An Iterative Interpolated DFT to Remove Spectral Leakage in Time-Domain Near-Field Scanning

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Abstract—Time-domain near-field scanning is gaining more and more interest within EMC engineering to analyse electromagnetic near-fields of e.g. quasi-stationary devices. When using a digital oscilloscope to scan the near-fields of an electronic device, the oscilloscope measures time-domain signals that comprise in most cases a large number of frequency components. For many of these components, non-coherent sampling occurs, resulting in spectral leakage when calculating the frequency spectrum of the time-domain signals with the Discrete Fourier Transform. This paper proposes an improved method to detect the presence of such non-coherently sampled signals as well as an iterative algorithm to obtain accurate approximations of all the frequency components with their accompanying amplitudes and phase angles. The algorithm excels over existing algorithms in obtaining these values especially in situations where several sinusoidal components are close to each other in the spectrum. This is achieved thanks to an iterative process of removing the influence of the multiple sinusoidal components on each other. This paper contains the mathematical description of the algorithm and a numerical example evaluating the accuracy of the algorithm. The algorithm has a higher accuracy than the existing approaches e.g. multipoint IpDFTs, with only a slight increase of the computational cost.

Index Terms—Near-Field Scanning, Spectral Leakage, Discrete Fourier Transform, Frequency estimation, Amplitude estimation, Phase estimation.

I. INTRODUCTION

USING a Discrete Fourier Transform (DFT) to calculate the frequency spectrum of a sampled time-domain signal is widely used due to its simplicity and ease of implementation [1]. Interpreting the frequency spectrum with the DFT works fine when coherent sampling is used, i.e. the sampled data contain exactly an integer number of periods of the signal [2]. In case of non-coherent sampling, spectral leakage occurs. The energy from the fundamental spectral line is spread into neighbouring frequency bins. Dedicated algorithms are needed to obtain the correct frequencies, amplitudes and phase angles of the sinusoidal components in the signal [3]–[7].

Obtaining coherent sampling is difficult or even impossible when using a time-domain based measurement device

(oscilloscope) for Electromagnetic Interference (EMI) near-field scanning [8], [9]. The frequencies at which the electronic device unintentionally radiates are usually not known in advance. Exact values of the frequencies, the corresponding amplitudes and phases of the frequency components are however required for post-processing algorithms, such as a near-to-far-field transformation [10]–[16]. An additional problem arises when the spectral leakage patterns, originating from different sinusoidal components, start to overlap. This cannot be avoided by increasing the sampling frequency F_s , while maintaining the total sampling interval (i.e. the measurement time), because the resolution of the frequency spectrum stays the same. Only by increasing the number of samples N and consequently increasing the total measuring interval $\Delta T = \frac{N}{F_s}$ (in s), the frequency resolution is increased, which requires more memory and time.

Several algorithms [4], [6], [7], [17]–[19] exist, with the focus on single sinusoidal components, neglecting the spectral leakage of other components. Anyhow, they serve as a state of the art to evaluate the iterative algorithm. The comparison with windows is not made in this paper, since this has been done in [7]. Here, the algorithm proposed in [7] is significantly improved in four ways: (I) in finding the multiple frequency components, (II) by using less assumptions in the analytical solution for the amplitude and phase, (III) by iteratively removing the impact from other spectral leakage patterns, instead of assuming a negligible influence and (IV) iteratively searching for other hidden frequency components.

This paper is organized as follows. In Section II the analytical solution and the behaviour of the real and imaginary part of the DFT is recapitulated. From this solution, the equation to calculate the frequency f and improved equations for amplitude A and phase θ are derived. Section III describes an improved non-iterative version of the algorithm [7] and a new iterative algorithm. An evaluation of the different algorithms is given in Section IV. Finally, Section V draws concluding remarks. In what follows, $\Re$ and $\Im$ are used for the real and imaginary part, respectively. $\bar{X}$ represents the complex conjugate of X and X^* represents the estimated value of X . $\lfloor X \rfloor$ and $\lceil X \rceil$ are the floor and ceiling of X , respectively.

