Continual Learning with Quasi-Newton Methods

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Abstract—Catastrophic forgetting remains a major issue when neural networks learn tasks sequentially. To alleviate catastrophic forgetting, Elastic Weight Consolidation (EWC) introduces a regularization loss, which, based on Bayesian inference, considers the posterior distribution over the previous tasks. However, when estimating the posterior with Laplace approximation, EWC approximates the Hessian by the diagonal of the Fisher information matrix, assuming the model parameters to be approximately independent. Very rarely will this be the case, and as such, the estimation of the Hessian and, consequently, of the posterior might be very poor. To overcome this, we propose Continual learning with Sampled Quasi-Newton (CSQN), which considers Quasi-Newton methods to obtain better Hessian approximations. Similar to EWC, CSQN uses these Hessian approximations to estimate the posterior and, as such, to encourage the model to update the model parameters in such a way that they remain optimal to both the current and previous tasks. Experimental results show that CSQN outperforms EWC as well as other baselines.

I. INTRODUCTION

Since the 2010s, in the so-called Deep Learning Revolution, Artificial Neural Networks (ANN) have been able to match or even surpass the performance of humans on a wide variety of tasks. Yet when presented with a set of tasks, to be learned sequentially - a setting which is referred to as Continual Learning (CL) - ANNs suffer from catastrophic forgetting [1]. Unlike humans, they seem unable to extend their knowledge without forgetting what they learned previously. Naively adapting the ANN to a new task generally leads to a deterioration of the performance of the network on previous tasks.

To overcome catastrophic forgetting, many CL methods have been proposed. One of the most well-known is Elastic Weight Consolidation (EWC) [2], which looks at CL from a Bayesian point of view. After training a task, it uses Laplace approximation [3] to estimate a posterior for the model parameters with respect to this task. When training a next task, this posterior is used by means of a regularization loss to prevent the model from catastrophically forgetting the previous task. To estimate the Hessian, which is needed in Laplace approximation to measure the (un)certainly of the model parameters, EWC uses the Fisher Information Matrix (FIM). Moreover, to simplify, EWC assumes that the FIM is approximately diagonal. Consequently, EWC uses the diagonal of the FIM to estimate the Hessian.

While EWC might often be very effective, its assumption of an approximately diagonal FIM (and, consequently, Hessian) will almost never hold. As a result, EWC's Hessian

approximation might be poor and, consequently, so will be its approximation of the posterior, which could severely hurt EWC's performance in Continual Learning.

To improve EWC's approximation of the Hessian, we use Quasi-Newton (QN) methods [4]. QN methods, being second-order optimization algorithms, involve (inverse) Hessian approximations which are updated at each iteration (but not computed explicitly) to find local optima. Two QN methods are BFGS (Broyden, Fletcher, Goldfarb, Shanno) and SR1 (Symmetric Rank-1), which apply a rank-2 and rank-1 update, respectively, to the Hessian approximation at each iteration. As we are only interested in QN methods because of their Hessian approximations, it would be disadvantageous would we have to train the model with the QN methods in order to obtain these approximations. Therefore, we propose to use Sampled Quasi-Newton (SQN) methods [5]. In SQN methods, the Hessian approximation is obtained by sampling around the current estimate for the local optimum. As such, the Hessian is re-computed at each iteration from scratch and is not based on previous iterations. Moreover, to further improve our method and to make it a literal extension to EWC, we use EWC's estimate of the Hessian, namely the diagonal of the FIM, as the initial approximation of the Hessian. Using SQN, we then improve this Hessian approximation, which is no longer diagonal. Similar to EWC, this improved Hessian approximation is used in the regularization loss to prevent forgetting of previously acquired knowledge when learning new tasks.

This paper is structured as follows. We start by reviewing related work in Section II. Next, we move on to Section III, in which we explain our proposed method (Section III-D) after a short review of EWC (Section III-B) and (Sampled) Quasi-Newton methods (Section III-C). In Section IV, we present the experimental results and compare our method to EWC and other baselines. Finally, we conclude the paper with Section V.

II. RELATED WORK

In general, there exist three large classes of CL methods: regularization-based methods, rehearsal-based methods and parameter-isolation methods [6].

Regularization-based. To prevent catastrophic forgetting, regularization-based methods regularize the learning of new tasks. Following [6], two classes of regularization-based methods are the prior-focused methods and data-focused methods.

EWC, using the diagonal of the FIM to estimate the importance of parameters to previous tasks, belongs to the prior-focused methods. Similar to EWC are Synaptic Intelligence (SI) [7] and Memory-Aware Synapses (MAS) [8], which compute the importance weights of the model parameters differently. While SI estimates the importance of parameters by accumulating each parameter's contribution to the reduction in the loss, MAS considers the gradients of the squared norm of the model's output with respect to each parameter. Similar work includes [9], [10], [11].

