See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/330894042>

Multiple zero group velocity Lamb modes in an anisotropic plate: propagation along different crystallographic axes

Article in Canadian Journal of Physics · February 2019

DOI: 10.1139/cjp-2018-0348

CITATIONS

0

READS

154

5 authors, including:

Soufiane Karous

KU Leuven

1 PUBLICATION 0 CITATIONS

[SEE PROFILE](#)

Souhail Dahmen

7 PUBLICATIONS 78 CITATIONS

[SEE PROFILE](#)

M. S. Bouhdima

Institut Supérieur des Sciences Appliquées et de Technologie de Gabès

6 PUBLICATIONS 7 CITATIONS

[SEE PROFILE](#)

Ben Amor Morched

Institut Préparatoire aux Etudes d'Ingénieurs de Sfax

26 PUBLICATIONS 130 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:

wave propagation in PM/PE layers [View project](#)

Analysis of the vibration behavior of a functionally graded piezoelectric plate at ZGV resonance frequency [View project](#)

Multiple zero group velocity Lamb modes in an anisotropic plate: propagation along different crystallographic axes.

Soufien Karous^{(a)*}, Souhail Dahmen^(a), Mohamed Shili Bouhdima^(a), Morched Ben Amor^(a), and Christ Glorieux^(b)

^a Laboratory of Physics of Materials, Faculty of Science of Sfax, BP 1171, 3000 University of Sfax, Tunisia.

^b Laboratory of Acoustics, Division of Soft Matter and Biophysics, Department of Physics and Astronomy, KU Leuven, Celestijnenlaan 200D, B3001 Heverlee, Belgium.

* Corresponding author:

E-mail address: soufien.karous1a@gmail.com

Abstract:

The paper studies the propagation of symmetric and anti-symmetric Lamb waves along ionic acid plate (HIO_3), 1 mm thick, in the 1 to 50 MHz frequency range. The Lamb modes propagation along three crystallographic planes was theoretically investigated, for two mutually orthogonal propagation directions. Several frequencies were found which correspond to Lamb modes with zero group velocity (ZGV) and non-null phase velocity values. The first symmetric Lamb mode, S_1 , was found to possess only one ZGV point, regardless of the propagation direction; higher order symmetric and anti-symmetric modes with up to four ZGV points were found, depending on the propagation plane. The dependence of the ZGV frequencies on each elastic constant (c_{11} , c_{13} , c_{33} , c_{55}) of the HIO_3 plate material was also investigated by changing the constant values by 5% and 10%. It was found that c_{33} and c_{55} affect the number of the ZGV points, while c_{11} and c_{13} affect the frequency of the ZGV points. The existence and frequencies of the ZGV points are strongly dependent on the proximity of standing longitudinal and transverse waves at nearby cutoff frequencies.

Keywords: Lamb waves, anisotropic plate, HfO_3 , multiple zero-group velocity points, elastic properties.

I. INTRODUCTION

One of the important ultrasonic approaches for nondestructive evaluation in industry makes use of the dependence of the experimentally accessible dispersive behavior of the velocity of Lamb waves on the elastic properties of the plates in which they are propagating, making their behavior a widely addressed topic of investigation [1-4]. The possibility to measure the dispersion characteristics of Lamb modes has also often been exploited to extract the mechanical properties of materials [5-9]. Recently, the existence of extrema in the phase velocity dispersive curve, which correspond to Lamb waves with zero group velocity, has attracted quite some attention [10-12]. When a range of frequencies in the neighborhood of such a zero group velocity (ZGV) point are excited, then a standing wave pattern induced by the interference of forward and backward waves can be observed, for which there is no net outflow of energy from the excitation region, resulting in a locally very strong resonance phenomenon [13-16]. The typically very high Q-factor of this resonance yields a quite accurate way to extract the local mechanical properties of the plate from the resonance frequency [17-22]. This feature has been exploited for local and non-contact measurement of mechanical properties of plates [23, 24] and for in-service nondestructive evaluations [25].

An incentive to researchers to study the behavior of ZGV points to characterize anisotropic plates comes from the increased use of fiber reinforced materials for automotive and aviation applications where the damages evaluation is a crucial case study and the need to characterize them has been steadily growing in the recent years [19].