II. SPECTRAL LEAKAGE

The spectral leakage phenomenon occurs due to a mismatch between the measuring interval and an integer number of periods of the sinusoidal components in the signal. This mismatch does not lead to a discretisation error, but it leads to an analysis error.

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The proposed algorithm uses the spectrum, which may contain this error, to obtain accurate information about the frequencies, amplitudes and phase angles of the sinusoidal components in the measured signal. The signal $x(l)$, e.g. a time-discrete signal (1), consists of M different cosines having their own frequency f (in Hz), amplitude A (in the unit of the physical quantity, e.g. V/m or A/m in NF-scanning) and phase angle θ (in rad). The signal $x(l)$ has a measuring interval $\Delta T = \frac{N}{F_s}$ (in s) and contains N samples (l ranges from 0 to $N-1$). The signal can be described in its exponential form with $f_m = -f_{-m}$, $A_m = A_{-m}$ and $\theta_m = -\theta_{-m}$.

$$x(l) = \sum_{\substack{m=-M \\ m \neq 0}}^M \frac{A_m}{2} e^{j\theta_m} e^{j2\pi f_m l \frac{\Delta T}{N}} \quad (1)$$

There is no specific relation between the different frequencies. Each non-coherently sampled sinusoidal component leads to spectral leakage. Using this spectral leakage pattern, containing the real and the imaginary parts, an accurate approximation of the frequency, amplitude and phase can be obtained. In what follows the analytical result for a DFT on the input signal (1) is given.

A. Analytical Spectrum

Applying the amplitude corrected DFT (2) on the input signal (1) results in the frequency spectrum $X(k)$ with k ranging from $-\lceil \frac{N-1}{2} \rceil$ to $\lfloor \frac{N-1}{2} \rfloor$.

$$X(k) = \frac{1}{N} \sum_{l=0}^{N-1} x(l) e^{-j2\pi \frac{kl}{N}} \quad (2)$$

The integers k correspond with their frequencies $f = \frac{k}{\Delta T}$ and thus k represents the number of periods in the measuring interval ΔT . The sinusoidal input frequencies f_m correspond to their number of periods a_m (3).

$$a_m = f_m \Delta T \quad (3)$$

In the case of coherent sampling a_m is integer and will coincide with a certain k . Non-coherent sampling leads to a non-integer a_m . In what follows k_m and $k_m + 1$ are the k values that enclose the correct number of periods a_m (4).

$$k_m = \lfloor a_m \rfloor < a_m < k_m + 1 = \lceil a_m \rceil \quad (4)$$

Using (3), the input signal becomes, with $a_m = -a_{-m}$:

$$x(l) = \sum_{\substack{m=-M \\ m \neq 0}}^M \left(\frac{A_m}{2} e^{j\theta_m} e^{j2\pi \frac{a_m l}{N}} \right) \quad (5)$$

Applying the DFT (2) on the input signal (5) and defining $\omega_m(k)$ as $e^{j\pi \frac{a_m - k}{N}}$ results in the complete spectrum $X(k)$:

$$X(k) = \frac{1}{N} \sum_{l=0}^{N-1} \left(\sum_{\substack{m=-M \\ m \neq 0}}^M \left(\frac{A_m}{2} e^{j\theta_m} (\omega_m(k))^{2l} \right) \right) \quad (6)$$

Fig. 1: Asymptotic behaviour [7]

Because $X(k)$ does not only consists out of positive components, m ranges from $-M$ to M without 0. The separate frequency components $X_m(k)$, negative or positive, are described by:

$$X_m(k) = \frac{1}{N} \sum_{l=0}^{N-1} \frac{A_m}{2} e^{j\theta_m} (\omega_m(k))^{2l} \quad (7)$$

Using the Euler equation and trigonometric equations (as in [7]), (7) leads to:

$$X_m(k) = \frac{A_m e^{j\theta_m}}{2N} \frac{\sin(\pi(a_m - k))}{\sin(\pi \frac{a_m - k}{N})} \omega_m(k)^{N-1} \quad (8)$$

When $a_m = k$ in (8) the term $\frac{\sin(\pi(a_m - k))}{\sin(\pi \frac{a_m - k}{N})}$ goes to N according to L'Hôpital's rule.