Many works have built on EWC's ideas. Online EWC [12] allows EWC to be used in an online way, without requiring task boundaries. [13] proposes Riemannian Walk, which combines the regularization losses of Online EWC and SI. Moreover, to address EWC's assumption of an approximately diagonal FIM, R-EWC (Rotated-EWC) [14] rotates the parameter space layer-wise such that the corresponding FIM is indeed approximately diagonal. While R-EWC improves the performance of EWC when adapting one task to another, R-EWC only applies the reparametrization to the previous task and has no mechanism to combine the importance weights of all tasks learned so far. Meanwhile, [15] proposes to use Kronecker factored approximations of the FIM [16], which allows to compute and store the FIM efficiently for each layer, thus obtaining a block diagonal approximation of the FIM. As such, unlike EWC, they consider interactions between model parameters within the same layer, but not across layers.

A second class of regularization-based methods are the data-focused methods, whose main idea is knowledge distillation [17], to transfer knowledge from the previous to the new model. Examples include Learning Without Forgetting (LWF) [18], but also [19], [20].

Instead of using a regularization loss, other methods constrain the gradients when learning new tasks. This includes Orthogonal Gradient Descent (OGD) [21], which updates the gradient used in Stochastic Gradient Descent (SGD) such that it is orthogonal to the gradients of previous tasks; Gradient Projection Memory (GPM) [22], which partitions the gradient space into two orthogonal subspaces, forcing the gradient to lie in the subspace least harmful to previous tasks; and the work of [23], [24].

Rehearsal-based. Rehearsal-based methods assume access to a memory of representative samples of previous tasks. Examples include [25], [26]. Gradient Episodic Memory (GEM) [27] can be considered an overlap between rehearsal-based and regularization-based methods [28], as it introduces a regularization loss based on a set of representative samples, assuring SGD's gradient updates do not increase the loss of the representative samples. GEM has been extended in [29], [30]. Similar to GEM is [31].

Within the rehearsal-based methods, the pseudo-rehearsal methods, instead of requiring access to a memory of representative samples, train a generator to simulate data of previous tasks. Examples include [32], [33], [34].

Parameter-isolation. Parameter-isolation methods expand the network when tasks are added, to avoid interference between tasks. Examples include [35], [36], [37], [38], [12]. The latter proposes Progress & Compress (P&C) [12], which uses

online EWC in the 'progress' stage. Another method on the intersection between parameter-isolation and regularization-based methods is AR1 [28], which combines Copy Weight with Reinit (CWR) [39] with SI and EWC.

III. CONTINUAL LEARNING WITH SAMPLED QUASI-NEWTON

In this section, we explain our proposed CL method. However, we start this section by the problem formulation, followed a review of EWC and (Sampled) Quasi-Newton methods.

A. Problem Formulation

We have a sequence of T tasks represented by data $D_1, D_2, \dots, D_T$ with, for $(x, y) \in D_i$, $x \in \mathbb{R}^d$ is the input to the model and y the class label. The tasks are learned in sequence and, during training of any task t , the model has access to D_t only. The goal is to learn task t without forgetting the knowledge extracted from $D_1, D_2, \dots, D_{t-1}$, i.e. if $\theta \in \mathbb{R}^N$ are the model's parameters, and $\theta^{(t)}$ are the parameters after learning the first t tasks in sequence, then we seek: $\theta^{(T)} = \arg \max_{\theta} P(\theta | D_1, D_2, \dots, D_T)$ where $\theta^{(t)}$ is obtained by training $\theta^{(t-1)}$ on D_t without access to $D_1, D_2, \dots, D_{t-1}$.

Moreover, denote by $f_{\theta}(\cdot)$ the model with parameters θ . $p_{\theta}(y|x)$ is its softmax output for class y given input x , while $L_t(\theta)$ is its cross-entropy loss on task t .

B. Elastic Weight Consolidation

EWC looks at CL from a Bayesian point of view. When learning a second task, the optimal parameters $\theta^{(2)}$ can be found as follows:

$$\begin{aligned} \theta^{(2)} &= \arg \max_{\theta} \log P(\theta | D_1, D_2) \\ &= \arg \max_{\theta} \log P(D_2 | \theta) + \log P(\theta | D_1) \end{aligned} \quad (1)$$

This result is obtained by applying Bayes' rule to $P(\theta | D_1, D_2)$ assuming that D_1 and D_2 are independent [2]. The first term on the right hand side of Equation 1 corresponds to optimizing the cross-entropy loss for D_2 , i.e. $L_2(\theta)$; the second term, the posterior after training on task 1, is intractable. However, it can be estimated using Laplace approximation [3]:

$$\log P(\theta | D_1) = -\frac{\lambda}{2} (\theta - \theta^{(1)})^T \Lambda (\theta - \theta^{(1)}) \quad (2)$$

In Equation 2, Λ is the Hessian of $L_1(\theta^{(1)})$. Laplace approximation thus approximates the posterior $P(\theta | D_1)$ by a normal distribution $N(\theta^{(1)}, \Lambda^{-1})$. To approximate the Hessian Λ , EWC considers the FIM, which is defined as:

$$F_{\theta} = \mathbb{E}_{(x,y) \sim D} \left[\left(\frac{\partial \log p_{\theta}(y|x)}{\partial \theta} \right) \left(\frac{\partial \log p_{\theta}(y|x)}{\partial \theta} \right)^T \right] \quad (3)$$

To simplify, moreover, EWC approximates the FIM by its diagonal. Combining Equations 1, 2 and the assumption that the FIM is approximately diagonal, the loss while training task 2 becomes the following:

$$\text{Loss}_2(\theta) = L_2(\theta) + \frac{\lambda}{2} \sum_{i=1}^N \Omega_i^{(1)} \left(\theta_i - \theta_i^{(1)} \right)^2 \quad (4)$$

with $\Omega^{(1)} \in \mathbb{R}^N$ the diagonal of $F_{\theta^{(1)}}$, θ_i the i th parameter of $f_{\theta}(\cdot)$ and λ a hyper-parameter, determining the weight of the regularization. $\Omega_i^{(1)}$ can be seen as an estimate of how important $\theta_i^{(1)}$ is to $f_{\theta^{(1)}}$ with respect to task 1.

