Sound transmission loss of two-layered pyramidal structure lined with porous material

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A theoretical model is presented that allows to calculate the sound transmission loss of two-layered pyramidal lattice sandwich structure lined with poroelastic material in the presence of external stationary flow. An approach that decomposes the sound field in spacial harmonics is adopted to describe the wave propagation in the structure with periodic cores, while Biot theory is applied to model the porous material. Acoustic-vibrational systems with different types of filling configurations of porous material are derived and solved numerically. The model results are validated by comparison with finite element simulations and results in literature. The influence of the porous material parameters, the external flow and the lattice structure are discussed. Conclusions are drawn the physical mechanisms that determine the acoustic insulation performance of the multifunctional structure, which are helpful for practical design.

Nomenclature

E	=	Young's modulus of structure material	ρ_0	=	density of air
D	=	bending stiffness of panel	c	=	sound speed
ρ	=	density of structure material	k	=	wave number
ν	=	Poisson's ratio of panel material	k_x, k_y, k_z	=	wave number components
η	=	loss factor of structure material	Φ	=	wave potential
h_1, h_2	=	thickness of panel	θ	=	elevation angle of incident wave
d	=	height of the core	φ	=	azimuthal angle of incident wave
L	=	size of unit cell	$\mathbf{v}$	=	velocity vector of external flow
K_{rx}, K_{ry}, K_z	=	equivalent stiffness of beam	V	=	velocity of external flow
θ_b	=	elevation angle of beam	ϕ_v	=	azimuthal angle of external flow

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φ_b	= azimuthal angle of beam	Ma	= Mach number of external flow
l_b	= length of beam	I	= amplitude of incident wave
A_b	= cross-sectional area of beam	R_{nm}	= amplitudes of reflected wave components
M_b	= mass of beam	T_{nm}	= amplitudes of transmitted wave components
$A_1^{nm} \sim A_4^{nm}$	= amplitudes of wave components in the core	j	= imaginary unit
I_b	= moment of inertia of beam	N	= bound limit
F_{ix}, F_{iz}	= forces applied on panel by beam	t	= time variable
M_{ix}, M_{iz}	= moments applied on panel by beam	ω	= angular frequency
P_{iz}	= total pressure on panel by air and foam	N_{pr}	= Prandtl number of air
δ	= Kronecker function	τ_{tr}	= transmission coefficient
u_i, v_i, w_i	= displacements of panel	STL	= sound transmission loss
$U_i^{nm}, V_i^{nm}, W_i^{nm}$	= amplitudes of displacement components	f_{st}	= standing wave resonance frequency
f_{air}	= mass-spring-mass resonance frequency	d_f	= thickness of foam
s_{air}	= equivalent stiffness of air in the core	ρ_1	= bulk density of foam
s_f	= equivalent stiffness of foam	E_m	= bulk Young's modulus of foam
s_{co}	= equivalent stiffness of core	N_s	= shear modulus of foam
e_s	= volumetric strain of solid phase in foam	η_s	= loss factor of foam material
ϵ	= volumetric strain of fluid phase in foam	ν	= bulk Poisson's ratio of foam
ω	= rotational strain of solid phase in foam	σ_f	= flow resistivity of foam
$\mathbf{u}_s$	= displacement vector of solid phase in foam	ϵ'	= geometrical structure factor of foam
$\mathbf{u}_f$	= displacement vector of fluid phase in foam	ϕ	= porosity of foam
k_1, k_2, k_3	= wave numbers in foam	σ	= forces in solid phase of foam
$C_1 \sim C_6$	= complex amplitude in foam	s	= forces in fluid phase of foam
e_{ij}	= strain of solid phase in foam	τ	= shear force in solid phase of foam
Subscripts			
p	= panel	in	= incident
b	= beam	re	= reflected
g	= gap	tr	= transmitted
co	= core		

I. Introduction

Lightweight sandwich structures have been widely used in many areas thanks to their superior mechanical properties in terms of high stiffness, high strength and low weight [1–3]. Many kinds of cores for the sandwich structure exist, such as air cavity, foam, honeycomb, corrugation, lattice. Compared to other structures, lattice geometries, such as pyramids, tetrahedrons, 3D-kagomes, etc., encompass a high stiffness-to-mass ratio as well as an additional multifunctional potential by virtue of the open space in the core, such as energy absorption, thermal control and sound insulation, etc.

In view of their functionality as acoustically insulating separators, sound transmission through periodic sandwich structures has been studied by many authors. One of the most useful methods to assess the acoustic behavior of these structures is the space harmonic method (SHM) proposed by Mead and Pujara [4]. Lee [5, 6] extended the model and studied the sound transmission loss (STL) of a periodically rib-stiffened plate and a cylindrical shell, and they investigated the role of different design parameters. Wang [7] applied SHM on lightweight double-leaf partitions and compared the results to those of the homogeneous method. Also, Fourier transform based approaches are appropriate to address this kind of study, and essentially, they are equivalent to SHM. Shen [8, 9] studied sound transmission through and radiation from an orthogonally stiffened laminated composite plate. Yin [10] studied the acoustic radiation from a point-driven, fluid-loaded laminated plate.