The existence of a ZGV resonance in the S_1 mode for some materials was first pointed out by Tolstoy and Usdin in 1957 [26]. Negishi experimentally observed the negative group velocity of the S_2 mode travelling along an aluminum plate, 1 mm thick, in 1978: he also predicted the existence of the negative group velocity of the S_4 mode [27]. Many papers on the behavior of Lamb waves in various isotropic materials have been published [14,15, 28-33]. Parygin et al. [34] found that there are modes with multiple ZGV points in an anisotropic paratellurite crystal plate. Prada *et al.* [19] found ZGV resonances in the S_1 and A_2 Lamb modes in a silicon plate with cubic symmetry. Hussain and Ahmad [10] revealed the existence of a multitude of ZGV points in the dispersion curves of Lamb waves propagating along one of the crystallographic axes in the P_{13} -plane of an orthotropic HfO_3 plate [10]. In reference [35] Hanyin Cui *et al.* have studied the influence of the outer-to-inner radius ratio in hollow cylinders in the group velocity of a backward mode to be used in acoustic logging tools. Hanyin Cui *et al.* [36] investigated the influences of a fluid-loading on ZGV resonances in pipes for the possible application of integrity inspections of oil transportation pipelines.

In this paper, we extend the work of Hussain and Ahmad [10], by investigating the ZGV points in an orthotropic HfO_3 plate for two mutually orthogonal propagation directions in three different crystallographic planes. The influence of the elastic constants on the number of the ZGV points and on their frequencies is also investigated.

II. THEORETICAL STUDY

A. The Wave propagation theory

The theory of wave propagation in orthotropic materials has been elaborated in a number of research works [37-45]. The theory starts from the Hooke's law,

$$T_{ij} = c_{ijkl} \cdot S_{kl} \quad (1)$$

that defines the linear relationship between stress T_{ij} and strain S_{kl} through the elastic constants c_{ijkl} , and the wave propagation equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j} \quad (2)$$

where $S_{kl} = \frac{1}{2} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right)$, ρ is the mass density of the plate material, and u_i is the wave displacement component along the i direction ($i = 1, 2, 3$). By combining equations 1 and 2, and solving for a plane wave solution

$$u_i = u_i^0 \cdot \exp j(\omega t - kx_1) \quad (3)$$

where x_1 is the propagation direction of the wave, ω [rad/s] is the angular frequency, and u_i^0 is the wave amplitude, we obtain the three Christoffel equations [46]

$$\sum_{l=1}^3 |\Gamma_{il} - \rho V^2 \delta_{il}| \cdot P_l = 0 \quad (4)$$

where δ_{il} is Kronecker's symbol, $\mathbf{V} = \omega/\mathbf{k}$ is the phase velocity of the wave, $\mathbf{P}$ is the polarization vector whose components are the eigenvectors corresponding to each eigenvalue (i.e. the three phase velocities) and Γ_{il} are the components of the 3x3 symmetric matrix:

$$\Gamma_{il} = \sum_{j=1}^3 \sum_{k=1}^3 c_{ijkl} \tilde{n}_j \tilde{n}_k \quad (5)$$

where $\tilde{\mathbf{n}}$ is the unit vector in the slowness direction. The system of three equations in three unknowns (4) admits non trivial solution if the determinant of the matrix of the coefficients is equal to zero, $\det(\Gamma_{il} - \rho V^2 \delta_{il}) = 0$: equation (4) describes a standard eigenvalue (ρV^2) problem with eigenvectors ($\mathbf{P}$) for the matrix $\mathbf{\Gamma}$. The final solutions consist in three velocities V [m/s], expressed in terms of c_{ijkl} and ρ , corresponding to the longitudinal, shear horizontal and shear vertical bulk waves velocities and polarization vectors. The equations system (4) can be solved for whatever propagation direction $\mathbf{n}$ with $\mathbf{n} = \tilde{\mathbf{n}}$, resulting in three positive values for V .

In the symmetry planes of orthotropic media, the determinant of Christoffel equations can be decomposed into linear and quadratic factors for V^2 . The roots of the quadratic factor correspond to quasi longitudinal and quasi-transverse bulk wave velocities: the polarization of the modes has no shear horizontal component.

B. Dispersion curves of Lamb waves in a plate

Lamb modes travel along finite thickness plates, hence the boundary conditions must be added to the problem of the waves propagation in unbonded media, i.e. by assuming that the top and bottom surfaces of the plate are traction-free surfaces,

$$T_{i3} = 0, \quad \text{at } x_3 = 0 \text{ and } x_3 = h \quad (6)$$

according to fig.1. We assume that Lamb waves propagate in the x_1 -direction in an orthotropic plate of thickness h with the sagittal plane lying in the (010) crystallographic plane denoted as P_{13} , as shown in Fig.1. The Lamb modes propagation was also investigated along the x_1x_2 and x_2x_3 sagittal planes, for two mutually orthogonal propagation directions. We adopted the following convention to identify the crystallographic plane under study, P_{ij} : the index i refers to the propagation direction and the index j represents the axis aligned to the plate depth.