To extend EWC to more than two tasks, a regularization loss can be introduced for each term. However, [40] suggests combining these regularization losses in one term. When learning task t , one then obtains:

$$\text{Loss}_t(\theta) = L_t(\theta) + \frac{\lambda}{2} \sum_{i=1}^N \left(\sum_{j=1}^{t-1} \Omega_i^{(j)} \right) \left(\theta_i - \theta_i^{(t-1)} \right)^2 \quad (5)$$

As such, it suffices to store $\sum_{j=1}^{t-1} \Omega^{(j)}$ and $\theta^{(t-1)}$ in order to learn task t with EWC.

While EWC might often alleviate catastrophic forgetting, its assumption that the FIM is approximately diagonal will almost never hold. On the contrary, in many cases the diagonal of the FIM might often capture but a small amount of the total weight. As such, EWC's Hessian approximation might be poor, and, consequently, so will be its approximation to the posterior. By considering better Hessian approximations, we can improve the approximations to the posterior and, therefore, the performance on continual learning.

C. (Sampled) Quasi-Newton Methods

To improve EWC's Hessian approximation, QN methods can be used. However, let us first start by reviewing QN and SQN methods.

Quasi-Newton. Given a function $f(\cdot)$ to be optimized with respect to $x \in \mathbb{R}^n$. Let $f_k = f(x_k)$ with x_k the current solution at iteration k and ∇f_k its gradient. In order to iteratively optimize f , QN methods consider the quadratic model:

$$m_k(p) = f_k + \nabla f_k^T p + \frac{1}{2} p^T B_k p \quad (6)$$

with B_k an estimate of the Hessian at the current solution x_k . QN methods move to the next iteration by taking a step in the direction of the quadratic model's minimizer, $p^* = -B_k^{-1} \nabla f_k$. Next, the Hessian estimate is updated and the process is repeated until convergence.

QN methods differ in how this Hessian estimate is computed. Two well-known QN methods are BFGS and SR1, which respectively apply a rank-two and rank-one update to the Hessian estimate at each iteration. For BFGS, the Hessian estimate at iteration k is computed as follows [4]:

$$B_k = B_0 - \begin{bmatrix} B_0 S_k & Y_k \end{bmatrix} \begin{bmatrix} S_k^T B_0 S_k & L_k \\ L_k^T & -D_k \end{bmatrix}^{-1} \begin{bmatrix} S_k^T B_0 \\ Y_k^T \end{bmatrix} \quad (7)$$

with $S_k = [s_0, s_1, \dots, s_{k-1}]$ and $Y_k = [y_0, y_1, \dots, y_{k-1}]$ where $s_i = x_{i+1} - x_i$ and $y_i = \nabla f_{i+1} - \nabla f_i$. Moreover, D_k is diagonal matrix with $(D_k)_{i,i} = s_i^T y_i$ and L_k is a lower diagonal matrix whose element $(L_k)_{i,j} = s_i^T y_j$ only when $i > j$, else it is zero. Finally, B_0 is an initial estimate for the Hessian, commonly $B_0 = \gamma I$ with I the $n \times n$ identity matrix. Similarly, for SR-1, the Hessian estimate at iteration k can be computed as follows [4]:

$$B_k = B_0 + (Y_k - B_0 S_k)(D_k + L_k + L_k^T - S_k^T B_0 S_k)^{-1} (Y_k - B_0 S_k)^T \quad (8)$$

While it is not guaranteed that the Hessian estimates produced by SR1 are positive definite, SR1 has proven to yield better Hessian estimates than BFGS [4].

Furthermore, to limit the memory requirements of the QN methods, only the M most recent vectors are stored in S_k and Y_k .

Sampled Quasi-Newton. To further improve QN methods, [5] proposes Sampled Quasi-Newton methods. While in QN methods S and Y depend on (s, y) pairs from the current iteration and previous $M - 1$ iterations, SQN methods "forget the past" and, at each iteration, build the S and Y matrices from scratch [5]. As shown in [5], this yields better Hessian approximations. Moreover, as S and Y are obtained from scratch at each iteration, it does not take M iterations until S and Y contain M components, which is the case for QN methods. Once S and Y are sampled, SQN proceed in the same way as QN methods.