Filling of sandwich structures with absorptive material, mostly porous material, has been implemented with the goal to improve their acoustic properties. The equivalent fluid model (EFM) has been widely used by virtue of its simplicity. Delany [12], Johnson [13], Allard and Champoux [14] introduced several semi-empirical models to describe a rigid-frame porous material. Xin [15, 16] applied the model on an orthogonally rib-stiffened double-panel structure. Wang [17] used the equivalent model to investigate sound transmission of pyramidal sandwich structure with porous material. This approach is easily used but can not match Biot theory [11] in accuracy and applicability. Biot theory takes into account the elasticity of the frame. Bolton [18] used the theory to study a stratified multi-panel structure lined with elastic porous materials. Zhou [19] extended Bolton's work to three dimensions. Liu [20] studied the effect of the flow on a double-wall structure. Shen [21] studied the sound transmission through a finite clamped double-panel structure with foam and flows. Qiao [22] analyzed a rib-stiffened structure with porous cores. Talebitooti [23, 24] investigated the STL of an infinite doubly curved shell and a cylindrical shell filled with a porous material.

The external mean flow was subsequently studied to simulate the realistic situations, for example aircraft fuselages. Meng [25] and Zhou [19] discussed the mean flow on the structures with fibrous material.

Among the aforementioned investigations, the transmission loss of a lattice structure filled with porous material has not been addressed, although it has potential for several multifunctional applications. Equivalent theory of porous material in the previous studies is less accurate and not applicable for non-full filled configurations. In previous work [17], we have calculated the transmission loss of two-layered pyramidal structure filled with porous material using the

EFM. In the present contribution, we elaborate and refine this by modeling using the Biot theory. Using this theory, we now can consider different kinds of filling configurations in the lattice structure. In addition, we enter the details of the interaction mechanism between the beams and the porous material.

II. Vibro-acoustic system

A sandwich structure with two-layered pyramidal cores of infinite extent is considered. The structure is lined with poroelastic foam material and immersed in a medium with density ρ_0 , as shown schematically in Fig.1. A right-hand coordinate system is assumed on the outer surface of the top panel where the x and y axes are on the outer surface and the z -axis is perpendicular to the panel and pointing inside. Two panels segregate the space into three regions, the incident medium, the middle core and the transmitted medium. Two-layered pyramidal cores and poroelastic material with different boundary configurations are present in the core. A plane wave impinges upon the top panel of the structure in the presence of an external mean flow with constant velocity vector $\mathbf{v}$. The mean flow represents situations such as aircraft and high-speed rail and is assumed to be tangential to the top panel. The incident wave travels through the structure with elevation angle θ_{in} and azimuthal angle φ , generating a transmitted wave on the other side with the angles θ_{tr} and φ . These two acoustic fields are characterized by (c_{in}, k_{in}) and (c_{tr}, k_{tr}) in terms of the speed of sound and wave number.

Fig. 1 Schematic illustration of (a) two-layered pyramidal lattice structure with porous material (not shown here) and (b) the three filling configurations of in a unit cell.

Three classic filling configurations of porous material, illustrated in Fig.1 have ben considered: in the bonded-bonded (BB) configuration, both sides of foam are bonded to the panels. In the bonded-unbonded (BU) configuration one side of the foam is bonded to one panel and the another side is separated from the second panel. The unbonded-

unbonded (UU) configuration has both sides of the foam loose.

In the considered geometry, two-layered pyramidal cores are uniformly distributed along the x and y axes with a spacing L . The elevation and azimuthal angles of beam are θ_b and φ_b , respectively. The length of beam is $l_b = d/(2 \sin \theta_b)$ where d is the height of core.

A. The vibro-acoustic equations of the structure

The plane wave in the incident and transmitted mediums can be introduced as [16, 17]

$$\Phi_1 = \Phi_{\text{in}} + \Phi_{\text{re}} = I e^{-j(k_x x + k_y y + k_z z)} + \sum_{nm} R_{nm} e^{-j(k_{xn} x + k_{ym} y + k_{\zeta, z}^{nm} z)} \Phi_2 = \sum_{nm} T_{nm} e^{-j(k_{xn} x + k_{ym} y + k_{\zeta, z}^{nm} z)} \quad (1)$$

where $k_x = k_{\text{in}} \sin \theta_{\text{in}} \cos \varphi$, $k_y = k_{\text{in}} \sin \theta_{\text{in}} \sin \varphi$ and $k_z = \sqrt{k_{\text{in}}^2 - k_x^2 - k_y^2}$ are the components of incident wavenumber along the corresponding axes. $j = \sqrt{-1}$. $k_{xn} = k_x + 2n\pi/L$ and $k_{ym} = k_y + 2m\pi/L$ are used to denote the discrete wavenumbers of harmonic waves. The wavenumber along the z -axis is calculated from the equality $(k_{\zeta, z}^{nm})^2 = k_{\zeta}^2 - k_{xn}^2 - k_{ym}^2$, ζ is “in” or “tr”. I is the amplitude of the incident wave. R_{nm} and T_{nm} are the amplitudes of the reflected and transmitted wave components, respectively. The symbol $\sum_{nm}$ represents $\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty}$ to alleviate the notion. In all expressions, the time dependent factor $e^{j\omega t}$ is implicit and has been omitted for convenience. The velocity vector of the mean flow is defined as $\mathbf{v} = (V \cos \varphi_v, V \sin \varphi_v, 0)$ in which φ_v is the azimuthal angle with x -axis.

