CLASS CONFLICT AND FAIRNESS IN
"DEMOCRATIC CAPITALISM" +

by

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ABSTRACT

This paper studies the distribution of income, class formation, deformation and conflict, and fairness in a political economy combining basic features of capitalism with certain democratic institutions determining the wage level by voting.

0. Introduction
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In this paper we study the distribution of income in a political economy whose design combines basic features of capitalism in a most simple way with certain democratic institutions. We take a private-ownership economy with capitalists and workers. The capitalists share the ownership of the available capital goods and are responsible for deciding on the level of employment and production. The workers contribute their labor when offered employment, but the wage is determined through one or another voting system by a majority rule with universal suffrage. Both capitalists and workers are expected-utility maximizers.

Given the relations governing production and the preferences governing behavior, we obtain the distribution of income as a function of a (proportional) profit tax under an open ballot and two types of secret ballot voting systems determining the wage level and, for each of these three systems, we compute the fair tax rate. Then we compare the outcomes under the various voting systems from the viewpoint of each politico-economic participant and identify a certain class conflict, also examining the three voting systems from the viewpoint of their fairness.

The study is motivated by two concurrent interests. On a general level, we wish to begin investigating social

systems whose basic behavioral relations and determined, in the standard fashion of economic theory, by "rational" participants maximizing expected utility, but where certain parameters constraining and affecting the behavior of these agents are set through mechanisms and institutions such as voting. On a different plane, we are motivated by a variety of interrelated questions concerning the foundations and consequences of politico-economic syndromes such as "democratic capitalism" and the possible forms of arisal of syndromes such as "people's capitalism", some concerning the formation and deformation of classes and the nature of class conflict under such syndromes, and some concerning the multitude of associated issues of equity and fairness.

While we are spurred by all these interests and questions, we certainly do not set out here to uncover any general theory or principles responding to these, but only to give account of a modest exploration. In a way, this is evident from the simplicity of the model we use. It is entirely static and incorporates no capital market. Perhaps more important, its labor market is quite simplistic with a perfectly elastic supply of labor up to the point of full employment, the underlying preferences of potential labor-suppliers (workers) being completely insensitive to the pleasures of leisure. Given all these simplifications, our analysis and results

are no more than suggestive. One may hope, however, that this study will stimulate researchers in the area to find ways of relaxing these simplifications.

1. The Political Economy of Democratic Capitalism

The economy consists of the set $\Omega = [0, 1]$ of economic agents; a production function $F: \mathcal{B}(R_+) \times \mathcal{B}(R_+) \rightarrow R_+$ such that $F(K, M) = f(k, m)$, where $k = \mu(K)$, $m = \mu(M)$, and f is of the Cobb-Douglas form

$$(1) \quad f(k, m) = k^{1-\alpha} m^{\alpha} \quad \text{with } \alpha \in (0, 1);$$

and a family $U: \Omega \rightarrow R^R$ of preferences for income, where the preference of a typical agent $\omega \in \Omega$ is represented by a utility function $U(\omega)$ such that¹ denoting $u_{\omega} = U(\omega)$,

$$(2) \quad u_{\omega}(x) = \text{sign}(x) (|x|)^{\gamma(\omega)}$$

with $\gamma(\omega)$ described by a positive measurable function $\gamma: \Omega \rightarrow R_+^0$.

Here K stands for a Borel set of capital goods with fixed positive measure $k = \mu(K) > 0$. Its ownership is shared uniformly by a "capitalist class" $C = [0, c]$ with $c \in (0, \frac{1}{2})$, so that each capitalist (i.e., each agent $\omega \in C$) owns a share $\frac{1}{c}$ of K .

The rest of the population is the "labor class" $L = \Omega \setminus C = [c, 1]$.

Production-employment decisions are made by the capitalists.

¹ Taking any function $d: \Omega \rightarrow R_+$ and specifying the utility functions as $d(\omega) u_{\omega}^+$ might give a more general appearance to our model, but the reader can check throughout that it would make absolutely no difference to our analysis.

There is a proportional profit tax, the only tax in this economy, and the tax rate is $t \in [0, 1)$. Thus, each capitalist maximizes $\frac{1-t}{c} \pi$, his share in after-tax profit, where π is the total profit of the capitalist class, given by

$$\pi(m) = f(k, m) - wm,$$

and w stands for the wage and m is the measure of the population M employed with the constraint that $M \subset L$ (and that M is a Borel set). Hence, the capitalist class will be unanimous in its production-employment decision for any given wage level $w \in R_+$, choosing m so as to maximize π . The tax $t\pi$ is distributed uniformly over the labor class so that each worker (i.e., each agent $\omega \in L$) receives $\frac{1}{1-c} t\pi$ from this source. The employed workers $\omega \in M$ each receive also w . The income distribution arising in this way may be perturbed only through voluntary transfer payments of disposable income.