To obtain S_k and Y_k at iteration k from scratch, SQN methods sample points $\tilde{x}_i \in \mathbb{R}^n$ with $i = 1, \dots, M$ in the neighbourhood of x_k . Then, they compute $s_i = x_k - \tilde{x}_i$ and $y_i = \nabla f(x_k) - \nabla f(\tilde{x}_i)$ and, similarly to QN methods, store these in $S_k = [s_1, s_2, \dots, s_M]$ and $Y_k = [y_1, y_2, \dots, y_M]$.

The process of obtaining S and Y by sampling is summarized in Algorithm 1. Note that here, $\tilde{x}_i$ is sampled from $N(x_k, \Sigma)$ with Σ a diagonal matrix. Moreover, note that y is computed using $\nabla^2 f(x_k)s$ instead of $y = \nabla f_k - \nabla f(\tilde{x}_i)$. The advantage of the former case is that it is invariant to the scale of Σ , whereas in the latter case the scale of Σ must be carefully chosen.

Algorithm 1 Computation of S and Y in SQN methods [5]

```

1: function COMPUTE_SY( $x_k, M, \Sigma$ )
2:   Initialize  $S, Y \leftarrow [], []$ 
3:   for  $i = 1, \dots, M$  do
4:     Sample  $\tilde{x}_i \sim N(x_k, \Sigma)$ 
5:      $s \leftarrow x_k - \tilde{x}_i$ 
6:      $y \leftarrow \nabla^2 f(x_k)s$ 
7:      $S, Y \leftarrow [S \ s], [Y \ y]$ 
8:   end for
9:   return  $S, Y$ 
10: end function

```

To assure that the Hessian approximations for BFGS are positive definite, an (s, y) pair may only be added to S and Y if $s^T y > \kappa \|s\|^2$ [5]. On the other hand, in order for Equation 8 to be well-defined for SR1, $|s^T(y - Bs)| \geq \kappa \|s\|^2$ must hold for all (s, y) pairs [5]. Therefore, only (s, y) pairs which satisfy the required condition are added to S and Y . Note that this occasionally may result in S and Y containing less than M components. However, [5] shows that, if κ is sufficiently small (e.g. $\kappa = 1e-8$), the condition is extremely rarely violated.

D. CSQN

Let us now combine EWC and SQN into our proposed CL method: Continual learning with Sampled Quasi-Newton (CSQN).

Given Equation 2, we consider SQN methods to find an estimate of the Hessian of $L_1(\theta^{(1)})$, which, unlike in EWC,

does not assume the model parameters to be independent. As SQN methods obtain S and Y at each iteration from scratch, Hessian approximation can be obtained without having to optimize the ANN with a (S)QN method. Given $\theta^{(1)}$, we can thus use Algorithm 1 to sample $S^{(1)}$ and $Y^{(1)}$. Next, Equations 7 and 8 return an estimate of the Hessian, $B^{(1)}$, which can then be used in Equation 2, resulting in the following loss function for training the second task:

$$\text{Loss}_2(\theta) = L_2(\theta) + \frac{\lambda}{2}(\theta - \theta^{(1)})^T B^{(1)}(\theta - \theta^{(1)}) \quad (9)$$

Dropping the superscripts for convenience, note that B can be computed from S , Y and B_0 , hence, it suffices to store these three objects, which combined are $2M + 1$ vectors of size N .

Moreover, as SR1 with M (s, y) pairs results in a rank- M (without the initial Hessian estimate B_0) approximation of the Hessian, it should suffice to store $M + 1$ instead of $2M + 1$ vectors. To this end, we compute $X \in \mathbb{R}^{N \times M}$ and $A \in \mathbb{R}^{N \times N}$ such that we can write Equation 8 as $B = B_0 + XA^{-1}X^T$. Next, we apply the Cholesky factorization to A^{-1} , such that $A^{-1} = LL^T$, which allows us to write $B = B_0 + ZZ^T$ with $Z = XL$. However, the Cholesky factorization requires A to be positive definite. If A is not positive definite, which may be problematic because in that case the regularization loss of Equation 9 might be negative, we can compute the Eigenvalue Decomposition (EVD) of A^{-1} : $A^{-1} = V\Gamma V^T$. Next, we obtain Γ' by setting the negative eigenvalues in Γ to zero, after which we compute the QR-factorization of $(V\sqrt{\Gamma'})^T$ such that $A^{-1} \approx V\Gamma'V^T = (QR)^TQR = R^TR = LL^T$ with $L = R^T$. As such, we find $Z = XL$ and it suffices to store $Z \in \mathbb{R}^{N \times M}$ and $B_0 \in \mathbb{R}^N$. Moreover, it is now assured that the regularization loss for CSQN with SR1 will always be positive.

Algorithm 2 summarizes the procedure of obtaining Z from S and Y .