The wave equation in the incident medium can be expressed as [17]:

$$\frac{D^2 \Phi_{\text{in}}}{Dt^2} = \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right)^2 \Phi_{\text{in}} = c_{\text{in}}^2 \nabla^2 \Phi_{\text{in}} \quad (2)$$

Substituting Φ_{in} into Eq.(2), we get the wavenumber

$$k_{\text{in}} = \frac{\omega/c_{\text{in}}}{1 + Ma \sin \theta \cos(\varphi - \varphi_v)} \quad (3)$$

where $Ma = V/c_{\text{in}}$ is the Mach number of the external flow. According to the continuity condition $k_x = k_{\text{in}} \sin \theta_{\text{in}} \cos \varphi = k_{\text{tr}} \sin \theta_{\text{tr}} \cos \varphi$, θ_{tr} is obtained as

$$\theta_{\text{tr}} = \arcsin \left[\frac{c_{\text{tr}}}{c_{\text{in}}} \frac{\sin \theta_{\text{in}}}{1 + Ma \sin \theta_{\text{in}} \cos(\varphi - \varphi_v)} \right] \quad (4)$$

Obviously, a critical value of θ_{tr} exists and it depends on the incident angle and Mach number.

Considering the continuity of the trace wavelengths on the fluid-solid interfaces, the displacements of panel u_i, v_i, w_i

can be expressed as:

$$(u_i, v_i, w_i) = \sum_{nm} (U_i^{nm}, V_i^{nm}, W_i^{nm}) e^{-j(k_{xn}x + k_{ym}y)} \quad (5)$$

where $i=1, 2$ denote the top and bottom panels respectively, $U_i^{nm}, V_i^{nm}, W_i^{nm}$ the corresponding component amplitudes of displacements.

The equations of motion of the infinite panels can then be expressed as [22]

$$D_i \nabla^4 w_i + \rho h_i \frac{\partial w_i}{\partial t^2} = P_{iz} + F_{iz} \delta_L \quad (6)$$

$$\rho h_i \frac{\partial u_i}{\partial t^2} - \frac{E_p h_i}{1 - \nu^2} \frac{\partial}{\partial x} \left(\frac{\partial u_i}{\partial x} + \mu \frac{\partial v_i}{\partial y} \right) - G_p h_i \frac{\partial}{\partial y} \left(\frac{\partial u_i}{\partial y} + \mu \frac{\partial v_i}{\partial x} \right) = P_{ix} \quad (7)$$

where $D_i = E_p(1 + j\eta)h_i^3/12(1 - \nu^2)$ is the bending stiffness of panel, $\nabla^4 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)^2$, η the loss factor accounting for the mechanical damping associated with the motion of material, ν Poissons ratio, ρ the mass density, E_p the Youngs modulus of the panels, $G_p = E_p/2(1 + \nu)$ the shear modulus, h_i the thickness of panel, $\delta_L = \delta(x - nL)\delta(y - mL)\delta$ the Kronecker delta function. P_{iz} is the sum of the normal fluid pressure by the air and porous material. (F_{ix}, F_{iz}) and (M_{ix}, M_{iz}) are respectively the forces and moments applied on the panel by beams.

The main deformations of the beams in the core are compression and bending. The connecting points of the eight beams in the unit cell are assumed in the mid-surface of the core. The bending and compressive stiffnesses of per beam are shown as [17]

$$K_z = \frac{E_b A_b l_b^2 \sin^2 \theta_b + 12 E_b I_b \cos^2 \theta_b}{l_b^3} \quad (8a)$$

$$K_{rx} = K_{ry} = \left[\left(1 - 3 \sin^4 \theta_b \right) \frac{E_b I_b}{l_b} + \frac{\sin^2 \theta_b \cos^2 \theta_b}{4} E_b A_b l_b \right] \cos^2 \varphi_b \quad (8b)$$

where E_b, A_b, θ_b and I_b are Young's modulus, the cross-sectional area, the elevation angle and the second moment of inertia, respectively. The force F_{iz} is the sum of the inertial and deformation forces, and can be expressed as

$$F_{iz} = -2K_z(w_1 - w_2) + 2K_{rx} \frac{\partial^2(w_1 - w_2)}{\partial x^2} + 2K_{ry} \frac{\partial^2(w_1 - w_2)}{\partial y^2} - 4M_b \omega^2 \quad (9)$$

where M_b is the mass of per beam.

In the present model, the effect of the interactions between the beams and the foam are ignored, especially in the BB and BU configurations. This is because the direct structural connections between the beams and the panels are much stronger than those between the beams and the foam. In addition, the interfaces between the panels and the foam are much larger than those between the slender beams and the foam.

B. Wave propagation in porous model

The equations of motion in a poroelastic material in terms of strains and displacements are expressed as [18, 19]

$$\nabla^4 e_s + A_1 \nabla^2 e_s + A_2 e_s = 0, \quad \nabla^2 \omega + k_t^2 \omega = 0 \quad (10)$$

where $e_s = \nabla \cdot \mathbf{u}_s$ and $\omega = \nabla \times \mathbf{u}_s$ are respectively the volumetric and rotational strains of the solid phase, $\mathbf{u}_s$ the displacement vector of the solid phase. In Eq.(10), A_1 , A_2 and k_r are complex functions of the bulk density of foam ρ_1 , the bulk Young's modulus E_m , the porosity ϕ , the angular frequency ω , the loss factor of the solid phase η_s , Poisson's ratio of the solid phase ν_s , the geometrical structure factor ε' , the flow resistivity σ_f and the Prandtl number of air N_{pr} ; N_s is the shear modulus of the solid phase. The wavenumbers in the foam belong to two longitudinal waves k_1 , k_2 and one transverse wave k_3 . The expressions of these coefficients have been given in Ref.[18, 19] and hence are not repeated for brevity.