An important simplification we make in our specification of the economy relates to the supply of labor. While the wage w is determined "democratically" in a way which we will soon describe, it is agreed that whatever positive wage bill w is thus adopted, a worker offered employment at that wage has no choice but to accept, and that, as long as the set of workers offered employment has full measure $(1-c)$, a worker has to accept employment even at a zero wage. Thus, the supply of labor is perfectly inelastic, as depicted in Diagram 1 below.

This brings us to the demand for labor, which we obtain through profit maximization. Given any wage w , profit is

$$\pi(m) = k^{1-\alpha} m^\alpha - wm,$$

which is concave, so the profit maximizing level of employment is obtained through first order conditions as

$$m(w) = \alpha^{1/1-\alpha} k w^{1/\alpha-1},$$

and this is the demand for labor. Thus, for any institutionally or otherwise given wage w , the actual employment level $m^*(w)$ in the economy will be the lower of the supply $1-c$ and the demand $m(w)$:

$$m^*(w) = \inf \{1-c, m(w)\}.$$

We see that

$$\bar{w} = \alpha k^{1-\alpha} (1-c)^{\alpha-1}$$

defines the highest wage at which full employment ($m^*(w) = 1-c$) obtains and that we may explicitly write

$$(3) \quad m^*(w) = \begin{cases} 1-c & \text{for } w \leq \bar{w}. \\ \alpha^{1/1-\alpha} k w^{1/\alpha-1} & \text{for } w \geq \bar{w}. \end{cases}$$

(See Diagram 1 above.)

So much describes the purely economic features of our political economy. To complete the description, we now turn to the institutions through which the wage w is to be

determined.

Now in ordinary capitalism the institution of the market mechanism, operating through the "dollar vote", would determine the wage by equating supply and demand as $\bar{w}$. As already remarked, however, we wish to consider "democratic" institutions to set the wage. In particular, we will have the wage determined through one or another voting system with universal suffrage where each agent $w \in \Omega$ casts one vote and the majority rule obtains.

Before we describe the details of these various voting systems and how voting is to take place, let us first spell out exactly what the majority rule means here. For each wage "bill" $w \in R$, let $\Omega(w)$ be the set of agents $w \in \Omega$ who cast their votes for w . A wage w is instituted as majority winner iff

$$(4) \quad \mu(\Theta) \geq \frac{1}{2} \text{ for some closed set } \Theta \subset \Omega.$$

As voting systems we will consider the open ballot (OB) and two types of secret ballot (SB). The voting is to take place in a milieu resembling a Kantian state of nature in the sense of Rawls [3]. Before the voting takes place, tables of the employment function (3) are made public. A worker may then compute the probability $\frac{m^*(w)}{1-c}$ (as a function of the wage w emerging as majority winner) of being employed, but does not in general know for certain from $m^*(w)$ whether he himself will be employed, and hence has no certain

knowledge of his income as a function of w , the majority-winning wage. Nevertheless, each agent votes for a wage maximizing his expected utility of income. The actual voting behavior that takes place will depend on the particular voting system in operation, i.e., on the rules governing the voting. So let us now describe the various voting systems we are about to consider.

The essential difference between the open ballot (OB) and the secret ballot (SB) voting systems is that vote-purchasing is effectively ruled out in the latter, for the simple reason that with a secret ballot there is no way of checking whether the "goods" (votes) purchased are delivered. In the case of the OB, of course, post-voting disposable income may be contracted in pre-voting agreements, and votes favorable to one's interests may be purchased in this way. Now while, in the case of the SB, the voting outcome cannot be affected by a voting market, there are methods other than the direct purchasing of votes that may be used to affect the voting outcome. For if, prior to the voting, agents come to own shares in the post-voting disposable income of other agents, their preferences regarding wage bills would in general become biased by sharing the latter agents' interests in the matter. For example, if a worker were donated a large enough share in the after-tax profit to accrue to the capitalist class, this worker may find that "what is in the interest of GM" is in his interest too, and his voting may be thus affected. If enough workers' interests could

be so aligned with those of the capitalist class, the outcome of the voting could be influenced in favor of this class. Whether it would pay the capitalists to undertake such political entrepreneurship as to actually donate shares in post-voting after-tax profit to segments of the labor class is one of the questions we address in the next section. In what follows we will distinguish between two types of secret ballot, the non-discriminatory (NDSB) and the discriminatory (DSB). In the NDSB, capitalists will be allowed to make only "non-discriminatory" donations to the workers, i.e., they will be allowed to make donations of shares in their disposable income which are equal for each worker. This restriction will be relaxed in the case of the DSB.