FIG.1. The geometry of the homogeneous plate, showing the coordinate system.

In this case the Lamb waves travelling along the HfO_2 plate are decoupled from the shear horizontal (SH) wave [43] ($n_2=0$) and the elements of the equation (5) are reduced to:

$$\Gamma_{ik} = c_{ilk1} n_1^2 + (c_{ilk3} + c_{ikl3}) n_1 n_3 + c_{ikl3} n_3^2 \quad (7)$$

Performing a root search of $\det(\Gamma)=0$, then the components of the polarization vector along the x_1 and x_3 directions can be solved. The most general method for solving the propagation of Lamb waves is by the partial wave technique where the principle of superposition of two upward and two downward travelling plane waves are assumed while satisfying the boundary conditions. At the two stress-free plate surfaces, $x_3 = 0$ and $x_3 = h$, the reflection of waves results in the combination of the motion of upward and downward moving quasi-longitudinal QL and quasi-shear QT waves. The top and bottom surface boundaries reflect the waves and the combination of these reflections going towards the upper or lower interfaces results in the propagating guided waves along the plate. The combination of all four waves defines a single guided wave mode. The total particle displacement vector $\mathbf{u}$ of Lamb waves can thus be expressed as a linear combination of 4 of such partial waves:

$$\vec{u} = \sum_{j=1}^4 A^{(j)} \vec{u}^{(j)} \quad (8)$$

where $\vec{u}^{(j)} = \exp i \omega (\vec{n}^{(j)} \cdot \vec{x} - t) \vec{P}^{(j)}$ are the displacement vectors of the 4 partial heterogeneous plane waves with polarization vectors $\mathbf{P}^m = (P_1^m, P_3^m)$, $\mathbf{n} = (n_1, n_3)$ is the slowness vector, $\mathbf{x} = (x_1, x_3)$ is the spatial coordinate vector; the index (j) denotes the partial wave (j=1, 2, 3, or 4 for modes QL^{down} , QL^{up} , QT^{down} or QT^{up} , respectively). $A^{(j)}$, the coefficients of the partial waves constituting the Lamb wave, are to be determined by satisfying the boundary conditions equation (6) at the plate interfaces. In the absence of a source, setting the stresses equal to zero yields a homogeneous set of 4 equations in the 4 unknowns $A^{(j)}$, inferring that in order to have non-trivial solutions, the determinant of the matrix should be zero. The latter condition can be

used to find the values of the Lamb wave phase velocity v_{ph} that are roots of the determinant. After finding the v_{ph} -values by means of a root search, for each solution v_{ph} , the corresponding partial wave coefficients $A^{(m)}$ can be determined by arbitrarily choosing one of the 4 coefficients, and thus reducing the 4x4 set of equations to a set of 3 linearly independent equations in 2 coefficients, which can be solved, yielding an expression of each of those 3 coefficients as a function of the chosen one.

This procedure thus allows to determine, for each ω value, the possible phase velocities, the corresponding coefficients, and the corresponding displacement vectors $\mathbf{u}$.

Given the symmetry of the problem, the determinant M of the coefficients of the boundary conditions equations system can be rewritten as the product of 2 determinants, one for the symmetric and one for the antisymmetric Lamb modes (i.e. with displacements symmetric and antisymmetric with respect to the midplane of the plate), $M = M_s \cdot M_a$, resulting in separate determinant equations $\det(M_s)=0$ and $\det(M_a)=0$. These equations can be solved analytically, yielding the following implicit dispersion relations for symmetric modes

$$\frac{tg(\omega^{Q_{Tv}} n_3(h))}{tg(\omega^{Q_L} n_3(h))} - \frac{(c_{13} n_1^{(2)} P_1 + c_{33}^{(2)} n_3^{(2)} P_3) \times i \omega c_{55}^{(1)} n_3^{(1)} P_1 + n_1^{(1)} P_3}{(c_{13} n_1^{(1)} P_1 + c_{33}^{(1)} n_3^{(1)} P_3) \times i \omega c_{55}^{(2)} n_3^{(2)} P_1 + n_1^{(2)} P_3} = 0 \quad (9)$$

and for antisymmetric modes, as function of the unknown v_{ph} [47,48]:

$$\frac{tg(\omega^{Q_{Tv}} n_3(h))}{tg(\omega^{Q_L} n_3(h))} - \frac{(c_{13} n_1^{(1)} P_1 + c_{33}^{(1)} n_3^{(1)} P_3) \times i \omega c_{55}^{(2)} n_3^{(2)} P_1 + n_1^{(2)} P_3}{(c_{13} n_1^{(2)} P_1 + c_{33}^{(2)} n_3^{(2)} P_3) \times i \omega c_{55}^{(1)} n_3^{(1)} P_1 + n_1^{(1)} P_3} = 0 \quad (10)$$

These equations remain valid for any material of higher symmetry class, assuming that the direction of propagation is collinear with a principal axis [41]. Solutions to these equations are coupled ω and v_{ph} values: they are obtained using a numerical process based on the dichotomy method to find initial solutions at specific angular frequencies, plus the Newton–Raphson method used to follow these roots as the angular frequency changes [45, 49].

