded stable configuration that can be reached through fluidic actuation (see Fig. S23 for details) and indicate the existence of a threshold value of $\Delta^{max}/||AB||$ (dependent on materials and fabrication process) that marks the transition from monostable to bistable behavior.

Whereas the connection of two different triangles side by side enables us to design inflatable and bistable structures, it limits us to star-like shapes. To expand the range of shapes, we next arrange the two triangles on top of each other in the flat configuration and mirror them twice to form an inflatable cavity (we refer to these structures as Design III—see Fig. 2e). This leads to geometries comprising eight triangles that are initially flat and transform into wedge-like shapes upon deployment. As for Designs I and II, geometric compatibility for Designs III requires a pair of triangles with $w_A^{c(1)} = w_A^{c(2)}$ and $||AB^{(1)}|| = ||AC^{(2)}||$, but the closure of the cavity is only possible if $\alpha_{III}^{(1)} = \alpha_{III}^{(2)}$. By imposing these constraints and

$\log h \geq 0$, we find that inflatable Designs III can be realized by combining two triangles with $\log h_{ABC} < 0$ and, therefore, substantially larger Δ_{max}^{ABC} (see areas highlighted in yellow in Fig. 2f). This is because the internal volume of Designs III is defined by the difference between $V_{ABC}^{(1)}$ and $V_{ABC}^{(2)}$ (i.e. $V = 4(V_{ABC}^{(1)} - V_{ABC}^{(2)})$) instead of their sum as for Designs I–II. Importantly, by plotting Δ^{max} vs. h for 500,000 randomly chosen Designs III, we find that these geometries are characterized by much larger maximum incompatibility in the inflatable domain. As a result, when we fabricate and test Design III-C with $(\alpha_{III-C}^{(1)}, \beta_{III-C}^{(1)}) = (37.1^\circ, 30.0^\circ)$ and $(\alpha_{III-C}^{(2)}, \beta_{III-C}^{(2)}) = (37.1^\circ, 40.6^\circ)$, (for which $\Delta^{max}/||AB|| = 9.93 \times 10^{-2}$), we record even larger values of negative pressure (i.e. larger energy barrier preventing the snap back) compared to the previous bistable Design II-B (see Fig. 2j and Video S2).

So far, all identified designs (i.e. Designs I–III) have been realized by assembling triangles that initially lie in the xy -plane and recover their angle α on such plane for $w_A = w_A^c$. However, the triangle in the xy -plane can also be seen as the projection of a triangle with internal angles α and β that has been initially rotated around its edge BC to have a height w_A^i and projected angles α_{xy}^i and β_{xy}^i (see Fig. 2g). In this case, if $\beta_{xy}^i \in [\pi/4 - \alpha_{xy}^i/2, \pi/2 - \alpha_{xy}^i]$, the angle α_{xy}^i is preserved for two distinct deployment heights, w_A^i and w_A^c (see inset in Fig. 2g and Supplementary Materials, Section S1 for details). As such, we can use these triangles as building blocks to realize star-like origami shapes with two expanded stable configurations in correspondence of w_A^i and w_A^c . An interesting feature of this family of structures (which we refer to as Designs IV) is that, if we select $\alpha_{xy}^i = \pi/n$ ($n = 3, 4, \dots$) and

$$\beta_{IV} \geq \beta_{IV}^* = \tan^{-1}(\sqrt{2} \tan \beta_{xy}^i), \quad [5]$$

the resulting origami are bistable and inflatable even if made out of a single triangular building block (see Supplementary Materials, Section S1 for details). This is because, for $\beta_{IV} \geq \beta_{IV}^*$, V_{ABC} monotonically decreases when deploying the triangle from w_A^i to w_A^c , so the compatible state corresponding to w_A^c can always be reached by deflation. To demonstrate the concept, in Fig. 2i, we consider 500,000 different geometries of Design IV for which $\alpha_{xy}^i = \pi/4$, $\beta_{xy}^i \in [\pi/4 - \alpha_{IV}/2, \pi/2 - \alpha_{IV}]$, and $\beta_{IV} \in [\beta_{xy}^i, \pi/2]$ and find that all geometries with $\beta_{IV} \geq \beta_{IV}^*$ are inflatable. Further, since β_{IV}^* is not affected by α_{xy}^i (see map of β_{IV}^* in Fig. 2h), inflatable origami structure can be realized by assembling highly incompatible triangles lying near the upper bound of their design space. This results in inflatable designs exhibiting a maximum incompatibility, Δ^{max} , much higher than Designs I–III and with a flat stable state in the xz -plane (since for $\beta = \pi/2 - \alpha$ the compatible deployment height is such that the deployed triangle lies in the orthogonal xz -plane with $w_A^c = ||AC||$ —see Supplementary Materials, Section S1 for details). In full agreement with these findings, when we fabricate and test Design IV-D with $(\alpha_{IV-D}, \beta_{IV-D}) = (29^\circ, 56^\circ)$ and $(\alpha_{xy}^i, \beta_{xy}^i) = (45^\circ, 33^\circ)$, (for which $\Delta^{max}/||AB|| = 2.05 \times 10^{-1}$), we record the largest negative pressure and energy barrier in the deployed stable state (see green curve in Fig. 2j and Video S2).