Given the waves in the foam and the space harmonic waves in the structure, the solutions for the displacements are accordingly expressed in the form of

$$(\mathbf{u}_s^T, \mathbf{u}_f^T)^T = [u_{sx}, u_{sy}, u_{sz}, u_{fx}, u_{fy}, u_{fz}]^T = \sum_{nm}^e \mathbf{A}_{nm} \mathbf{e}_{nm} \mathbf{c}_{nm} \quad (11)$$

where $C_1 \sim C_6$ are complex amplitudes and $\mathbf{u}_f = [u_{fx}, u_{fy}, u_{fz}]^T$ is the displacement vector of fluid phase, $\mathbf{e}_{nm} = \text{diag} \left(e^{-jk_{1z}^{nm}z}, e^{jk_{1z}^{nm}z}, e^{-jk_{2z}^{nm}z}, e^{jk_{2z}^{nm}z}, e^{-jk_{3z}^{nm}z}, e^{jk_{3z}^{nm}z} \right)$, $\mathbf{c} = [C_1^{nm}, C_2^{nm}, C_3^{nm}, C_4^{nm}, C_5^{nm}, C_6^{nm}]^T$. The coefficient matrix $\mathbf{A}_{nm}$ is shown in A.

The forces in the porous material are given via [18, 19]:

$$\sigma_{ij} = 2N_s e_{ij} + (Ae_s + Q\epsilon)\delta_{ij}, \quad s = Qe_s + R\epsilon, \quad i, j = 1, 2, 3 \quad (12)$$

where σ_{ij} and s are respectively the forces per unit area in solid and fluid, e_{ij} the solid strain, $\epsilon = \nabla \cdot \mathbf{u}_f$ the volumetric strain of fluid phase, A the first Lamé constant, R relating fluid stress and strain, Q the coupling between the volume change of the solid and that of the fluid.

C. Boundary conditions and configurations

The boundary conditions between the foam and the panel involve the displacement conditions, the vibration conditions and the continuity of stress and velocity. When the porous material is directly bonded to the panel, the

boundary conditions are [19, 21]

$$\begin{aligned}
& \text{(i)} \quad -\frac{\partial \Phi}{\partial z} = \frac{\partial w}{\partial t} \quad \text{(ii)} \quad u_z = w \quad \text{(iii)} \quad U_z = w \\
& \text{(iv)} \quad u_x = u \mp \frac{h}{2} \frac{\partial w}{\partial x} \quad \text{(v)} \quad u_y = v \mp \frac{h}{2} \frac{\partial w}{\partial y} \quad \text{(vi)} \quad P_{ix} = \pm \tau_{xz} \\
& \text{(viii)} \quad P_z = \pm \rho \frac{\partial \Phi}{\partial t} \pm (\sigma_z + s) + \frac{h}{2} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right)
\end{aligned} \tag{13}$$

The first three conditions express the continuity of velocity and displacement. The next two require that the in-plane displacements of panel are equal to the tangential displacements of porous material. The last two are the equations of motion of Euler-Bernoulli panel taking the shear forces into account. In these equations, the first signs are appropriate when the porous material is bonded to the positive z -facing surface and the second signs are valid when the porous material is bonded to the negative z -facing surface. Note that the derivative operator in condition (i) should be replaced by the one in Eq.(2) when it refers to the incident field in the presence of mean flow.

When the foam is loose from the panel, an additional air gap exists in the core which is expressed as Φ_g . The related boundary conditions of the continuity of stress and velocity at the open surfaces are [19, 21]

$$\begin{aligned}
& \text{(i)} \quad -\phi \rho_0 \frac{\partial \Phi_g}{\partial t} = s \quad \text{(ii)} \quad -(1-\phi) \rho_0 \frac{\partial \Phi_g}{\partial t} = \sigma_z \\
& \text{(iii)} \quad (1-\phi) \frac{\partial u_z}{\partial t} + \phi \frac{\partial U_z}{\partial t} = -\frac{\partial \Phi_g}{\partial z} \quad \text{(iv)} \quad \tau_{xz} = 0 \quad \text{(v)} \quad \tau_{yz} = 0
\end{aligned} \tag{14}$$

In addition, the boundary conditions of the panels at the mid-surface satisfy

$$\begin{aligned}
& \text{(i)} \quad -\frac{\partial \Phi}{\partial z} = \frac{\partial w}{\partial t} \quad \text{(ii)} \quad -\frac{\partial \Phi_g}{\partial z} = \frac{\partial w}{\partial t} \quad \text{(iii)} \quad P_z = \pm \rho_0 \left(\frac{\partial \Phi}{\partial t} - \frac{\partial \Phi_g}{\partial t} \right)
\end{aligned} \tag{15}$$

In the BB configuration, the foam is directly bonded to both panels, as shown in Fig.1. The conditions in Eq.(13) are employed twice for the two interfaces. Finally, the conditions can be expressed in a matrix form as

$$\mathbf{Y}_{n'm'} \mathbf{x}_{n'm'} + \sum_{nm} \mathbf{Z}_{nm} \mathbf{x}_{nm} = \begin{cases} I, & k = l = 0 \\ 0, & \text{else} \end{cases} \tag{16}$$

where $\mathbf{x}_{nm}$ is a 14×1 vector, $\mathbf{x}_{nm} = \{C_1^{nm}, C_2^{nm}, C_3^{nm}, C_4^{nm}, C_5^{nm}, C_6^{nm}, U_1^{nm}, V_1^{nm}, W_1^{nm}, U_2^{nm}, V_2^{nm}, W_2^{nm}, R_{nm}, T_{nm}\}^T$.