B. Asymptotic behaviour

In order to determine a_m , A_m and θ_m either the positive terms and/or the negative terms in the spectrum can be used. There are two methods to find the different sinusoidal components:

- 1) Finding local maxima in $|X(k)|$. When using the local maxima technique, k_{max} (k at a local maximum) is found. The correct order is situated between $k_{max} - 1$ and $k_{max} + 1$ as described in [20].
- 2) Using the asymptotic behaviour (introduced in this paper) combined with sign flips (introduced in [7]) of the real and/or the imaginary part of the spectrum, results in finding the frequency bins above and below the correct frequency (4).

In this subsection the asymptotic behaviour of the real and imaginary part of (8) is discussed (a picture of the asymptotic behaviour is shown in Fig. 1. It shows X containing two sinusoids with the correct orders a_m and a_n . It also shows the separate and combined real values of X surrounding a_m and a_n).

The asymptotic behaviour of the spectrum can only be studied using the real and/or imaginary part and not $|X(k)|$. The

TABLE I: Sign pattern surrounding a_m

γ	$k \leq k_m$	$k \geq k_m + 1$
$-\frac{\pi}{2} < \gamma < \frac{\pi}{2}$	$\Re(X_m(k)) > 0$	$\Re(X_m(k)) < 0$
$\frac{\pi}{2} < \gamma < \frac{3\pi}{2}$	$\Re(X_m(k)) < 0$	$\Re(X_m(k)) > 0$
$0 < \gamma < \pi$	$\Im(X_m(k)) > 0$	$\Im(X_m(k)) < 0$
$\pi < \gamma < 2\pi$	$\Im(X_m(k)) < 0$	$\Im(X_m(k)) > 0$

first characteristic of an asymptote is the sign flip. All possible sign flips are shown in Table I, using $\gamma = \pi(a_m - k_m) + \theta_m$.

In Table I, four situations are not mentioned. When γ equals $\frac{\pi}{2}$ or $\frac{3\pi}{2}$, the real part of $X_m(k)$ equals zero and only the imaginary part of $X_m(k)$ is usable to find sign flips. When γ equals 0 or π , the imaginary part of $X_m(k)$ equals zero and only the real part of $X_m(k)$ is usable to find sign flips. Since it does not occur that the real and the imaginary part both equal zero at the same time, every sign flip can be found.

When multiple spectral leakage patterns overlap, e.g. due to multiple frequency components, it could happen that other sign flips occur in the spectrum. A second condition has to be satisfied to allow the sign flip to be an asymptote. These conditions are shown in Table II, using the definitions of $\Delta X_r(k)$ and $\Delta X_i(k)$ given in (9a) and (9b).

$$\Delta X_r(k) = \Re(X(k)) - \Re(X(k-1)) \quad (9a)$$

$$\Delta X_i(k) = \Im(X(k)) - \Im(X(k-1)) \quad (9b)$$

TABLE II: Asymptote pattern surrounding a_m

γ	$k = k_m$	$k = k_m + 2$
$-\frac{\pi}{2} < \gamma < \frac{\pi}{2}$	$\Delta X_r(k) > 0$	$\Delta X_r(k) > 0$
$\frac{\pi}{2} < \gamma < \frac{3\pi}{2}$	$\Delta X_r(k) < 0$	$\Delta X_r(k) < 0$
$0 < \gamma < \pi$	$\Delta X_i(k) > 0$	$\Delta X_i(k) > 0$
$\pi < \gamma < 2\pi$	$\Delta X_i(k) < 0$	$\Delta X_i(k) < 0$

In Section III these conditions are implemented and further explained.

C. Calculating the correct number of periods a_m

In order to create an analytical expression for a_m , two assumptions are made to rewrite (8) as (10). First, $1 - \frac{1}{N} \approx 1$ when N is large. Secondly, $\sin\left(\pi \frac{a_m - k}{N}\right) \approx \pi \frac{a_m - k}{N}$ when $(a_m - k)$ is significantly smaller than N , both conditions are satisfied in the region of interest.

$$X_m(k) \approx \frac{A_m}{2} \frac{\sin(\pi(a_m - k))}{\pi(a_m - k)} e^{j(\pi(a_m - k) + \theta_m)} \quad (10)$$

Once the region, k_m and $k_m + 1$, of the correct order a_m is obtained (using Table I and II), the correct order a_m can be estimated. This can be verified by using (10) in (11).