2. Determination of the Wage: Voting Behavior

In analyzing agents' preferences for the wage to be instituted and in studying their resulting voting behavior, as well as in examining how their preferences and behavior may be influenced through a vote market or as a consequence of agents' owning claims to each others' disposable income, we will make a crucial assumption regarding the ability of agents to coalesce. Namely, we will assume that the capitalist class is perfectly organized and that the labor class is perfectly unorganized. [Technically, the coalition structure of our political economy is

$$(5) \quad C = \{C\} \cup \{S \subseteq L \mid \mu(S) = 0\},$$

so that the coalitions are C and all sets of measure zero consisting of workers.] Thus, the capitalists always cooperate amongst themselves and workers always behave non-cooperatively.²

The notion of a donation will be central to our analysis, so first we define this notion. Technically, what we mean by a donation is a simple function³ $\delta: L \rightarrow R_+$ such that

² This, of course, is far from being descriptive of those instances of modern industrial capitalism with strong labor parties or trade unions, but it probably does approximate early capitalism and perhaps instances of fascism.

³ Any non-negative real-measurable function on L can be arbitrarily closely approximated, in the sense of the point-open topology, by simple functions; in fact, see Theorem 5.2, p. 104, of [2] or Proposition A.20, p. 395, of [1].

$\int_L \delta \leq 1$. The role of a donation will be that it will assign to each worker $\omega \in L$ a share $\delta(\omega)$ of the after-tax profit $(1-t)\pi$, entitling him therefore to $\delta(\omega)(1-t)\pi$ extra income above and beyond possible wage income (earned if employed) and the quantity $\frac{t}{1-c} \pi$ of taxes channeled to him. A donation is to be understood, of course, as being from the capitalist class to the labor class.

In examining how agents will vote, the simplest task is to determine how capitalists will vote. For the actual profit $\pi^* = \pi \circ m^*$ (i.e., $\pi^*(w) = \pi(m^*(w))$) is clearly a monotonically decreasing function of the wage w instituted, so all capitalists prefer $w = 0$ to every other wage and vote for it.

Now we turn to the task of determining workers' preferences regarding w , toward understanding how they will vote. The key to this, of course, is a worker's expected utility V_ω of income, and this depends, in the following fashion, on the wage w instituted and on the donation δ :

$$(6) \quad V_\omega(w, \delta) = \frac{m^*(w)}{1-c} \left[w + \frac{t}{1-c} \pi^*(w) + \delta(\omega)(1-t)\pi^*(w) \right]^{\gamma(\omega)} \\ + \left(1 - \frac{m^*(w)}{1-c} \right) \left[\frac{t}{1-c} \pi^*(w) + \delta(\omega)(1-t)\pi^*(w) \right]^{\gamma(\omega)}$$

PROPOSITION 1: Every worker $\omega \in L$ receiving a share $\delta(\omega)$ less than [resp., greater than] $\frac{1}{1-c}$ strictly prefers higher to lower [resp., lower to higher] wages in the interval $[0, \bar{w}]$, and if $\delta(\omega) = \frac{1}{1-c}$, ω is indifferent between all wages in this interval.

Proof: On the interval $[0, \bar{w}]$ there is full employment ($\pi^*(w) = 1-c$), so that the probability of a worker w being unemployed is nil, and his expected utility of income reduces to

$$(7) \quad V_w(w, \delta) = \left[w + \frac{t}{1-c} \pi^*(w) + \delta(w)(1-t)\pi^*(w) \right]^{\gamma(w)}.$$

Now

$$(8) \quad \pi^*(w) = \begin{cases} k^{1-\alpha}(1-c)^\alpha - (1-c)w & \text{if } 0 \leq w \leq \bar{w}, \\ k(\alpha^{1-\alpha} - \alpha^{1/\alpha-1})w^{\alpha/\alpha-1} & \text{if } w \geq \bar{w}. \end{cases}$$

Thus, defining $\beta(w) = \frac{t}{1-c} + \delta(w)(1-t)$, on $[0, \bar{w}]$ we have

$$V_w(w, \delta) = \left[(1-\beta(w)(1-c))w + \beta(w)k^{1-\alpha}(1-c)^\alpha \right]^{\gamma(w)},$$

which is increasing/decreasing/constant on $[0, \bar{w}]$, iff $\beta(w)$ is, resp., less than/greater than/equal to $\frac{1}{1-c}$ and clearly $\beta(w) \begin{cases} \leq \\ > \end{cases} \frac{1}{1-c}$ iff $\delta(w) \begin{cases} \leq \\ > \end{cases} \frac{1}{1-c}$. $\diamond$