$\mathbf{Y}_{nm}$ and $\mathbf{Z}_{nm}$ are 14×14 coefficient matrices.

In the BU configuration, the porous material is bonded on one side and unbonded on the other one, generating an air gap characterized by (c_g, k_g) in the core. $k_g = \omega/c_g$ because there is no flow in the gap. Similar to the wave

potentials in the external acoustic fields, the wave in the gap is expressed as

$$\Phi_g = \sum_{nm} A_1^{nm} e^{-j(k_{xn}x + k_{ym}y + k_{gz}^{nm}z)} + \sum_{nm} A_2^{nm} e^{-j(k_{xn}x + k_{ym}y - k_{gz}^{nm}z)} \quad (17)$$

where A_1^{nm} and A_2^{nm} are the amplitudes of the positive and negative traveling waves in the air gap. The conditions in Eq.(13) are used for the bonded side and Eqs.(14) and (15) for the unbonded one. Assume that the incident panel is lined with foam. Then the governing equations of the vibro-acoustic system are obtained in the similar form. The vector $\mathbf{x}_{nm} = \{C_1^{nm}, C_2^{nm}, C_3^{nm}, C_4^{nm}, C_5^{nm}, C_6^{nm}, U_1^{nm}, V_1^{nm}, W_1^{nm}, A_1^{nm}, A_2^{nm}, W_2^{nm}, R_{nm}, T_{nm}\}^T$.

In the UU configuration, two air gaps $\Phi_{gi}, (i = 1, 2)$ exist in the core. The corresponding amplitudes of positive and negative waves are (A_1, A_2) and (A_3, A_4) , respectively. Conditions Eqs.(14) and (15) are employed for each side. The governing equations are accordingly obtained with the unknown vector $\mathbf{x}_{nm} = \{C_1^{nm}, C_2^{nm}, C_3^{nm}, C_4^{nm}, C_5^{nm}, C_6^{nm}, A_1^{nm}, A_2^{nm}, W_1^{nm}, A_3^{nm}, A_4^{nm}, W_2^{nm}, R_{nm}, T_{nm}\}^T$.

The transmission coefficient τ_{tr} is defined by the ratio of the sound intensities [20]:

$$\tau_{tr}(\theta_{in}, \varphi) = \sum_{nm} \frac{|T_{nm}|^2 \omega^2}{I^2(\omega - V \cos \varphi_v k_{xn} - V \sin \varphi_v k_{ym})^2} \quad (18)$$

where $\text{Re}(\cdot)$ is the real operator of a complex value.

The sound transmission loss is represented as

$$\text{STL}(\theta_{in}, \varphi) = -10 \log_{10} \tau_{tr}(\theta_{in}, \varphi) \quad (19)$$

III. Numerical results and discussions

In this section, results on the sound transmission loss of the sandwich structure lined with foam and pyramidal cores are reported. The section begins with convergence check of the truncated equations. Next, the theoretical model is validated by comparing with results in literature and finite element simulations. Finally, some parameters are discussed to investigate the acoustic properties of the structure. The related parameters of the system are listed in Table 1.

A. Convergence check and validation

The dimension of the governing set of equations Eq.(16) is infinite. It is necessary to choose reasonable bounds of the wavenumbers considering accuracy and complexity. We have set the bounds as $(n, m) \in [-N, N]$. The harmonic waves are thereby comprised of $(2N + 1)^2$ terms and the dimension of the matrix in Eq.(16) is $14 \times (2N + 1)^2$. The iterative algorithm and the results of convergence check at the upper limit of frequency (10 kHz in the paper) are shown in Fig.2(a) and 2(b) respectively. The convergence points where the errors of STL value are less than 0.1 dB have been

Table 1 Parameters of the sandwich structure lined with foam

Parameter	Value	Parameter	Value
E_p, E_b	2.72 GPa	ρ	1110 kg/m ³
ν	0.4	η	0.01
h_1, h_2	2 mm	d	50 mm
L	38.77 mm	θ_b	45°
φ_b	45°	A_b	2×2 mm ²
ρ_0	1.21 kg/m ³	c_{in}, c_{tr}, c_g	340 m/s
d_f	25 mm	θ_{in}, φ	45°
ρ_1	30 kg/m ³	E_m	8 × 10 ⁵ Pa
η_s	0.265	ν_s	0.4
ϕ	0.9	ε'	7.8
σ_f	25000 MKS Rayls/m		

marked in the curves. Different numbers of term have been applied for the different configurations.

Fig. 2 (a) Iterative algorithm of searching N . (b) Convergence of STL versus N at 10 kHz.

The models with different configurations can be reduced to the double-leaf structure lined with foam in Ref.[18] by setting φ , V and the beam's parameters to zero. A comparison between the results is presented in Fig.3(a) and shows excellent consistency.