III. RESULTS AND DISCUSSION

A. Numerical investigation

The accuracy of our calculation was confirmed by the correspondence between our results and those of Hussain et al. [10]; the HfO_2 elastic properties were taken from Ref. 46 and are listed in Table I.

Elastic parameter	c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	c_{44}	c_{55}	c_{66}	ρ
HfO_2	30.1	16.1	11.1	58.0	8.0	42.9	16.9	20.6	15.8	4.64

TABLE I: Elastic parameters c_{ij} [GPa] and density [kg/m³] of HfO_2 -plate [46].

Standing waves occur when half of the wavelength of longitudinal or transverse waves travelling back and forth between the two plate boundaries fits an integer number n of times in the plate thickness. Fundamental and higher order modes appear that are characterized by cutoff frequencies f_c . For symmetric modes, the cutoff frequencies are given by [15, 42]:

$$f_c \cdot h = n \cdot V_T, \text{ for mode } S_{2n} (n \geq 1) \quad (11)$$

$$f_c \cdot h = \frac{(m+1)}{2} \cdot V_L, \text{ for mode } S_{2m+1} (m \geq 0) \quad (12)$$

and for antisymmetric modes:

$$f_c \cdot h = n \cdot V_L, \text{ for mode } A_{2n} (n \geq 1) \quad (13)$$

$$f_c \cdot h = \frac{(m+1)}{2} \cdot V_T, \text{ for mode } A_{2m+1} (m \geq 0) \quad (14)$$

where $n=1, 2, 3, \dots$, and $m=0, 1, 2, \dots$

It is noteworthy that, although the dispersion curves reported by Hussain and Ahmad [10] for the same plane (P_{13} plane) are coinciding with the ones we have calculated, however, their choice of indexing, based on sorting of modes according to their order of appearance versus frequency, is not consistent with the indexing rule we have adopted which is based on the number of nodes present throughout the plate cross-section [15, 49]. E.g. the symmetric mode with cutoff frequency equal to 1.673 MHz, which is denoted by Hussain and Ahmad as S_{13} , [10] corresponds to the mode denoted as S_{11} in literature [15, 42, 50] and in this work.

In Tables II and III we summarize respectively the symmetric and antisymmetric modes up to the 42th order, according to the number of ZGV points these modes exhibit for propagation along the x_1 -axis in the P_{13} crystallographic plane in an iodic acid plate.

Modes with no ZGV point	Modes with one ZGV point	Modes with two ZGV points	Modes with three ZGV points
S_0, S_2, S_3, S_5, S_9 S_{10}, S_{13}, S_{16}	$S_1, S_4, S_6, S_7, S_8, S_{12}$ $S_{14}, S_{18}, S_{20}, S_{22}, S_{24}$ $S_{26}, S_{32}, S_{34}, S_{44}$	$S_{11}, S_{15}, S_{17}, S_{19}, S_{21}$ $S_{23}, S_{25}, S_{27}, S_{28}, S_{29}$ $S_{30}, S_{31}, S_{36}, S_{38}, S_{42}$	S_{39}, S_{40}, S_{46}

TABLE II : Classification of symmetric Lamb modes propagating along the x_1 -direction in the P_{13} plane, according to the number of ZGV points.

Modes with no ZGV point	Modes with one ZGV point	Modes with two ZGV points	Modes with three ZGV points
A_0, A_1, A_3, A_4 A_8, A_9, A_{12}, A_{29}	$A_2, A_5, A_{11}, A_{13}, A_{15}$ $A_{16}, A_{17}, A_{19}, A_{21}, A_{23}$ $A_{25}, A_{27}, A_{31}, A_{33}, A_{37}$ A_{39}	$A_6, A_{10}, A_{14}, A_{18},$ $A_{20}, A_{22}, A_{24}, A_{26},$ $A_{28}, A_{30}, A_{25}, A_{27},$ A_{31} $A_{33}, A_{32}, A_{34}, A_{36}$ A_{38}, A_{40}	A_{35}, A_{41}

Table III : Classification of antisymmetric Lamb modes propagating along the x_1 -direction in the P_{13} plane, according to the number of ZGV points.