For the proof of our next proposition we will need the following

Lemma 1:⁴ Let x_0, x_1, y_0, y_1, p and q be real numbers such that $y_0 < x_0 < x_1 < y_1$ and $p < q$ with $p, q \in [0, 1]$. If

$$(9) \quad px_0 + (1-p)x_1 \geq qy_0 + (1-q)y_1,$$

⁴ We thank Dr. Sidney Jacobs for suggesting the proof we use here.

then, for any twice-continuously differentiable non-decreasing concave function $V: R \rightarrow R$,

$$pV(x_0) + (1-p)V(x_1) \geq qV(y_0) + (1-q)V(y_1).$$

Proof: Assume (9) and that V is as described. Define $s = q-p$, so that

$$(10) \quad q(x_0 - y_0) + s(x_1 - x_0) \geq (1-q)(y_1 - x_1).$$

By the Mean Value Theorem there are numbers z_0 , $\bar{x}$ and z_1 with $z_0 \in [y_0, x_0]$, $\bar{x} \in [x_0, x_1]$ and $z_1 \in [x_1, y_1]$ such that

$$V(x_0) - V(y_0) = V'(z_0)(x_0 - y_0),$$

$$V(x_1) - V(x_0) = V'(\bar{x})(x_1 - x_0)$$

and

$$V(y_1) - V(x_1) = V'(z_1)(y_1 - x_1).$$

Multiplying (10) by $V'(z_1)$,

$$q(x_0 - y_0)V'(z_1) + s(x_1 - x_0)V'(z_1) \geq (1-q)(y_1 - x_1)V'(z_1),$$

since $V'(z_1) \geq 0$. But the concavity of V gives

$$V'(z_0) \geq V'(\bar{x}) \geq V'(z_1),$$

so

$$q(x_0 - y_0)V'(z_0) + s(x_1 - x_0)V'(\bar{x}) \geq (1-q)(y_1 - x_1)V'(z_1),$$

i.e.,

$$pV(x_0) + (1-p)V(x_1) \geq qV(y_0) + (1-q)V(y_1). \quad \diamond$$

PROPOSITION 2: Every worker $\omega \in L$ who is not risk-liking (i.e., for whom $\gamma(\omega) \leq 1$) strictly prefers lower to higher wages in the interval $[\bar{w}, \infty)$.

Proof: Take any $w \in [\bar{w}, \infty)$. Now substituting (3) for m^* and (8) for π^* in (6), the expected utility of income for any worker $\omega \in L$ is

$$(11) \quad V_{\omega}(w, \delta) = \frac{\alpha^{1/1-\alpha} k w^{1/\alpha-1}}{1-c} [w + g(w)]^{\gamma(\omega)} \\ + (1 - \frac{\alpha^{1/1-\alpha} k w^{1/\alpha-1}}{1-c}) [g(w)]^{\gamma(\omega)},$$

where

$$g(w) = \beta(\omega) k (\alpha^{1/1-\alpha} - \alpha^{1/\alpha-1}) w^{\alpha/\alpha-1}.$$

If $\gamma(\omega) = 1$, this becomes

$$(12) \quad V_{\omega}(w, \delta) = \frac{1}{1-c} \alpha^{1/1-\alpha} k w^{\alpha/\alpha-1} + g(w),$$

which is strictly decreasing on $[\bar{w}, \infty)$ for every donation δ , so that ω strictly prefers lower to higher wages in the interval $[\bar{w}, \infty)$. Take any $w, w' \in [\bar{w}, \infty)$ with $w < w'$. Now $g(w') < g(w) < w + g(w)$ and $g(w') < w' + g(w')$. If $w' + g(w') \leq w + g(w)$, then evidently

$$(13) \quad V_{\omega}(w', \delta) < V_{\omega}(w, \delta)$$

for every (positive) assignment of $\delta(\omega)$ and of $\gamma(\omega)$. If $w + g(w) < w' + g(w')$, then so long as $\gamma(\omega) \leq 1$, Lemma 1 applies (associate p with

$1 - \frac{m^*(w)}{1-c}$, q with $1 - \frac{m^*(w')}{1-c}$, x_0 with $g(w)$, x_1 with $w + g(w)$, y_0 with $g(w')$ and y_1 with $w' + g(w')$) and (13) holds again. $\circ$

For each agent $\omega \in L$ we define the secret ballot vote of ω as the correspondence $w_\omega: R_+ \rightarrow R_+$ given by

$$w_\omega(\delta(\omega)) = \{w \in R_+ \mid V_\omega(w, \delta) \geq V_\omega(w', \delta) \text{ for all } w' \in R_+\}.$$

The next proposition follows directly from Propositions 1 and 2, so we omit its proof.