The weighted average of the orders k_m and $k_m + 1$ with the norms of the components $|X_m(k_m)|$ and $|X_m(k_m + 1)|$ leads to a very accurate approximation of a_m . This is equal to the well known two-point IpDFT for rectangular windows

[21]. The approximation of a_m is called a_m^* , leading to the estimated frequency f_m^* using (3).

$$a_m^* = \frac{|X_m(k_m)|k_m + |X_m(k_m + 1)|(k_m + 1)}{|X_m(k_m)| + |X_m(k_m + 1)|} \quad (11)$$

D. Calculating A_m and θ_m

Equation (8) can be converted to equation (12).

$$A_m e^{j\theta_m} = \frac{2X_m(k)N}{\omega_m(k)^{(N-1)}} \frac{\sin\left(\frac{\pi(a_m - k_m)}{N}\right)}{\sin(\pi(a_m - k_m))} \quad (12)$$

The norm of (12) leads to the amplitude. To obtain θ_m , one must take the imaginary part of the natural logarithm of (12) divided by the amplitude. Using the correct order a_m and the measured spectral line $X_m(k_m)$ or $X_m(k_m + 1)$ in (12), (13a) and (13b) are obtained. (13b) is preferred when $|X_m(k_m)| < |X_m(k_m + 1)|$.

$$A_m = 2N|X_m(k_m)| \frac{\left|\sin\left(\frac{\pi(a_m - k_m)}{N}\right)\right|}{\left|\sin(\pi(a_m - k_m))\right|} \quad (13a)$$

$$A_m = 2N|X_m(k_m + 1)| \frac{\left|\sin\left(\frac{\pi(a_m - k_m - 1)}{N}\right)\right|}{\left|\sin(\pi(a_m - k_m - 1))\right|} \quad (13b)$$

Again using the correct order a_m and the measured spectral line $X_m(k_m)$ or $X_m(k_m + 1)$ in (12) results in (14a) and (14b). (14b) is preferred when $|X_m(k_m)| < |X_m(k_m + 1)|$.

$$\theta_m = \Im \left(\ln \left(\frac{X_m(k_m)N}{\frac{A_m}{2} \frac{\sin(\pi(a_m - k_m))}{\sin\left(\frac{\pi(a_m - k_m)}{N}\right)} \omega_m(k)^{(N-1)}} \right) \right) \quad (14a)$$

$$\theta_m = \Im \left(\ln \left(\frac{X_m(k_m + 1)N}{\frac{A_m}{2} \frac{\sin(\pi(a_m - k_m - 1))}{\sin\left(\frac{\pi(a_m - k_m - 1)}{N}\right)} \omega_m(k)^{(N-1)}} \right) \right) \quad (14b)$$

Since (8) instead of (10) is used as the basis for equations (13) and (14), they differ from the ones used in [4] and [7], resulting in the exact expression for A_m and θ_m . Note that when the approximation a_m^* instead of the correct a_m is used in (13) and (14), the result is the approximations A_m^* of A_m and θ_m^* of θ_m .

E. Multiple frequencies

The influence of different spectral leakage patterns is always present. Even if only one component $M = 1$ is present in $x(l)$, multiple spectral leakage patterns occur and influence each other in $X(k)$. More specifically one component in $x(l)$ results in a set of negative $m = -1$ and positive $m = 1$ frequency components. If this single component is non-coherently sampled the negative spectral leakage components can influence the positive components and vice versa. For an indication of the effect of the negative frequency on the positive frequency, a closer look is taken at the error of a_m^* using (11), on the full spectrum (with $m = -1$ and $m = 1$) and on the positive spectrum (with only $m = 1$). Using $N = 933$,

Fig. 2: Error on estimated a_m Fig. 3: Error on A_m and θ_m

$F_s = 500\text{Hz}$, $A_m = 1$ and $\theta_m = \frac{\pi}{2}$, let f_m range from 1 to 100Hz . Fig. 2 shows the absolute error of a_m^* on a_m for the full spectrum and for the positive spectrum only. The error increases, in comparison with the positive spectrum only, with a factor of 10^5 for the full spectrum. Moreover the error increases when f_m decreases because its negative and positive frequency (spectral leakage pattern) are closer to each other. Note that a spectrum with only positive (or only negative) frequency components, as shown in this example, does not exist.