In the following paragraphs of this paper, we take a closer look at the existence and features of ZGV points, for symmetric and antisymmetric Lamb waves propagating in HIO_3 plates for differ crystallographic planes. For some cases, a study is performed to get a deeper insight on the influence of the elastic parameters on the features of the ZGV points, i.e. there existence and their frequency. Distinction is made between symmetric (Section III.B) and antisymmetric (Section III.C) modes.

B. Symmetric modes

1. Effect of propagation direction on ZGV existence and features

Fig.2 presents the dispersion curves of a selection of symmetric Lamb modes (S_1, S_{11}, S_{26} and S_{39}) propagating in the P_{31} plane of an iodic acid plate, as an example. With increasing the order of the modes, the number of ZGV points, characterized by a minimum or a maximum in the phase velocity branch, increases: S_1, S_{11}, S_{26} and S_{39} modes have 1, 2, 3 and 4 ZGV points,

211 respectively. The appearance of a mode with four ZGV points S_{39} is the most interesting result
212 in P_{31} , as illustrated in Fig. 2.

213
214 **FIG.2.** Dispersion curves of S_1 , S_{11} , S_{26} and S_{39} modes travelling in the P_{31} plane of an iodic acid plate.
215

FIG.3. Graphs of S_1 , S_4 , S_{26} and S_{31} modes comparing plane P_{31} to plane P_{13} .

Figure 3 shows the dispersion curves of the same modes, S_1 , S_4 , S_{26} and S_{31} , travelling along the x_1 and x_3 directions of the P_{13} and P_{31} planes in HfO_3 plate. The ZGV points are very sensitive to the anisotropy. For example, the S_{26} mode has one ZGV point in the P_{13} plane, which becomes three ZGV points in the P_{31} plane (see Fig. 3). The same behavior was found in all other modes.

Table 4 shows a list of the symmetric Lamb modes according to the number of ZGV points in the P_{13} and P_{31} planes, respectively: this proves the strong dependencies of ZGV points on the propagation direction. For example, the S_{44} mode has one ZGV point in the P_{13} plane, which becomes three ZGV points in the P_{31} plane (see Table 4). The ZGV frequency is more sensitive to the crystallographic plane in which the waves travel than to the cutoff frequency. While the cutoff frequency is only dependent on the change of propagation plane when N , the number of nodes across the thickness of the plate, is odd, the ZGV frequencies are different in all cases.

Can. J. Phys. Downloaded from www.nrcresearchpress.com by TULANE UNIVERSITY on 02/08/19
For personal use only. This Just-IN manuscript is the accepted manuscript prior to copy editing and page composition. It may differ from the final official version of record.

E.g. in case of the S_4 mode, even the number of ZGV points changes when changing the plane from P_{13} to P_{31} .

plane	Modes with no ZGV point	Modes with one ZGV point	Modes with two ZGV points	Modes with three ZGV points	Modes with four ZGV points
P_{31}	S_0, S_2, S_4, S_5 S_9, S_{16}	S_1, S_3, S_6, S_7, S_8 $S_{10}, S_{12}, S_{14}, S_{18},$ $S_{20}, S_{22}, S_{24}, S_{30}$ S_{32}	$S_{11}, S_{13}, S_{17}, S_{19},$ $S_{21}, S_{23}, S_{25}, S_{27},$ $S_{33}, S_{30}, S_{35}, S_{37},$ S_{40}	S_{26}, S_{28}, S_{36} S_{31}, S_{38}, S_{42} S_{44}, S_{46}	S_{39}
P_{13}	S_0, S_2, S_3, S_5, S_9 S_{10}, S_{13}, S_{16}	S_1, S_4, S_6, S_7, S_8 $S_{12}, S_{14}, S_{18}, S_{20}$ $S_{22}, S_{24}, S_{26}, S_{32}$ S_{34}, S_{44}	$S_{11}, S_{15}, S_{17}, S_{19},$ $S_{21}, S_{23}, S_{25}, S_{27},$ $S_{28}, S_{29}, S_{30}, S_{31},$ S_{36}, S_{38}, S_{42}	S_{40}, S_{46}	

TABLE IV: Classification of symmetric Lamb modes according to the number of ZGV points in the P_{13} and P_{31} plane, respectively.