Meter-scale functional structures. As a next step, we use the simple geometries presented in Fig. 2 as basis to design functional and easily deployable structures for real-world applications and build them at the meter scale.

Fig. 3. Meter-scale inflatable archway. (a). The two Design III units used to construct the arch. (b). Schematics illustrating an inflatable archway comprising six Design III-C and seven Design III-C' units. (c). To facilitate inflation, we create a single cavity by cutting all units through their xz mirror plane, separating the two resulting parts by a distance t , and connecting them with rectangular facets. (d). Fabricated meter-scale inflatable arch in its flat and deployed stable configurations.

To begin with, we use the expanded wedge-like shapes of Designs III as building blocks to realize an inflatable archway. Focusing on Design III-C, we find that in the expanded stable state it has an opening angle $\theta_{\text{III-C}} = 40^\circ$ (see Fig. 3a). To design a deployable arch, we couple this unit with a different geometry of the same design family (which we referred to as Design III-C') that (i) is bistable and deployable through inflation; (ii) has an edge AB of equal length; (iii) has an opening angle $\theta_{\text{III-C'}}$ such that, when we alternate $m + 1$ units of Designs III-C' with m units of Designs III-C, we span an angle of 180° in the expanded configuration; and (iv) has the larger triangle (referred to as triangle 1 in Fig. 2) identical to that of Design III-C but mirrored (i.e. $\alpha_{\text{III-C}}^{(1)} = \beta_{\text{III-C'}}^{(1)}$ and $\beta_{\text{III-C}}^{(1)} = \alpha_{\text{III-C'}}^{(1)}$ —see Fig. 3b) to ensure compactness in the flat state. By inspecting the database of Fig. 2j, we find that for $m = 6$ all above requirements are satisfied when Design III-C' is characterized by $(\alpha_{\text{III-C'}}^{(1)}, \beta_{\text{III-C'}}^{(1)}, \alpha_{\text{III-C'}}^{(2)}, \beta_{\text{III-C'}}^{(2)}) = (30^\circ, 37^\circ, 30^\circ, \text{ and } 51^\circ)$. However, since the resulting archway comprises 13 inflatable cavities, multiple pressure inputs would be needed to inflate it. To simplify the deployment process,

we modify the structure by cutting it through the xz mirror plane, separating the two resulting parts by a distance t , and connecting them with rectangular facets (see insets in Fig. 3c). Since this procedure does not affect the geometric deployment of the triangles, we expect the additional facets to have negligible impact on the structure's multistability, but to facilitate its inflatability by creating a single cavity. In Fig. 3d, we show a meter-scale version of this archway with $\|AB\| = 30$ cm and $t = 10$ cm constructed out of corrugated plastic sheets (clear 8 ft $\times$ 4 ft, 4-mm thick sheets from Corrugated Plastics). To build this structure, we use a digital cutting system (G3 cutter from Zünd) to cut two parts (each comprising both triangular building blocks and rectangular facets— see Fig. S20) and pattern the hinges by scoring the sheets to locally reduce the thickness of the material. We then connect the two digitally cut parts using adhesive tape to form an airtight cavity (see Supplementary Materials, Section S2 for details). In the folded configuration, the structure has a height of 20 cm and a width of 30 cm. Upon pressurization, it inflates into a 60 cm tall and 150 cm wide archway that, because of its multistability, preserves its shape even when the pressure is suddenly released (see Video S3). Finally, it can be folded back to the initial flat state by applying vacuum to overcome the energy barrier (see Video S3).