Algorithm 2 Computation of Z from S and Y

```

1: function COMPUTE_Z_FROM_SY( $S, Y$ )
2:   Compute  $X, A$  from Eq. 8 (or 7) s.t.  $B_0 = B + XA^{-1}X^T$ 
3:   Compute  $A^{-1} = V\Gamma V^T$  ▷ EVD
4:   if  $A^{-1}$  is positive definite then
5:     Compute  $A^{-1} = LL^T$  ▷ Cholesky factorization
6:     return  $Z = XL$ 
7:   else
8:     Set  $\Gamma' = \max(\Gamma, 0)$  ▷ Element-wise  $\max(\cdot)$ 
9:      $(V\sqrt{\Gamma'})^T = QR$  ▷ QR factorization
10:    return  $Z = XR^T$ 
11:  end if
12: end function

```

To further improve CSQN, we use $\Omega^{(1)}$, the diagonal of the FIM, in two ways. First, as covariance matrix Σ to compute S and Y (see Algorithm 1), we set $\Sigma = (\Omega^{(1)} + \epsilon \max(\Omega^{(1)})I)^{-1}$ with $\max(\Omega^{(1)})$ the maximum of $\Omega^{(1)}$ and I the $N \times N$ identity matrix. The idea is that knowing the Hessian would lead to better sampling, and the diagonal of the FIM is an initial estimate of the Hessian. This has proven to work well

in our experiments. However, note that this introduces a new hyper-parameter ϵ to avoid numerical problems when some elements of $\Omega^{(1)}$ are (close to) zero. Second, we take $\Omega^{(1)}$ as our initial estimate of the Hessian, i.e. $B_0^{(1)} = \Omega^{(1)}$. This means that the regularization loss of CSQN can be split into two terms: one term is related to B_0 and therefore equal to EWC's regularization loss, while the second term is related to S and Y and the corresponding rank- $2M$ or rank- M Hessian approximations. As such, CSQN literally extends EWC.

Moreover, note that Algorithm 1 requires computing the gradient of $L_1(\theta^{(1)})$, which is also required to compute $\Omega^{(1)}$. Therefore, computing $\Omega^{(1)}$ does not entail an extra forward or backward pass over the task's data.

To adapt a model trained on $t - 1$ tasks to a t th task, we follow the reasoning of [40] and consider the following loss:

$$\text{Loss}_t(\theta) = L_t(\theta) + \frac{\lambda}{2} \left(\theta - \theta^{(t-1)} \right)^T \left(\sum_{j=1}^{t-1} B^{(j)} \right) \left(\theta - \theta^{(t-1)} \right) \quad (10)$$

A disadvantage of our method as currently proposed is that the memory requirements increase linearly with the number of tasks. To overcome this, a straightforward way would be to use Algorithm 2 to compute $Z^{(i)}$ for each task i , after which these matrices could be concatenated into one large matrix $Z^{(\leq t)}$, to which we can then apply Singular Value Decomposition (SVD) to reduce its number of columns to e.g. M (SR1) or $2M$ (BFGS). This could be done after each task, and as such the number of columns $Z^{(\leq t)}$ could be kept constant (at M or $2M$) as t increases. Moreover, to compensate for the reduction of regularization loss (as we reduce the number of columns of $Z^{(\leq t)}$ from M_b to M_a), we multiply the reduced $\tilde{Z}^{(\leq t)}$ by $\sqrt{M_b}/\sqrt{M_a}$. We call this strategy *CT*.

One problem of this strategy might be that at task $t + 1$, when $\tilde{Z}^{(\leq t)}$ and $Z^{(t+1)}$ are concatenated and reduced with SVD, $\tilde{Z}^{(\leq t)}$ represents t times more tasks than $Z^{(t+1)}$, yet both contain the same number of columns. Consequently, one can expect $\tilde{Z}^{(\leq t+1)}$ to contain less and less information about older tasks as t increases. To overcome this, we could give the columns of $\tilde{Z}^{(\leq t)}$ a weight of $\sqrt{t}$ before applying SVD. We refer to this strategy as *CT-W*. Another option would be to group the tasks into a binary tree. All tasks start at the bottom of the tree and are grouped two-by-two. Once a task and its neighbour in the tree are both learned by the model, their Z matrices are concatenated and reduced with SVD into a Z matrix of size M for SR1 and $2M$ for BFGS. The resulting matrix moves up the tree and the process is repeated. As such, unlike in the CT strategy, any two Z matrices that are concatenated and reduced represent an equal number of tasks. With this strategy, which we refer to as *BTREE*, the number of components to store is not constant, but increases with only $\log_2(T)$.

Algorithm 3 summarizes our method, CSQN. Note that here, *reduce*($\cdot$) could be any user-defined function to reduce $Z^{(\leq t)}$. If it is defined, or if it is not (because reducing the number of components is not desired) but the considered QN method is SR1, then Algorithm 2 is used to compute $Z^{(t)}$ (Line 10). In the former case, by applying *reduce*($\cdot$) to $Z^{(t)}$ and $\tilde{Z}^{(< t)}$,

$\tilde{Z}^{(\leq t)}$ is obtained (Line 14), which is used in the regularization loss of Line 4 when learning task $t + 1$. In the latter case, on the other hand, $Z^{(t)}$ is stored along $Z^{(1)}, \dots, Z^{(t-1)}$, which are all required to compute the regularization loss for task $t + 1$. Finally, if it is not desired to reduce the number of components and the considered method is BFGS, then it is not required to compute $Z^{(t)}$, and it suffices to store $S^{(t)}$ and $Y^{(t)}$. To learn task $t + 1$, then, $S^{(1)}, \dots, S^{(t)}, Y^{(1)}, \dots, Y^{(t)}$ and Equation 7 are used.