Fig. 4 shows the occurrence of multiple ZGV points limited to Lamb waves propagating in the $P_{13}, P_{31}, P_{12}, P_{21}, P_{23}$, and P_{32} planes, with less recurrence in P_{12}, P_{21} and P_{23} . The S_1 mode

possesses a single ZGV point, regardless of the plane of propagation. However, as far as the other modes are concerned, the number of their ZGV points differs from one plane to another. This is due to the influence of the propagation direction on the ZGV phenomenon. E.g. in case of the S_{11} mode, even the number of ZGV points changes when changing the plane.

FIG.4. Dispersion curves of S_1 , S_{11} , S_{28} and S_{36} symmetric Lamb waves running along an iodic acid plate in different crystallographic planes.

2. Effect of elastic constants on ZGV frequencies

Continuing earlier research on ZGV modes [7, 19, 51], in the following, we have investigated the dependence of the ZGV frequencies on the values of the different elastic constants (c_{11} , c_{13} , c_{33} , c_{55}). Each constant was successively varied by 5% and 10% with respect to the values with respect to the values listed in Table 1, and propagation in the P_{13} plane was considered. A general observation is that each ZGV frequency depends on specific constants. Fig.5, for

instance, displays the dispersion of the S_1 mode for different variations of the elastic constants. The elastic constants that affect the S_1 -ZGV frequency mostly are c_{11} and c_{33} . Variations in c_{13} and c_{55} hardly result in any effect.

FIG.5. Effect of variations of the elastic constants c_{11} , c_{13} , c_{33} , c_{55} on the S_1 mode propagating in the P_{13} plane of an iodic acid – like plate (0% original plate properties, +5% and +10% compared to the original values). Interestingly, in case of the S_7 mode (Fig.6), which possesses a single ZGV point in a genuine iodic acid plate, a change of c_{33} and c_{55} does not only considerably affect the ZGV frequency, but, when large enough, it leads to the emergence of an additional ZGV point (for a 10% increase of c_{33}), or the disappearance of one (for a 5% or 10% increase of c_{55}). Variations in c_{11} and c_{13} only lead to a moderate change of ZGV frequency.

Also for the S_{30} mode (Fig.7), changes in c_{11} and c_{13} hardly result in any effect, while changes in c_{33} and c_{55} lead to the emergence or disappearance of ZGV points and to substantial changes of ZGV frequencies.

FIG.6. Effect of variations of the elastic constants c_{11} , c_{13} , c_{33} , c_{55} on the S_7 mode propagating in the P_{13} plane of an ionic acid plate (0%: original plate properties, +5% and +10% compared to the original values).

FIG.7. Effect of variations of the elastic constants c_{11} , c_{13} , c_{33} , c_{55} on the S_{30} mode propagating in the P_{13} plane of an ionic acid plate (0%: original plate properties, +5% and +10% compared to the original values) .

The observation that variations of c_{33} and c_{55} have a remarkably large effect on the existence and features of the S_7 and S_{30} mode can be understood from the fact that the concerned ZGV frequencies are quite close to cutoff frequencies, which in turn are directly linked to c_{33} and c_{55} via the velocities of longitudinal ($v_L=(c_{33}/\rho)^{0.5}$) and transverse ($v_T=(c_{55}/\rho)^{0.5}$) bulk waves, which determine the standing wave conditions across the plate thickness. The very high sensitivity of the ZGV features to c_{33} and c_{55} infer the efficiency of ZGV analysis for solving the inverse problem, i.e. determining c_{33} and c_{55} by detecting ZGV modes.

C. Antisymmetric case

The analysis presented for symmetric Lamb waves in section III.B was also carried out for antisymmetric waves. Similar tendencies were found. Figs.8 and 9, for instance, display a pronounced difference in ZGV behavior between waves propagating in different crystallographic planes.

FIG. 8. Dispersion curves of A_4 , A_{27} , A_{29} and A_{37} Lamb waves propagating in the P_{31} and P_{13} crystallographic planes.

Table V lists a classification of antisymmetric Lamb modes propagating in the P_{13} plane and in the P_{31} plane of an iodic acid plate, according to the number of ZGV points. Again the number of ZGV points depends on the crystallographic plane of propagation. E.g. the A_{16} mode has a single ZGV point in the P_{13} plane, compared to two in the P_{31} plane.