Using Comsol Multiphysics software, we have also employed a finite element method (FEM) to validate the analytical results. The comparisons of the theoretical and the FEM results are shown in Fig.3. Fig.3(b) illustrates the results of the two-layered sandwich structure with and without foam inside. Two curves follow the same trend. Most peaks and dips coincide. The small discrepancies can be mainly attributed to the fact that the theoretical model ignores the coupling between beams and foam, and simplifies the motions of beam. These reasons can also explain the phenomenon that the curves of the structure without foam match a little better with each other than those lined with foam. In addition, the comparison of the BU configuration in Fig.3(c) shows satisfactory agreement. Fig.3(d) depicts UU configuration. The match between the curves in the UU case is not as good as other configurations at intermediate

frequencies. This shows that ignoring the only solid connection in the core, i.e. the coupling between foam and beam, affects the results to some degree. The curves, as well as the dips and peaks at low and high frequencies match well. Although there is some discrepancy, the analytical predictions are acceptable and they take as little as 10% of computational time compared to FEM.

Fig. 3 Validation of the model by comparing with Ref.[18] and FEM simulations

B. The influence of the foam

Acoustic waves propagate in the composite structure by two ways: via the beams and through the foam. Foam absorbs a part of the airborne wave energy by viscous drag forces and converts it into heat. Beams transmit the waves rather unattenuated and “efficiently” but they are needed for mechanic stability. The conflicting effects are worthy investigation and weighting. The aim of this section is to discuss the effect of foam on overall sound insulation in different configurations. Fig.4(a) and (b) illustrate differences in STL between different filling configurations in beam-free and beam-relying structures, respectively. In the beam-free structures (Fig.4(a)), BB performs slightly better in the low frequency range with an increase of 2.5~11 dB. This can be explained by the STL at low frequencies being mainly determined by the mass law. The structural contact made by the foam between the two panels also increases the stiffness of their connection, leading to the increase of the mass-spring-mass resonance frequency. The structural contact also acts as a transmission for structure borne waves, thus deteriorating the overall acoustic insulation.

Contrary to the BB configuration, in the UU configuration, due to the loose mounting of the foam, it does not

generate a mechanical transmission path, leading to a better isolation performance in the medium and high frequency range. Transmission is limited to the airborne wave in the pores [26], which is hampered due to the viscous drag forces, which increase with increasing frequency. The BU configuration combines advantages of both configurations: the partially loose foam does not make a structural transmission path and attenuates airborne waves in the cavity.

Fig.4(b) gives an overview of STL performance of the configurations with beam-connections. As expected, except for low frequencies, where the added mass of the beams increases the acoustic isolation with about 6~15 dB, the structural contact made by the beams between the two panels deteriorates the STL. Their presence evokes a variety of structural vibration modes that shortcut structure borne sound between the two panels, resulting in deep dips in the STL spectrum above 1kHz [7, 27]. This deterioration is somewhat mitigated at medium frequencies in the BB and BU configurations, in which the mechanical connection of the foam to one of the panels brings vibrational energy to the foam, where it is partially dissipated due to friction with air. The lack of mechanical connection with the other panel in the BU avoids an additional panel to panel structural transmission path via the foam. The beams also adds mechanical stiffness, shifting the mass-spring-mass dip to higher frequencies.

Fig. 4 Effect of foam in (a) beam-free and (b) beam-relying structures

An important advantage of placing foam in the cavity in the three configurations is that the airborne standing wave resonances, marked by “▲”, are quenched due to dissipative effects at the air-pore interfaces. The resonance frequencies of the cavity are given by

$$f_{st} = \frac{c_g n}{2d \cos \theta_{co}}, n = 1, 2, 3, \dots \quad (20)$$

where θ_{co} is the elevation angle of the wave in the core and equal to θ_{in} in the absence of foam. The indicated dips at 4824 and 9659 Hz match well with the values 4808 and 9616 Hz yielded by Eq.(20). In addition, the resonance marked by “▼” appears when the two panels move in opposite phase, bouncing on the “spring-damping system“. For the air case, the equivalent stiffness of the core is $s_{air} = \rho_0 c_g^2 / d$ and the damping can be considered zero. The frequency of

the “mass-spring-mass” resonance is given as [7]:

$$f_{\text{air}} = \frac{1}{2\pi \cos \theta_{\text{in}}} \sqrt{\frac{s_{\text{air}}(h_1 + h_2)}{\rho h_1 h_2}} \quad (21)$$

The predicted result 357 Hz matches well with the numerical result 355 Hz.

Lumped model equivalents of the structures are shown in Fig.5(a), in which m is half of the total mass, c_{co} the damping coefficient and s_{co} the equivalent stiffness of the core (i.e. the stiffness of air s_{air} or foam s_f in this case). The “mass-air-mass” resonance frequency f of the structure is roughly proportional to $\sqrt{s_{\text{co}}/m}$. Because the stiffness ratio s_f/s_{air} is larger than the mass ratio m_f/m_{air} , the resonance frequency of the foam filled BB configuration is larger than the one with empty cavity.

Fig. 5 Sketches of the equivalent systems of (a) the air/BB and (b) BU/UU cases

In the cases where the core space is filled with both air and foam (BU and UU configurations), the equivalent stiffness of the core is formed by the serial combination of stiffnesses s_{air} and s_f , as shown in Fig.5(b). A calculation, showed that the rate of stiffness increase by filling the cavity with foam is slightly less than the rate of mass increase, and thus the “mass-air-mass” resonance frequency of the BU and UU configurations is lower than that of the air case.

In addition to the above filling configurations, we have investigated some variants. In the “BU-UB” configuration in Fig.6(a), each panel was lined with foam with half thickness compared to the above BU case, leaving an air gap in the middle of the core. In the “UU-UU” variant, two identical layers of foam were arranged in parallel in the core, with 5 cm interdistance. The total thickness of the two layers was equal to the one in the UU case.