PROPOSITION 3: Every worker $\omega \in L$ who is not risk-liking (i.e., for whom $\gamma(\omega) \leq 1$) has the following secret ballot vote (depicted also in Diagram 2):

$$w_\omega(\delta(\omega)) = \begin{cases} \{\bar{w}\} & \text{if } \delta(\omega) \in [0, 1/(1-c)], \\ [0, \bar{w}] & \text{if } \delta(\omega) = \frac{1}{1-c}, \\ \{0\} & \text{if } \delta(\omega) \in (1/(1-c), \infty). \end{cases}$$

Now we make the following important

ASSUMPTION: $\omega \in [c, c + \frac{1}{2}] \Rightarrow \gamma(\omega) \leq 1.$

This assumption says that the workers in $[c, c + \frac{1}{2}]$ all have elasticity of utility with respect to income no greater than unity or, expressed in yet another way, that none of them is risk-liking.

Given all the above information about how workers' preferences regarding wage bills w and, hence, their votes may be influenced by donations, we now put ourselves in the capitalists' boots and compare the advisability of various possible donations in the case of the secret ballot. From Proposition 1, we know that no worker ω receiving $\delta(\omega) < \frac{1}{1-c}$ will vote for a wage bill $w < \bar{w}$. If the capitalists choose the zero donation $\delta_0^* = 0$, then the workers in $[c, c + \frac{1}{2}]$ all vote for the wage bill $\bar{w}$ and this is instituted as the majority-winning wage. If the capitalist class chooses the zero donation, each capitalist receives

$$\Pi_0(t) = \frac{1}{c}(1-t) \pi^*(\bar{w}) = \frac{1}{c}(1-t) (1-\alpha) k^{1-\alpha} (1-c)^\alpha.$$

Now they can not guarantee themselves any better outcome by choosing a donation δ which is positive on a set of positive measure, unless $\delta(\omega) > \frac{1}{1-c}$ on a set containing a closed set of measure greater than or equal to $\frac{1}{2}$.

Letting ϵ be an arbitrarily small positive number, however, focus attention on the donation δ_1^* defined by

$$\delta_1^*(\omega) = \begin{cases} 0 & \text{if } \omega \in (\frac{1}{2}, 1] \\ \frac{1}{1-c} + \epsilon & \text{if } \omega \in [c, \frac{1}{2}]. \end{cases}$$

Now the donation δ_1^* induces the workers, by Proposition 3 and our Assumption, to vote for $w = 0$; as the capitalist class $C = [0, c)$ votes for $w = 0$ in any case, this brings about a vote in favor of $w = 0$ from the whole closed interval $[0, \frac{1}{2}]$. Thus, if the capitalist class chooses the donation δ_1^* , then by the majority rule (4) the wage $w = 0$ is instituted and each capitalist receives

$$\begin{aligned} \Pi_1(t) &= \frac{1}{c} (1 - \int_L \delta_1^*(\omega) d\omega) (1-t) \pi^*(0) \\ &\approx \frac{1/2}{c(1-c)} (1-t) k^{1-\alpha} (1-c)^\alpha. \end{aligned}$$

Thus, we have the following

THEOREM 1: In the case of the secret ballot where discriminatory donations are allowed (DSB), the capitalists' optimal charity plan Δ depends on α and, indeed,

$$\Delta(\alpha) = \begin{cases} \delta_0 & \text{if } \alpha \leq \frac{1/2-c}{1-c} \\ \delta_1^* & \text{if } \alpha > \frac{1/2-c}{1-c} \end{cases}$$

and the wages $\bar{w}, 0$ are instituted as majority-winners accordingly as $\alpha \leq \frac{1/2-c}{1-c}$ or $\alpha > \frac{1/2-c}{1-c}$.

Similarly, in the case of NDSB, the capitalist class need only compare the donation δ_0^* with a constant

(non-discriminatory) donation $\delta_2^*(\omega) = \frac{1}{1-c} + \varepsilon$, where ε is an arbitrarily small positive number. Now when the donation $\delta_2^*(\omega)$ is instituted, each capitalist receives

$$\pi_2(t) = \frac{1}{c} (1 - \int_L \delta_2^*(\omega) d\omega) (1-t) \pi^w(0) < 0.$$

Thus, we conclude that

THEOREM 2: In the case of the secret ballot where discriminatory donations are not allowed (NDSB), the capitalists' optimal charity plan Δ is independent of α and, indeed,

$$\Delta = \delta_0^*$$

and the wage $\bar{w}$ is instituted as majority-winner.

It is important to note that, in case of the secret ballot, full-employment obtains regardless of the value of α . Note also that, when $\alpha > \frac{1/2-c}{1-c}$, the discriminatory secret ballot gives rise to 'people's capitalism', in the sense of a political-economic syndrome where a sufficiently large portion of the population (in our case, a majority $[0, 1/2]$ of the population) receives income accruing to capital in a significant way, i.e., in a way affecting the recipients' politico-economic behavior.