A similar error behaviour, shown in Fig. 3, can be seen in the calculation of A_m and θ_m . In this figure the exact a_m is used to calculate A_m and θ_m which results in an exact (only a discretization error exists in some cases) result for the positive spectrum.

The effect of two frequencies on each other will be similar as with its own negative and positive frequency. This shows that the influence of the multiple spectral leakage patterns do have an effect on each other and a method is needed to remove the influence of the multiple leakage patterns.

III. ALGORITHM

This section discusses the various steps of the non-iterative and iterative algorithm. Both algorithms start by searching the different k_m and $k_m + 1$ using the asymptotic behaviour. When a_m^* is calculated using k_m and $k_m + 1$, the accompanying f_m^* , A_m^* and θ_m^* can be calculated. This section contains three main parts: the search technique, the non-iterative and the iterative algorithm.

A. Search technique

Searching for the asymptotic behaviour in $\Re(X(k))$ or $\Im(X(k))$ is faster than searching for a local maximum. To find the different k_m , the following steps are required:

- 1) Search for sign flips in $\Re(X(k))$ and $\Im(X(k))$. The index is called k_m (the bin before a_m).
- 2) Since not every sign flip indicates a sinusoidal component in the spectrum (sign flips can be caused by noise for example), the asymptotic behavior has to be checked. For every sign flip in $\Re(X(k))$ or $\Im(X(k))$, the k values surrounding k_m have to fulfil the conditions in Table II. Exceptions can occur when two adjacent components are very close to each other. A first exception is when the second condition (Table II) of asymptotic behaviour is no longer valid, but the sign flip, on the other hand, stays present. A specific piece of code has been added to detect 3 sign flips over 2 bins to indicate two very close components. The first and the third sign flip are then used for further processing. A second exception happens if one of the two very close components is much larger than the other one. In this case it could happen that the sign flip of the smaller component doesn't occur and the search technique only finds one component. To solve this an extra step has been added to the iterative algorithm: by removing the large component, the sign flip of the smaller component can be found (step 7 in the explanation of the iterative algorithm in section III-C).

This technique works for the positive as well as the negative frequencies. The non-iterative algorithm only uses the positive frequencies, while the iterative algorithm uses both.

B. Non-iterative improvement of [7]

Once k_m is found, a_m^* is calculated using (11). (13) and (14) are used to calculate A_m^* and θ_m^* . These calculations are only done using the positive a_m^* . The remaining errors exist due to:

- 1) The discretization error (This error is very low and invisible in the evaluation of the algorithm in Section IV).
- 2) The influence of the multiple frequency components on each other. Note that the negative frequencies can have an influence on the positive frequencies and vice versa.
- 3) a_m^* is an approximation of a_m which introduces errors.

The algorithm is performed twice, once with N data points and once with $N - 1$ data points, to ensure that every sinusoidal component leads to spectral leakage.

C. Iterative Algorithm

The major point of improvement is discussed in this subsection and tries to eliminate the influence of different spectral leakage patterns on each other. The increased accuracy when calculating a_m^* , A_m^* and θ_m^* is obtained by estimating and removing the influence of the separate spectral leakage patterns on each other.

The flowchart of the iterative algorithm is shown in Fig. 4, providing the reader with a better understanding of the algorithm.

The initial steps are the same as in Section III-B but instead of only finding k_m for the positive frequencies, the k_m for the negative frequencies are also sought and used to calculate a_m^* , A_m^* and θ_m^* . The number of k_m values should always be even. If there is an extra k_m found in only either the positive or negative range, the wrong k_m should be deleted (there should always be a negative k_m having the same absolute value as a positive k_m).