Algorithm 3 Continual Learning with SQN

```

1: Initialize  $\theta$ , select  $M, \epsilon$ , method (BFGS or SR1), set  $B_0 = 0$ , define reduce( $\cdot$ ) to reduce  $Z$  (if desired).
2: for  $t = 1, \dots, T$  do
3:   Determine optimal  $\lambda$  based on a validation set
4:   Train  $f_\theta$  with loss as in Eq. 10 on  $D_t$  to obtain  $\theta^{(t)}$ 
5:   Compute  $\nabla L_t(\theta^{(t)})$  and  $\Omega^{(t)}$ 
6:   Compute  $S^{(t)}$  and  $Y^{(t)}$  using Algorithm 1 with  $x_k = \theta^{(t)}$ ,  $\Sigma = (\Omega^{(t)} + \epsilon \max(\Omega^{(t)})I)^{-1}$ 
7:   Update  $B_0 = B_0 + \Omega^{(t)}$ 
8:   # For SR1, or to reduce, Z must be computed
9:   if method == SR1 or reduce( $\cdot$ ) is defined then
10:    Compute  $Z^{(t)}$  using Algorithm 2
11:    if reduce( $\cdot$ ) not defined then
12:      Store  $B_0, Z^{(t)}$ 
13:    else
14:      Compute  $\tilde{Z}^{(\leq t)} = \text{reduce}([\tilde{Z}^{(<t)} \ Z^{(t)}])$ 
15:      Store  $B_0, \tilde{Z}^{(\leq t)}$ 
16:    end if
17:  else
18:    Store  $B_0, S^{(t)}, Y^{(t)}$ 
19:  end if
20: end for
21: result model  $f_{\theta^{(T)}}$ 

```

IV. EXPERIMENTS

Datasets. We consider the Rotated MNIST [27], Split CIFAR-10/100 [7] and Vision Datasets [15] benchmarks. In the Rotated MNIST benchmark, similar to [21], the MNIST [41] images for task t are rotated by an angle of $\gamma(t - 1)$. To obtain a validation set, we split the MNIST training data into 55,000 training and 5,000 validation images. For the Split CIFAR-10/100 experiments, we split the CIFAR-100 data [42] into T subsets, each with $100/T$ non-overlapping classes. As the first task in this benchmark is the CIFAR-10 dataset, we thus have $T + 1$ tasks. We split the CIFAR-10 and CIFAR-100 training sets into 45,000 training and 5,000 validation images. Finally, in the Vision Datasets benchmark, the tasks are five vision datasets: MNIST, CIFAR-10, SVHN [43], FashionMNIST [44] and notMNIST [45]. Again, we split the training data for each dataset into 95% training and 5% validation images. In the Rotated MNIST experiments, the entire model is shared, while in the Split CIFAR-10/100 and Vision Datasets benchmarks, the classification layer is task-specific.

Baselines. The baselines are: (a) EWC, as explained in III-B; (b) Memory-Aware Synapses (MAS) [8]: similar to

EWC but computes $\Omega^{(t)}$ differently, without requiring the class labels; (c) Learning without Forgetting (LWF) [18]: uses knowledge distillation between $f_{\theta^{(t)}}$ and f_θ (the current model) on data from task $t + 1$ when learning task $t + 1$; (d) Kronecker-Factored approximated Laplace (KF) [15]: uses Kronecker-factored approximations of the FIM to estimate the posterior with Laplace approximation; (e) Fine-Tuning: naïvely adapts the model to new tasks without using any CL algorithm - considered the lower bound.

Evaluation Metrics. To evaluate the performance of the methods, we consider the average accuracy (ACC) and backward transfer (BWT), as in [27]. The latter was defined as follows, assuming that R_{ij} is the accuracy on task j after learning task i and that T tasks are learned:

$$\text{BWT} = \frac{1}{T-1} \sum_{i=1}^{T-1} R_{Ti} - R_{ii} \quad (11)$$

Consequently, negative BWT indicates forgetting.

Models and training. For the MNIST benchmarks, we consider an MLP with two hidden layers of 256 neurons and ReLU activations and a dropout [46] of 25%. We train the model for 10 epochs for each task with Adam ($\beta_1 = 0.9, \beta_2 = 0.999$) with a learning rate of 0.001. For the Split CIFAR-10/100 experiments, we considered the ‘simple’ Resnet-32 that was used in [47] on the CIFAR-10 dataset, as it was implemented by [48]. The first task (the CIFAR-10 dataset) was trained for 15 epochs, while the subsequent Split CIFAR-100 tasks were trained for 10 epochs. The optimizer was the same as for the MNIST experiments. Finally, for the Vision Datasets experiments, we used a similar LeNet [49] as in [15], with two convolutional layers with convolutions of 5×5 and 20 and 50 channels, respectively, followed by a fully-connected layer with 500 neurons. However, we added another fully-connected layer of 84 neurons, to reduce the number of neurons in the classification layer, which is task-specific and therefore not subject to regularization. Each of the five tasks is trained for 50 epochs using the same optimizers as for the MNIST experiments.