The UB configuration gives the same STL as BU case, as shown in Fig.6(a). In the intermediate frequency range, the BU-UB configuration in the beam-free structure performs better than the BU case but it performs worse at high frequencies. This is because the foam hampers the motion of the second panel and makes the structure approach to the BB configuration at high frequencies. The difference between the BU-UB and BU cases substantially decreases in a beam-relying structure, due to the dominance of structure borne transmission, making the medium in the core less relevant. These observations also hold for the UU-UU case shown in Fig.6(b).

The parameters of the foam can affect the STL as well. These parameters can be separated into two groups, related to the fluid phase (such as flow resistivity, porosity, structure factor) and the solid phase (such as Young’s modulus, loss factor, Poisson’s ratio, density). Their influence is illustrated in Fig.7.

Fig. 6 STL curves of (a) the BU-UB and (b) UU-UU configurations

Fig. 7 Sensitivity of the STL in the BB, BU and UU configurations to the changes of: (a) E_m, ϵ' (BB); (b) ρ_1, σ (BB); (c) E_m, ϵ' (BU); (d) ρ_1, σ (BU); (e) d_f (BU); (f) all above parameters (UU). The units of the parameters are: Young's modulus E_m , MPa; flow resistivity σ , MKS Rayls/m; thickness of foam d_f , cm and density ρ_1 , kg/m³.

The STL of BB configuration is sensitive to the solid phase parameters [18]: increasing the foam modulus increases the structure borne transmission and therefore deteriorates the STL at high frequencies. In this case, due to the dominance of structure borne transmission, the fluid parameters do not play a role. In the mass-controlled low frequency region, an increase of foam density improves the transmission loss clearly. As shown in Fig.7(c) and (d), in the BU configuration, variations of both the solid and fluid phase parameters affect the STL, and their influences are interconnected. Large solid parameters reduce the impact of fluid ones, while large fluid parameters amplify the sensitivities to solid ones. The interactions can be explained by the fact that the solid parameters determining frame

borne transmission (similar to structure borne transmission via the beams), and thus crippling the transmission via the pore fluid. As shown in Fig.7(f), the same observation can be made for the UU configuration. The effect of the foam thickness in the BU configuration is depicted in Fig.7(e). Changes in the STL spectrum in beam-relying structures are small due to dominance of structure borne transmission, while significant effects appear in beam-free structures.

C. Influence of the external mean flow

Our consideration of a non-zero external mean flow in the incident field is inspired by its relevance in real-life situations, e.g. involving aircraft fuselages. For the simulations, the physical properties of air were taken as in the stationary case [19]. The air flow affects the wave equation in the incident medium as well as the angle of transmission. Eq.(4) shows that the critical value of θ_{tr} depends on the angle of incidence and the Mach number. In this section, the influence of the Mach number on the STL is studied for a fixed angle of incidence. Fig.8 presents the influence of the external flow along the downstream ($\varphi_v = 0^\circ$) direction in different configurations. In general, the STL values of all structures increase as the Mach number Ma increases. The curves of beam-free structures in Fig.8(d) follow the same trend. The positive correlation is due to the added mass of the converted fluid loading and the aerodynamic damping effect [25]. For the beam-relying structures, shifts of some peaks and dips to high frequency can be noticed. These resonance frequencies roughly proportional to $1/k_{in}^2$ [7]. Combining with Eq.(3), it is easy to see that the resonance frequencies are easily proved to increase as Ma rises downstream and decrease when the flow is upstream.

Results for upstream flow ($\varphi_v = 180^\circ$) are shown in Fig.9. Compared to downstream results, big differences appear. As Ma increases, the curves decrease a little and then rise a lot, especially for the curves in Fig.9(d). The rise is caused by the Mach number exceeding the critical value derived from Eq.(4). In this situation, the transmitted wave becomes evanescent, thus causing the STL to increase sharply. The critical value of Ma_{cr} is 0.586 for $\theta_{in} = \varphi = \pi/4$. The influence of the Mach number in the beam-relying structures shown in Fig.9(a)~(c) is similar to Fig.9(d) but not extreme. The drastic increase does not occur due to the existence of the periodically distributed beams. Waves in the periodic model are a series of harmonic waves with discrete wavenumbers, leading to not only evanescent but also harmonic waves in the transmitted field. The mechanism causing some dips to shift to lower frequencies is similar to the one in Fig.8.

D. Influence of the sandwich structure

The lattice sandwich structure is an important component of the whole system. The geometric and mechanical parameters of panel and beam affect the vibro-acoustic properties of system. The vibration of the bottom panel determines the radiated waves directly. The beams provide superior mechanical behavior but shortcut the beneficial effects of the foam.

The effect of the modulus of lattice structure in the BB configuration is illustrated in Fig.10(a). The subscripts p

Fig. 8 Effect of Mach number in the beam-relying (a) BB, (b) BU and (c) UU structures and (d) beam-free structures along the downstream direction.

and b denote *panel* and *beam*, respectively. There is almost no change in the low frequency range, which obeys the mass law. Enhancing the modulus of panel increases the resonance frequency of panel, shifting the corresponding dips to high frequencies. Varying the modulus of beam obtains the similar results. The influence of density in BB case is shown in Fig.10(b). The STL values increase visibly as the density of panel or beam increases in the low frequency range and vice versa. In addition, the density of the panel influences the results overall while the density of the beam has less influence at high frequencies. This is due to the fact that the panel dominates the radiated waves. An extreme case is shown in Fig.10(b) for the structure with beam of 10% density and 1% modulus. The disadvantages of the use of beams on the sound insulation are eliminated and the STL curve coincides with the one of the beam-free structure. The same phenomenon appears in the BU and UU beam-relying structures as shown in Fig.10(c)~(f). Thus, reducing the density or modulus of beam is an effective method to decrease the wave transmission through beams. However, it weakens the mechanical bearing capacity of the structure.