Can. J. Phys. Downloaded from www.nrcresearchpress.com by TULANE UNIVERSITY on 02/08/19
For personal use only. This Just-IN manuscript is the accepted manuscript prior to copy editing and page composition. It may differ from the final official version of record.

plane	Modes with no ZGV point	Modes with one ZGV point	Modes with two ZGV points	Modes with three ZGV points	Modes with four ZGV points
P_{31}	A_0, A_1, A_3, A_4 A_8, A_9, A_{12} A_{29}	A_2, A_5, A_{11}, A_{13} $A_{15}, A_{16}, A_{17}, A_{19}$ $A_{21}, A_{23}, A_{25}, A_{27}$ $A_{31}, A_{33}, A_{37}, A_{39}$	$A_6, A_{10}, A_{14}, A_{18},$ $A_{20}, A_{22}, A_{24}, A_{26},$ $A_{28}, A_{30}, A_{25}, A_{27},$ A_{31} $A_{33}, A_{32}, A_{34}, A_{36}$ A_{38}, A_{40}	A_{35}, A_{41}	A_{44}
P_{13}	A_0, A_1, A_3, A_5 A_6, A_8, A_{17}	$A_2, A_4, A_7, A_9,$ $A_{11}, A_{13}, A_{19}, A_{21},$ $A_{23}, A_{29}, A_{31}, A_{33},$ A_{37}, A_{39}	$A_{10}, A_{12}, A_{16}, A_{18}$ $A_{20}, A_{22}, A_{24}, A_{26}$ $A_{28}, A_{30}, A_{32}, A_{36}$	$A_{27}, A_{34},$ $A_{37}, A_{38},$ A_{39}, A_{40} A_{41}, A_{43}	

TABLE V: Classification of antisymmetric Lamb modes according to the number of ZGV points in the P_{13} and P_{31} plane, respectively.

Further on, the following observations can be made: (i) Changing the plane in which the waves are propagating affects cutoff frequencies if N , the number of the nodes across the plate thickness of the standing wave associated with those frequencies is even. In case N is odd, the corresponding cutoff frequency remains unaffected. This correlation is opposite in the case of symmetric Lamb waves. (ii) The modes that exhibit ZGV points exhibit either 2 or 4 of them when N is even and 1 or 3 when N is odd. Lamb modes with 4 ZGV points occur both in the symmetric and antisymmetric case. (iii) The lowest antisymmetric mode for which multiple ZGV points are present is A_6 . In the case of symmetric mode, the lowest order mode with more than one ZGV point is S_{11} . (iv) The influence of changing elastic constants on the ZGV features is similar to the symmetric case: C_{33} and C_{55} have an effect on the number of ZGV points, while C_{11} and C_{13} only influence the ZGV frequency (see Figs.10 and 11).

FIG. 9. Dispersion curves of A_2 , A_6 , A_{27} and A_{37} antisymmetric Lamb modes running along an iodic acid plate in different crystallographic planes.

FIG.10. Effect of variations of the elastic constants c_{11} , c_{13} , c_{33} , c_{55} on the A_6 mode propagating in the P_{13} plane of an ionic acid plate (0%: original plate properties, +5% and +10% compared to the original values).

FIG.11. Effect of variations of the elastic constants c_{11} , c_{13} , c_{33} , c_{55} on the A_{12} mode propagating in the P_{13} plane of an iodic acid plate (0%: original plate properties, +5% and +10% compared to the original values).

IV. CONCLUSION

The paper studies the propagation of symmetric and anti-symmetric Lamb waves along two mutually orthogonal propagation directions in three crystallographic planes. A detailed analysis is presented of dispersion curves in the P_{13} plane. Several zero group velocity (ZGV) points were found and the dependence of their frequencies on each elastic constant (c_{11} , c_{13} , c_{33} , c_{55}) of the HIO_3 plate material was investigated. It was found that c_{33} and c_{55} affect the number of the ZGV points, while c_{11} and c_{13} affect the frequency of the ZGV point. The study has demonstrated that the ZGV points are strictly related to the material plate constants and to its crystallographic properties: this suggests that the experimental assessment of ZGVs should give

an adequate approach to extract some of the elastic property of plates, and to detect in a very sensitive way defects that affect the plate elasticity.

ACKNOWLEDGMENTS

The authors are grateful for the funding provided to LPM laboratory by the Tunisian Ministry of Higher Education and Scientific Research. They also extend their thanks to Mrs. Leila Mahfoudhi and Mr. Wassim Hariz, English Language Teachers at the Faculty of Science of Sfax, for proofreading and polishing the language of the manuscript.

REFERENCES

[1] Lynnworth, C. Lawrence. Academic press, (2013).

[2] W. J. Staszewski, B. C. Lee, R. Traynor. Meas. Sci. Technol. **18**(3), 727, (2007).

[3] L. Martinez, N. Wilkie-Chancellier, C. Glorieux, B. Sarens, E. Caplain. J. Phys. Conf. Ser. **195**, 012009, (2009).