Implementation. For our method, CSQN, we considered $\epsilon = 1e-4$ and $\kappa = 1e-8$ in all experiments. Moreover, in all experiments, we try CSQN with both BFGS (referred to as CSQN-B) and SR1 (CSQN-S) with $M = 10$ and $M = 20$ (reported between brackets). For the hyper-parameter selection with respect to λ , the regularization weight, for both our method and the baselines, as well as for the other hyper-parameters for the baselines, see the appendix. All experiments were done in PyTorch [50]. All reported results are averages of five runs. The standard deviation of these five runs is, for all experiments, reported in the appendix.¹

A. Rotated MNIST experiments

For the Rotated MNIST experiments, we consider $T = 20$ and $\gamma = 5^\circ$.

As shown in Table I, CSQN outperforms EWC, MAS and LWF after 20 tasks. This is true for all four settings, between

¹For our code, see github.com/StevenVdEeck/CSQN.

which the differences are relatively small. Nevertheless, increasing the number of components from $M = 10$ to $M = 20$ seems to slightly improve the performance of CSQN with BFGS and SR1. Moreover, we find CSQN with SR1 slightly outperforming CSQN with BFGS.

Figure 1 shows the average accuracy after each task is learned, from which we can see that CSQN consistently performs better than EWC, LWF and MAS.

However, while CSQN outperforms EWC, it substantially underperforms KF, which reaches an impressive average accuracy of more than 94%, which is approximately 10% and 19% higher (in relative terms) than the best CSQN setting and EWC, respectively. We note that this is consistent with [15], in which in similar experiments (on the MNIST dataset with a similar model), KF performed considerably better than EWC. KF does, however, not attain the highest backward transfer, as LWF achieves a positive backward transfer of almost 8%. This means that LWF is able to improve the performance on previous tasks when learning new ones. However, LWF still reaches a relatively low average accuracy as it is unable to learn the new tasks well.

TABLE I: Average accuracy (ACC) and backward transfer (BWT) in % on the Rotated MNIST experiments with $\gamma = 5^\circ$ and $T = 20$.

Model	<i>Rotated MNIST</i>	
	ACC	BWT
Fine-Tuning	58.66	-41.15
EWC	79.29	-17.12
MAS	82.06	-12.48
LWF	73.59	7.94
KF	94.66	-1.92
CSQN-S (10)	85.49	-8.21
CSQN-S (20)	86.22	-7.46
CSQN-B (10)	85.37	-8.45
CSQN-B (20)	85.96	-7.96

Fig. 1: Average accuracy after each task for Rotated MNIST experiments.

B. Split CIFAR-10/100 experiments

We split the CIFAR-100 dataset into $T = 10$ tasks, thus obtaining a sequence of 11 tasks. Table II shows the final results for the Split CIFAR-10/100 experiments.

Similar conclusions can be drawn as for the Rotated MNIST experiments. CSQN substantially outperforms EWC, MAS and LWF, and this is true for all CSQN settings, except, in this case, CSQN with BFGS and $M = 10$. One of the reasons for this may be that CSQN-B (10) turns out to be very sensitive w.r.t. to λ and that none of the values of λ that we tried worked well.

Figure 2a shows the average accuracy after training each task. Note that until the fourth task, LWF performs better than CSQN. Nevertheless, from task 5 on, the performance of LWF drops significantly and, as a result, CSQN reaches the best performance after the 11 tasks. Similar to the Rotated MNIST experiments, we find that the performance improves when increasing the number of components from $M = 10$ to $M = 20$. Moreover, compared to the Rotated MNIST experiments, the increase observed here is much larger. While BFGS with $M = 20$ performs better than SR1, note that in terms of storage requirements, BFGS with $M = 10$ (and not BFGS with $M = 20$) is the equivalent of SR1 with $M = 20$, since for BFGS, each additional component applies a rank-2 update to the Hessian, while for SR1 this is only a rank-1 update. Considering this, SR1 seems to be preferred to BFGS. This was also true for the Rotated MNIST experiments.

In terms of backward transfer, EWC and MAS outperform CSQN. However, their average accuracy is much lower as they are unable to learn the new tasks as well as CSQN.

TABLE II: Average accuracy (ACC) and backward transfer (BWT) in % on the Split CIFAR-10/100 and Vision Datasets experiments.

Model	<i>Split CIFAR</i>		<i>Vision Datasets</i>	
	ACC	BWT	ACC	BWT
Fine-Tuning	18.07	-61.07	33.00	-73.00
EWC	50.68	-0.58	69.52	-13.98
MAS	51.21	-0.14	73.78	-8.96
LWF	39.32	-28.74	75.71	-9.44
KF			77.27	-7.53
CSQN-S (10)	54.39	-6.54	78.20	-6.70
CSQN-S (20)	55.81	-3.82	78.59	-5.90
CSQN-B (10)	50.81	-12.65	78.31	-6.55
CSQN-B (20)	56.29	-5.36	77.80	-7.39

C. Vision Datasets

Finally, let us turn to the Vision Datasets experiments. Table II shows the results. Once again, CSQN outperforms EWC, MAS and LWF. Compared to the Rotated MNIST experiments, CSQN now also performs better than KF. As shown by Figure 2b, this is true from task 2 on. Comparing the different CSQN settings, we find that increasing $M = 10$ to $M = 20$ results in improved average accuracy and backward transfer only for SR1, while for BFGS, the performance deteriorates, as BFGS

(a) Split CIFAR

(b) Vision Datasets

Fig. 2: Average accuracy after each task for Split CIFAR and Vision Datasets experiments.

with $M = 20$ reaches the lowest average accuracy of the four CSQN settings. As in the previous experiments, SR1 seems to be preferred to BFGS, though the difference between SR1 with $M = 20$ and BFGS with $M = 10$ is not large. Nevertheless, CSQN with SR1 and $M = 20$ reaches the highest average accuracy as well as backward transfer of all methods.