The iterative algorithm contains several steps following Fig. 4:

- 1) All sign flips are checked for the condition of an asymptote, resulting in a number of k_m indicating M^* different frequency components in $x(l)$.
- 2) For all k_m found, the quantities a_m^* , A_m^* and θ_m^* are calculated as in Section III-B.
- 3) The iterative algorithm starts by executing an inner loop for every k_m found with index i .
- 4) To find a better approximation of a_m^* , A_m^* and θ_m^* , all other $X_m(k_{i \neq m})$ and $X_m(k_{i \neq m} + 1)$ are excluded from the original data X using equations (15a) and (15b). This is done for both the positive as the negative components. If only the positive components would be used in the iterative algorithm, then the spectral leakage contribution from the negative bins could never be removed from the signal parameters that are estimated. This would result in a residual error which could never be removed by the iterative process.
- 5) This approach results in $X^{cor}(k_m)$ and $X^{cor}(k_m + 1)$, where the influence of all other components on the parameters a_m^* , A_m^* and θ_m^* , calculated as in Section III-B, is reduced. These values are directly used in the next calculation of the following k_m . Because the inner loop starts with negative components, the remaining error decreases faster for the positive components than for the negative components. The positive components will eventually be used as the output of the algorithm.
- 6) After one run on all the k_m ; a_m^* , A_m^* and θ_m^* are used to calculate X^{cor} (15c). This X^{cor} is the original data X excluding the first approximation of the M^* frequency components found.
- 7) X^{cor} is now checked for any remaining sign flips with the condition of an asymptote. If new k_m are found different from the original k_m , this original set is expanded with the new k_m and results in a new larger set of M^* . This step makes sure that small multiple signals close to each other are found. It might happen that the spectral leakage coming from one sinusoid masks other

Fig. 4: Improved Algorithm

sign flips and their asymptotic behaviour.

This process is re-executed until a_m^* converges. The convergence of a_m^* implies convergence of A_m^* and θ_m^* , because a_m^* is used to calculate A_m^* and θ_m^* . In this iterative algorithm, a_m^* is converged when the relative difference between the new and the previous a_m^* is smaller than a given value. Instead of waiting for convergence a fixed number of repetitions can be set, in the present evaluation this number of repetitions is set to 10. The remaining error on the results only originates from the assumptions used to obtain (11) and using a_m^* in the calculation of A_m and θ_m .

$$X^{cor}(k_m) = X(k_m) - \sum_{\substack{i=-M^* \\ i \neq m, i \neq 0}}^{M^*} X_i(k_m) \quad (15a)$$

$$X^{cor}(k_m + 1) = X(k_m + 1) - \sum_{\substack{i=-M^* \\ i \neq m, i \neq 0}}^{M^*} X_i(k_i + m) \quad (15b)$$

$$X^{cor} = X - \sum_{m=-M^*}^{M^*} X_m \quad (15c)$$

A similar algorithm [22] has already been presented for multiple non-coherent frequency components. Though, in [22] a different (non-iterative) search technique is used and a non-iterative algorithm is proposed. The accuracy of this algorithm would be similar with this iterative algorithm if only one single iteration would be performed. In [23] an iterative procedure has also been described for exponential signals, which is not convertible for the application of non-exponential inputs. If in [23] $\alpha = 0$ (infinite sinusoid) the equations (10) and (11) would not be the same closed formulations as the formulas (13) and (14) presented in this paper. The algorithm described in [23] would therefore never reach the accuracy of the iterative algorithm.

IV. EVALUATION

Similar as in [7] an example with two very close frequency components on two different tracks on a PCB can be used as an evaluation. This configuration is simulated in EMPRO [24] with two sources ($S1$, $S2$) on different tracks on a PCB of 10 cm by 10 cm (Fig. 5). The sources have two different frequency components with $f_1 = 100 \text{ MHz}$ and $f_2 = 100.120 \text{ MHz}$, each with the same amplitude.