The dependence of the STL on the elevation angle of beam is shown in Fig.11. Changes of θ_b influence the size of the unit cell as well as the stiffness of the core. The STL values rise a little at low frequencies and some dips move to high frequencies as θ_b increases. The former follows the mass law and the latter can be attributed to the decrease of L leading to an increase of the span between the discrete wavenumbers $2\pi/L$. Interestingly, decreasing the elevation

Fig. 9 Effect of Mach number in the beam-relying (a) BB, (b) BU and (c) UU structures and (d) beam-free structures along the upstream direction.

Fig. 10 Effects of the modulus and the density of lattice structure in BB (a,b), BU (c,d) and UU (e,f) configurations, respectively.

angle θ_b offers a way to eliminate the STL deterioration caused by structure borne transmission through connecting beams. Fig.11 shows that, as θ_b decreases, the STL curves tend to those of beam-free situations. This is because the

decrease of the stiffness of core weakens the connections between panels and makes less energy travel through them.

Fig. 11 Effect of the elevation angle of beam in (a) BB, (b) BU and (c) UU configurations

IV. Conclusion

A theoretical analysis of the sound insulation of two-layered pyramidal structure lined with different configurations of porous material was established using space harmonic approach and Biot theory. The model was validated by comparison of the results with reports in literature and finite element simulations. The transmission loss of the three classic filling configurations as well as some variant was studied. Parameters of porous material and lattice structure were discussed and the mechanism of acoustic insulation was analyzed. The main conclusions are:

- (1) The BB configuration, with the foam in the core connecting to both panels, performs well in the low frequency range, a loose foam (UU configuration) stands out in the high range. With the foam connected to only one of both panels (BU configuration), the best overall transmission loss is obtained. The presence of the foam eliminates airborne standing wave resonances in the core cavity. Connecting the two panels by beams (e.g. to obtain a stronger structure) introduces structure borne sound transmission and therefore deteriorates the acoustic isolation. Interestingly, a periodic distribution of beams leads to a series of isolation dips and peaks caused by bandgaps. Introducing two foams in the core has some effect on the STL spectrum in the case without connecting beams.
- (2) The effect of foam on the STL depends both on the parameters of its solid and fluid phase.
- (3) The presence of an external flow along the downstream direction causes a small overall increase and moves the resonances to high frequencies. An upstream flow reduces the acoustic insulation property when the Mach number is below the critical value, and leads to a sharp increase of STL when the Mach number exceeds the critical point.
- (4) Varying the modulus of the panel or beam has little effect on STL at low frequencies but affects some resonances in high frequency range. Adding mechanical attenuation to the beams can reduce its inhibition effect on the foam and thus improve the insulation performance of the whole system. Changing the density of the panel has more influence than changing the one of the beam. Reducing the elevation angle of beam is an alternative effective

approach to lower the structure-borne transmission.

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A. The elements of the matrix $\mathbf{A}_{kl}$ in Eq.(11)

$$\mathbf{A}_{nm} = \begin{bmatrix} A_{nm}^{u_l} & A_{nm}^{u_t} \\ A_{nm}^{U_l} & A_{nm}^{U_t} \end{bmatrix}$$

where

$$A_{nm}^{u_l} = \begin{bmatrix} j \frac{k_{xn}}{k_1^2} & j \frac{k_{xn}}{k_1^2} & j \frac{k_{xn}}{k_2^2} & j \frac{k_{xn}}{k_2^2} \\ j \frac{k_{ym}}{k_1^2} & j \frac{k_{ym}}{k_1^2} & j \frac{k_{ym}}{k_2^2} & j \frac{k_{ym}}{k_2^2} \\ j \frac{k_{1z}^{nm}}{k_1^2} & -j \frac{k_{1z}^{nm}}{k_1^2} & j \frac{k_{2z}^{nm}}{k_2^2} & -j \frac{k_{2z}^{nm}}{k_2^2} \end{bmatrix}, A_{nm}^{u_t} = \begin{bmatrix} \frac{jk_{xn}k_{3z}^{nm}}{k_3^2\sqrt{k_{xn}^2+k_{ym}^2}} & -\frac{jk_{xn}k_{3z}^{nm}}{k_3^2\sqrt{k_{xn}^2+k_{ym}^2}} \\ \frac{jk_{ym}k_{3z}^{nm}}{k_3^2\sqrt{k_{xn}^2+k_{ym}^2}} & -\frac{jk_{ym}k_{3z}^{nm}}{k_3^2\sqrt{k_{xn}^2+k_{ym}^2}} \\ -\frac{j\sqrt{k_{xn}^2+k_{ym}^2}}{k_3^2} & -\frac{j\sqrt{k_{xn}^2+k_{ym}^2}}{k_3^2} \end{bmatrix}$$

$$A_{nm}^{U_l} = A_{nm}^{u_l} \cdot \text{diag}\{b_1, b_1, b_2, b_2\}, \quad A_{nm}^{U_t} = g \cdot A_{nm}^{u_t},$$

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