Now we may turn to the corresponding analysis of the open ballot. In this case, not only are all donations allowed, but also each coalition may purchase votes from any other coalition. Now given the coalition structure

specified in (5), the only coalition of positive measure is C , hence C is the only coalition over which the integral of income density may be positive. Thus the capitalist class (C) is singled out as the only coalition which is able to transfer a uniformly positive density of income to a set of agents with positive measure, i.e., to buy the votes of a set $S \subset L$ with $\mu(S) > 0$. Furthermore, the votes of a coalition do not affect the overall voting outcome unless the coalition has positive measure, so that no individual worker's vote affects the overall voting outcome and, thus, each worker is willing to sell his vote, i.e., to vote for any wage bill demanded, in return for an arbitrarily small positive price. In sum, the capitalist class C is the only coalition able and willing to purchase votes from a set of positive measure; it is, in fact, able and willing to purchase a vote in favor of the wage 0 from all the workers by negotiating with all singleton coalitions $\{a\} \subset L$, securing the zero wage as a majority-winner, all at an arbitrarily small positive cost. Note that, in view of Proposition 3, the optimal charity plan of the capitalists in the case of the open ballot is δ_0^* , independently of α , for any vote switching that can be achieved through a donation may be achieved at less cost by vote-purchasing. Thus, we may record the following

THEOREM 3: In the case of the open ballot (OB), the capitalists' optimal charity plan is

$$\Delta = \delta_O^{\#}$$

and the wage 0 is instituted as majority-winner. [To achieve this outcome, the capitalist class pays a price $\rho(\omega)$ to each worker $\omega \in L$ in return for a vote from ω in favor of $w = 0$, where $\rho: L \rightarrow R_+^0$ is a measurable function bounded from above by an arbitrarily small positive number ε].

As in the case of the SB, note that full employment obtains again in the case of the OB.

3. Income Distribution, Class Conflict and Fairness in Democratic Capitalism

In this section we put together results of the last section to compare the three systems OB, DSB and NDSB from the viewpoints of three classes, $C = [0, c)$, $D = [c, \frac{1}{2}]$ and $P = (\frac{1}{2}, 1]$, and to see, en passant, the conflict of interest between these three. The class C, as the capitalist class, is already distinguished from the labor class L. In the original Marxian sense, C owns the means of production, i.e., capital goods. The distinction between D and P within L is not entirely originally Marxian: it is a distinction between a class D of possible defectors (a "comprador" class?) and a "pure" proletariat P. The class D turns out to share, to some extent, the interests of C, while P remains "true labor".

Apart from bringing out a class conflict, and on the way toward doing that, we also give a welfare analysis of "democratic capitalism". First, in Table 1, we present the income densities of the three classes (C, D and P) under each system (OB, DSB and NDSB) as a function of the tax rate t and the elasticity α of communal production w.r.t. employment. In the bottom row of this table we compute, for each system (OB, NDSB and DSB) the fair tax rate t^* , i.e., the (unique) tax rate that makes the worst-off person as well off (in terms of income density) as possible:

maximizing the minimal income as a function of t .⁵

INSERT TABLE 1

Now, using Table 1, we obtain Table 2 by substituting the fair tax rate t^* corresponding to each column into the income densities of that column.

INSERT TABLE 2

As we see, it is only in the case of the DSB that the size of α makes any difference. Also, as c tends to zero, this difference, too, washes out.

Thus, we arrive at Table 3. The notation $>$, $\sim$, $<$ here is the usual: we read ' $>$ ' as "is strictly preferred to", ' $\sim$ ' as "is equivalent to" and ' $<$ ' as "is strictly inferior to". The entries of Table 3 are derived in obvious fashion, by comparison of income densities at any tax rate $t \in [0, 1)$ not just at the fair tax rates computed in Table 2. Note that Table 3 shows, in the case of $\alpha \leq \frac{1/2-c}{1-c}$, conflict between L and C with a solidarity between D and P, the two "subclasses" of the labor class L. Econometric evidence places greater credibility on $\alpha \leq \frac{1/2-c}{1-c}$. In the case of $\alpha > \frac{1/2-c}{1-c}$ however, even the labor class has its internal conflict, the compradors ($\omega \in D$) sharing certain preferences (amongst OB, NDSB and DSB) with the capitalist class C.

INSERT TABLE 3

⁵ This is a "Sup Inf" interpretation of Rawls' [3] interpretation of fairness, as applied to our present model.