Two simulations, one for each frequency, are performed resulting in the correct amplitudes for the electric field z-component $E_z^1(y)$ and $E_z^2(y)$ and phases $\theta^1(y)$ and $\theta^2(y)$ along the y axis at a height of 5mm. The correct time-domain near-field signals are then calculated using the following equation:

$$E_z(l, y) = E_z^1(y) \cos(2\pi f_1 l + \theta^1(y)) + E_z^2(y) \cos(2\pi f_2 l + \theta^2(y)) \quad (16)$$

This near-field time-domain signal (16), along the y axis, is sampled at 500 Msamples/s with a total measurement length of $c * \frac{1}{(f_2 - f_1)}$ with $c = 3.5$. Here, c determines the number

Fig. 5: Test PCB

Fig. 6: Results on $|E_z^1|$

of bins between the two given frequency components. The minimum value of c for the new iterative algorithm is 3.5. This is because the new iterative algorithm needs a minimum of 3 bins between two frequency components. Also a minimum of 3 bins above the DC component and 3 bins under the Nyquist frequency are recommended. With $c = 3.5$ neither f_1 nor f_2 are coherently sampled. The iterative algorithm presented in Section III-C and the non-iterative algorithm from Section III-B used in [7] are compared with other algorithms. The other algorithms are indicated with 'Jain' for the algorithm described in [4], 'Agrez2' for two point interpolation, 'Agrez3' for three point interpolation and 'Agrez5' for five point interpolation described in [6], [17], [18].

The correct amplitudes $|E_z^1|$ for f_1 and phase, for each

Fig. 7: Results on $|E_z^2|$

frequency, along the y-axes are calculated in two FDTD simulations (one for each frequency)

In Fig. 6 and Fig. 7 the results from the iterative algorithm and the 3-point IpDFT from [17] are shown (other algorithms are not shown in these figures to keep a good perceptibility). The iterative algorithm clearly shows correct results over the complete plot. The 3-point IpDFT also follows the correct values except on the place where the track of the other frequency is present. At these places the IpDFT is not able to distinguish both sinusoids from S_1 and S_2 . Note that when an algorithm only finds one peak instead of two the result is the same for both frequencies. Hence, the electric field z-component shows two PCB-tracks for both frequencies for the 3-point IpDFT.

The errors of all the algorithms (in % compared with the correct values) are shown in Fig. 8a and Fig. 8b. The iterative algorithm shows exceptionally good results compared with the other algorithms.

Fig. 9: Results on $Arg(E_z^1)$ Fig. 10: Results on $Arg(E_z^2)$

In Fig. 9 and 10 the results from the iterative and the 3-point IpDFT from [17] are shown (again other algorithms are not shown in these figures to keep a good perceptibility). It is clearly shown that the iterative algorithm follows the correct values.

The errors of all the algorithms (in radians compared with the correct values) are shown in Fig. 11a and 11b. The iterative algorithm shows exceptionally good results compared with the other algorithms.

V. CONCLUSION

In case of non-coherent sampling, spectral leakage occurs when calculating a DFT. Especially when the signal contains multiple frequency components close to each other, it is a challenge to calculate the correct frequencies, amplitudes and phases for all components. In this paper an iterative method has been described for: (i) finding the frequency bins around a non-coherently sampled signal, and (ii) calculating the parameters of this signal. An iterative algorithm was proposed that outperforms other algorithms not only when two sinusoidal components are sufficiently far away from each other but especially, as shown in Section IV, when they are close to each other. The new algorithm outperforms others by reducing the relative amplitude error with a factor 10^4 to 10^5 and a phase error under 10^{-6} radians, succeeding in the presented challenge. The search technique that searches for asymptotes can provide the sinusoidal components faster than the local maxima search. The iterative algorithm enables the user to find sources of radiation, which are very close in the spectrum, in the near-field of a PCB. Not using this algorithm would lead to a wrong interpretation of the near-field. In the example of Section IV using another algorithm than the iterative algorithm, presented in this paper, would result in the wrong conclusion that both tracks radiate a single frequency of around 100 MHz.

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Fig. 8: Relative amplitude errors on (a) $|E_z^1|$ and (b) $|E_z^2|$ Fig. 11: Phase errors on (a) $\arg(E_z^1)$ and (b) $\arg(E_z^2)$

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