D. Limiting memory requirements

As discussed in Section III-D, a disadvantage of CSQN are its memory requirements, which increase linearly with T , the number of tasks. In the same section, we proposed three strategies to overcome this linear increase. We apply them to CSQN with SR1 and $M = 20$ for the Rotated MNIST and Split CIFAR-10/100 experiments. The results are shown in Table III.

CT, which uses SVD to assure that memory requirements remain constant even as the number of tasks increases, suffers from severe forgetting and even performs much worse than EWC on the Split CIFAR-10/100 experiments. On the Rotated MNIST experiments, however, it performs still better than EWC, though the performance degradation with respect to the non-reduced CSQN setting is clear. CT-W, which gives $\hat{Z}^{(\leq t)}$ a weight of $\sqrt{t}$ before it is being concatenated with $Z^{(t+1)}$ and later reduced with SVD, yields no satisfying improvement. The BTREE strategy, on the other hand, in which the Z matrices

of the tasks are arranged in a binary tree and reduced two-by-two, such that any two Z matrices involved in such operation represent an equal number of tasks, works much better. On the Rotated MNIST experiments, the performance degradation is, after 20 tasks, less than 1% (in terms of average accuracy). On the Split CIFAR-10/100 experiments, the difference is larger, but not so dramatic as for the CT and CT-W reduction strategies, and CSQN-S (20) with BTREE reduction strategy still outperforms EWC and MAS.

TABLE III: Average accuracy (ACC), backward transfer (BWT) and forward transfer (FWT) in % on Rotated MNIST and Split CIFAR-10/100 experiments with different reduction strategies applied to CSQN-S (20). Last column are the memory requirements with N the size of the (shared) model, T the number of tasks and M the number of components in BFGS and SR1.

Reduction	Rot. MNIST		Split CIFAR		Memory
	ACC	BWT	ACC	BWT	
None	86.22	-7.46	55.81	-3.82	$(M \cdot T + 2)N$
CT	82.44	-6.91	36.84	-16.17	$(M + 2)N$
CT-W	82.09	-8.49	40.05	-17.63	
BTREE	85.48	-8.36	52.70	-7.56	$\leq (M \cdot \log_2 T + 2)N$

E. Comparison between CSQN and KF

Both CSQN and KF are based on EWC's idea to use Laplace approximation to estimate the posterior with respect to previous tasks. Since EWC uses the diagonal of the FIM in Laplace approximation, it assumes that the parameters of the model are approximately independent. CSQN and KF both try to overcome this. To this end, KF considers Kronecker-factored approximations of the FIM [16], which allows the FIM to be efficiently and compactly obtained per layer. As such, KF's Hessian approximation is block diagonal and thus considers interactions between parameters in the same layer. CSQN, on the other hand, considers the Hessian approximations obtained in QN methods. To obtain these Hessian approximations, it uses the idea of SQN methods, which suggests to sample around the current model parameters. Consequently, CSQN's Hessian approximation is no longer diagonal, and interactions between any two parameters could be considered. Another advantage of CSQN is that it is very easy to implement for any network architecture, since sampling around the current model parameters and computing $M + 1$ gradients (at the current model parameters and the sampled model parameters) is all that is required to obtain the Hessian approximations. However, a disadvantage of CSQN is that the Hessian estimate is a very low rank approximation, as M usually very small with respect to N .

In the experiments, we find that KF is superior to CSQN on the Rotated MNIST benchmark, while CSQN outperforms KF on the more challenging Vision Dataset benchmark. These results are consistent with [15], in which the difference between KF and EWC was much larger on the MNIST experiments using an MLP than on the Vision Datasets experiments with a LeNet-architecture.

V. CONCLUSION

In this paper, we proposed CSQN, an algorithm for Continual Learning which, while using its reasoning, overcomes EWC's assumption of (approximately) independent model parameters. This is done by considering EWC's diagonal FIM an initial Hessian estimate and extending it with QN methods. Moreover, by considering SQN, the extended Hessian estimate can be obtained without using the QN methods as an optimizer. Simply sampling a number of points around the current model parameters in the parameter space and computing the gradients at these points with respect to a set of representative samples, is all that is required to improve EWC's Hessian approximation. In our experiments, we found CSQN significantly outperforming EWC as well as other baselines, including KF on the Vision Datasets experiments. Moreover, by introducing the BTREE reduction strategy, our method's main disadvantage, namely the linear increase of the memory requirements with T , the number of tasks, was alleviated.

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