Another strategy to realize functional shapes is to merge components of different design families together to form a single cavity. As an example, we can create an inflatable tent-like geometry by combining one layer of a Design I with another one of a Design IV (see Fig. 4a). To ensure successful merging, the two layers must have (i) outer edges BC of equal length, and (ii) the same xy -projection in the two compatible states (i.e. $\alpha_1^{(1)} = \alpha_1^{(2)} = \alpha_{xy}^i$ and $\beta_1^{(1)} = \beta_1^{(2)} = \beta_{xy}^i$). Further, to realize structures with a fully flat compatible state, we choose the triangles to lie on the upper boundary of the deployable domain (i.e. triangles with $\beta_1^{(1)} = \pi/2 - \alpha_1^{(1)}$ and $\beta_1^{(2)} = \pi/2 - \alpha_1^{(2)}$). By imposing these constraints, we can design tent-like structures that can be folded flat and expanded via inflation (see Fig. S24 for a centimeter-scale version), but their compactness is limited by the long AB edge of the Design IV. To further decrease the occupied volume in the compact state, we truncate the triangular building blocks of the Design IV layer into quadrilaterals and add additional layers of Design IV, some of which can be folded inwardly. As shown in Fig. 4b, these operations do not only reduce the initial volume, but also result in a more livable sheltered space in the deployed state. To demonstrate this strategy, we fabricate the structure shown in Fig. 4b at the meter-scale applying the same construction process used for the inflatable archway (see Supplementary Materials, Section S2 and Video S4 for details). As shown in Figs. 4c and d, the structure can be folded completely flat (with the ceiling folded inward) to occupy a space of $1 \times 2 \times 0.25$ m. When an input pressure is provided, the structure first expands to a stable configuration with the roof folded inward. Upon further pressurization the roof snaps outward and the final deployed shape of $2.5 \times 2.6 \times 2.6$ m is reached. Importantly, because of the multistability, at this point the door can be opened without impacting structural integrity, making the internal space accessible. Finally, using vacuum, the shelter can be folded back to the flat configuration (see Video S3).

Fig. 4. Meter-scale inflatable shelter. (a). A tent-like design can be created by merging one layer of a Design I with another one of a Design IV. Note in that the initial, zero-volume configuration of the tent, both layers are in their compatible expanded state, whereas in the final inflated configuration of the tent, both layers are in their initial state (flat for Design I and initially rotated for Design IV). (b). The initial volume can be further decreased by truncating the triangular facets into quadrilaterals and arranging successive layers of Design IV. (c). Schematics illustrating the deployment process. (d-e). The fabricated meter-scale inflatable shelter can be inflated from a compact state to a fully deployed state. Because of multistability, the door can be opened and make the internal space accessible.

Conclusion

In summary, we demonstrated how geometry can be efficiently exploited to realize pressure-deployable origami structures characterized by two stable configurations—one compact and one expanded. The design methodology presented in this work could be extended both to larger and smaller scales if properly accounting for loading conditions and fabrication challenges (4, 10, 11, 20). Since our functional structures are multistable, they can also be designed to achieve target deployment sequences (Fig. S25). In addition, by introducing building

blocks comprising more than two different facets, we expect to further expand the range of achievable shapes (Fig. S26). To that end, complementary to our geometric model and experiments, a mechanical model capable of predicting the full energy landscape (39, 50) could provide a useful tool to guide such exploration. Finally, building on our results, deployable structures able to switch between targeted stable states could be efficiently identified by generalizing our design rules to arbitrary origami polyhedra and, combining them with stochastic optimization algorithms, solve the inverse design problem.

Methods

Details of the design, materials, and fabrication methods are summarized in Supplementary Materials, Sections S1 and S2. The experimental procedure of the inflation with water to measure the pressure-volume curve is described in Supplementary Materials, Section S3. Finally, additional information about extending our methodology to more complex designs is provided in Supplementary Materials, Section S4.

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 Fig. S24. Inflatable tent-like design.
 Fig. S25. Multistable origami shapes.
 Fig. S26. Closed origami structure comprising eight different triangles.
 Fig. S27. Deployment of centimeter and meter-scale arches.
 Fig. S28. Deployment of centimeter and meter-scale shelters.
 Fig. S29. A deployable pagoda-like structure.
 Fig. S30. Deployable booms.
 Video S1. Fabrication of centimeter-scale structures.
 Video S2. Inflatable and bistable origami shapes.
 Video S3. Multistable inflatable origami structures at the meter-scale.
 Video S4. Fabrication of the meter-scale shelter.

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