This brings us to Table 4, our last, illustrated also in Diagrams 3-4. Here we identify the poorest class and record its income density under each voting system as a function of the tax rate t , considering two possible ranges of values for α , namely, $(0, \frac{1/2-c}{1-c}]$ and $(\frac{1/2-c}{1-c}, 1)$. To do this we use Table 1. In the first case where $\alpha \in (0, \frac{1/2-c}{1-c}]$, the labor class has one and the same income density with no difference between D and P. Taking any value of the tax rate $t \in [0, 1)$ we compare the three voting systems OB, DSB and NDSB and determine which systems give the highest income density to the poorest. For the value of t considered, we classify these systems as fair and the remaining systems as unfair. For example, when $t \in [0, 1 - \frac{c}{1-\alpha})$, workers have less income than capitalists, but it is under the secret ballot that they are best off. When t equals $1 - \frac{c}{1-\alpha}$, Table 1 tells us (see also Table 2) that the fair tax rate for the secret ballot obtains, so that it is the capitalist class which takes the position of poorest as the tax rate is increased beyond $1 - \frac{c}{1-\alpha}$ while adhering to the secret ballot system(s). The value $t = \frac{(1-\alpha)(1-c)}{(1-\alpha)(1-c)+c}$ is of interest. It is at this value of the tax rate that the fairness configuration of the voting system is reversed, the systems fair up to that point becoming unfair and the unfair ones becoming fair. Note also that this critical value of t is not one of the fair tax rates (see Table 1). As t is increased beyond this critical value, the lot of the capitalist class

continues to worsen steadily, becoming just as bad as the (continually improving) lot of the labor class at the OB fair tax rate $t = 1-c$.

The case of $\alpha \in (\frac{1/2-c}{1-c}, 1)$ is tabulated similarly. In this case it is clear from Table 1 that the labor class does split into two income density groups D and P under the DSB and that the pure proletariat is the worst off of these two subclasses of L under this voting system. Again the value $t = \frac{(1-\alpha)(1-c)}{(1-\alpha)(1-c)+c}$ of the tax rate occupies a critical position at which the fairness configuration ratios of voting systems is reversed, and again the lot of the capitalists is continually worsened as t is increased.

The values of t missing from Table 4 are, except for the critical value $t = \frac{(1-\alpha)(1-c)}{(1-\alpha)(1-c)+c}$ noted above, precisely the fair tax rates computed in Table 1 and used in Table 2. Thus Table 2 combined with Table 4 summarizes the complete fairness analysis of democratic capitalism in this study.

4. A Note on Interpreting the Model

The model we have worked with in this study, with all its underscored simplifications, may be interpreted as a model of a capitalistic economy. Before closing, we would like to give at least one other interpretation. Briefly, what we want to suggest is that the political economy we have described may be viewed as one where a set C of agents are appointed to be in charge of production-employment decisions and are incentivized by a decree entitling each of them to $\frac{1}{C}$ of the after-tax profit $(1-t)\pi$, while the rest of the population, i.e., the set L , are required by this very same decree to work whenever offered employment at a non-negative wage. In as much as the agents $\omega \in C$ in this political economy need not in fact own the means of production (i.e., K), these may be viewed as owned by the "state" in whose name the agents $\omega \in C$ manage the economy. In the lexicon of political economies, this may be found to approximate "democratic state capitalism".

DIAGRAM 1: SUPPLY OF AND DEMAND FOR EMPLOYMENT
 $(k = 1, c = .1)$

DIAGRAM 2:- THE SECRET BALLOT VOTE $w_{\omega}(\delta(\omega))$ OF
A NON-RISK-LIKING WORKER RECEIVING
SHARE $\delta(\omega)$

SYSTEMS AS A FUNCTION OF THE TAX RATE WHEN $\alpha > \frac{1/2-c}{1-c}$.
 $(\alpha = 1/2, c = 0.1).$

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TABLE 1: INCOME DENSITIES AND FAIR TAX RATES

($\bar{f} = k^{1-\alpha} (1-c)^\alpha$ is full-employment communal income.)

	OB	NDSB	DSB	
			$\alpha \leq \frac{1/2-c}{1-c}$	$\alpha > \frac{1/2-c}{1-c}$
$C = [0, c)$	$\frac{1}{c}(1-t)\pi^*(0)$ $= \frac{1}{c}(1-t)\bar{f}$	$\frac{1}{c}(1-t)\pi^*(\bar{w})$ $= \frac{1}{c}(1-t)(1-\alpha)\bar{f}$	$\frac{1}{c}(1-t)\pi^*(\bar{w})$ $= \frac{1}{c}(1-t)(1-\alpha)\bar{f}$	$\frac{1/2}{c(1-c)}(1-t)\pi^*(0)$ $= \frac{1/2}{c(1-c)}(1-t)\bar{f}$
$D = [c, 1/2]$	$\left\{ \begin{array}{l} \frac{1}{1-c} t\pi^*(0) \\ = \frac{t}{1-c} \bar{f} \end{array} \right.$	$\left\{ \begin{array}{l} \bar{w} + \frac{1}{1-c} t\pi^*(\bar{w}) \\ = \frac{\alpha+t(1-\alpha)}{1-c} \bar{f} \end{array} \right.$	$\left\{ \begin{array}{l} \bar{w} + \frac{1}{1-c} t\pi^*(\bar{w}) \\ = \frac{\alpha+t(1-\alpha)}{1-c} \bar{f} \end{array} \right.$	$\frac{1}{1-c} \pi^*(0)$ $= \frac{1}{1-c} \bar{f}$
$P = (1/2, 1]$				$\frac{1}{1-c} t\pi^*(0)$ $= \frac{t}{1-c} \bar{f}$
Fair Tax Rate t^*	$1 - c$	$1 - \frac{c}{1-\alpha}$	$1 - \frac{c}{1-\alpha}$	$\frac{1}{1+2c}$

TABLE 2: INCOME DENSITIES AT CORRESPONDING FAIR TAX RATE

($\bar{f} = k^{1-\alpha}(1-c)^\alpha$ is full-employment communal income.)

	OB	NDSB	DSB	
			$\alpha \leq \frac{1/2-c}{1-c}$	$\alpha > \frac{1/2-c}{1-c}$
$C = [0, c)$	}	}	}	$\frac{1/2}{(1-c)(1/2+c)} \bar{f}$
$D = [c, 1/2]$				$\frac{1}{1-c} \bar{f}$
$P = (1/2, 1]$				$\frac{1/2}{(1-c)(1/2+c)} \bar{f}$

TABLE 3: CLASS CONFLICT

	$a \leq \frac{1/2-c}{1-c}$	$a > \frac{1/2-c}{1-c}$
$C = [0, c)$	$OB > DSB \sim NDSB$	$OB > DSB > NDSB$
$D = [\hat{c}, 1/2]$	$\left. \begin{array}{l} \\ \end{array} \right\} OB < DSB \sim NDSB$	$OB < NDSB \leq DSB$
$P = [1/2, 1]$		$OB \sim DSB < NDSB$

TABLE 4: THE POOREST CLASS AND ITS INCOME DENSITY UNDER ALTERNATIVE
VOTING SYSTEMS AS A FUNCTION OF THE TAX RATE

(The income densities are expressed as a percentage of the
full-employment communal income $\bar{F} = k^{1-a}(1-c)^a$.)

	Poorest Class	Income Density of Poorest Class	Fairness	Poorest Class	Income Density of Poorest Class	Fairness	Poorest Class	Income Density of Poorest Class	Fairness	Poorest Class	Income Density of Poorest Class	Fairness	Poorest Class	Income Density of Poorest Class	Fairness
OB	L	$\frac{t}{1-c}$	No	L	$\frac{t}{1-c}$	No	L	$\frac{t}{1-c}$	Yes	L	$\frac{t}{1-c}$	Yes	C	$\frac{1-t}{c}$	Yes
DSB	P	$\frac{t}{1-c}$	No	P	$\frac{t}{1-c}$	No	P	$\frac{t}{1-c}$	Yes	C	$\frac{1-t}{2c(1-c)}$	No	C	$\frac{1-t}{2c(1-c)}$	No
NDSB	L	$\frac{a+t(1-a)}{1-c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	No	C	$\frac{(1-t)(1-a)}{c}$	No	C	$\frac{(1-t)(1-a)}{c}$	No
t	$0 \leq t < 1 - \frac{c}{1-a}$			$1 - \frac{c}{1-a} < t < \frac{(1-a)(1-c)}{c+(1-a)(1-c)}$			$\frac{(1-a)(1-c)}{c+(1-a)(1-c)} < t < \frac{1}{1+2c}$			$\frac{1}{1+2c} < t < 1-c$			$1-c < t < 1$		
OB	L	$\frac{t}{1-c}$	No	L	$\frac{t}{1-c}$	No	L	$\frac{t}{1-c}$	Yes	C	$\frac{1-t}{c}$	Yes			
DSB	L	$\frac{a+t(1-a)}{1-c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	No	C	$\frac{(1-t)(1-a)}{c}$	No			
NDSB	L	$\frac{a+t(1-a)}{1-c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	Yes	C	$\frac{(1-t)(1-a)}{c}$	No	C	$\frac{(1-t)(1-a)}{c}$	No			
t	$0 \leq t < 1 - \frac{c}{1-a}$			$1 - \frac{c}{1-a} < t < \frac{(1-a)(1-c)}{c+(1-a)(1-c)}$			$\frac{(1-a)(1-c)}{c+(1-a)(1-c)} < t < \frac{1}{1+2c}$			$1-c < t < 1$					

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