[4] L. Martinez, N. Wilkie-Chancellier, C. Glorieux. J. Acoust. Soc. Am. **123**(5), 3846-3846, (2008).

[5] W. Gao, C. Glorieux. AJT. Int. J. Eng. Sci. **41**, 219, (2003).

[6] M. Cès, D. Royer, C. Prada. J. Acoust. Soc. Am. **132**(1), 180, (2012).

[7] B. Verstraeten, X. Xu, L. Martinez, C. Glorieux. AIP Conference Proceedings. **1433**, 423–6, (2012).

[8] W. Gao, C. Glorieux, K. Van De Rostyne, M. Wevers, W. Lauriks, J. Thoen. AIP Conf. Proc. **454**, 1, (1999).

[9] L. Yuan, Z. Shen, X. Ni, J. Lu. J. Appl. Phys. **106**(2), 023529, (2009).

[10] T. Hussain, F. Ahmad. J. Acoust. Soc. Am. **132**(2), 641, (2012).

[11] O. Balogun, T. W. Murray, C. Prada. J. Appl. Phys. **102**(6), 1, (2007).

- 367 [12] C. Prada, O. Balogun, T. W. Murray. *Appl. Phys. Lett.* **87**(19), 1, (2005).
- 368 [13] M. Cès, D. Clorennec, D. Royer, C. Prada. *Rev. Sci. Instrum.* **82**(11), 114902, (2011).
- 369 [14] M. Cès, D. Clorennec, D. Royer, C. Prada. *J. Acoust. Soc. Am.* **130**(2), 689, (2011).
- 370 [15] C. Prada, D. Clorennec, D. Royer. *J. Acoust. Soc. Am.* **124**(1), 203, (2008).
- 371 [16] D. Clorennec, C. Prada, D. Royer. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control.*
- 372 **57**(5), 1125, (2010).
- 373 [17] S. D. Holland, D. E. Chimenti. *Ultrasonics.* **42**(1–9), 957, (2004).
- 374 [18] A. Gibson, J. S. Popovics. *J. Eng. Mech.* **131**(4), 438, (2005).
- 375 [19] C. Prada, D. Clorennec, T. W. Murray, D. Royer. *J. Acoust. Soc. Am.* **126**(2), 620,
- 376 (2009).
- 377 [20] C. Grünsteidl, T. W. Murray, T. Berer, I. A. Veres. *Ultrasonics.* **65**(10), 1, (2016).
- 378 [21] M. S. Bouhdima, M. Zagrouba, M. H. Ben Ghazlen. *Can. J. Phys.* **90**(2), 159, (2012).
- 379 [22] C. Prada, D. Clorennec, D. Royer. *J. Appl. Phys.* **101**(3), 1, (2007).
- 380 [23] O. Tofeldt, N. Ryden. *J. Acoust. Soc. Am.* **141**(5), 3302, (2017).
- 381 [24] C. Prada, D. Clorennec, D. Royer, T. W. Murray. *Appl. Phys. Lett.* **89**(2), 3, (2006).
- 382 [25] V. Tournat, S. Raetz, N. Chigarev, C. Mechri, J. Blondeau, B. Campagne, V. E. Gusev,
- 383 V. Tournat. *NDT E Int.* **85**, 13, (2017).
- 384 [26] I. Tolstoy, E. Usdin. *J. Acoust. Soc. Am.* **29**(1), 37, (1957).
- 385 [27] K. Negishi. 1978. *J. Acoust. Soc. Am.* **64**(6), S63, (1978).
- 386 [28] C. Prada, D. Clorennec, D. Royer. *Wave Motion.* **45**(6), 723, (2008).
- 387 [29] M. Zagrouba, M. S. Bouhdima. *Can. J. Phys.* **94**(11), 1189, (2016).
- 388 [30] M. Zagrouba, M. S. Bouhdima, M. H. Ben Ghazlen. *Adv. Acoust. Vib.* **2014**, 1, (2014).
- 389 [31] F. M. Werby, H. Uberall. *J. Acoust. Soc. Am.* **111**(6), 2686, (2002).
- 390 [32] A. H. Meitzler. *J. Acoust. Soc. Am.* **38**(5), 835, (1965).
- 391 [33] E. Kausel. *J. Acoust. Soc. Am.* **131**(5), 3601, (2012).

- 392 [34] V. N. Parygin, A. V. Vershoubskiy, V. G. Mozhaev, M. Weihnacht. Ultrasonics. **38**(1),
 393 594, (2000).
- 394 [35] H. Cui, W. Lin, H. Zhang, X. Wang, J. Trevelyan. J. Acoust. Soc. Am. **135**(4), 2419-
 395 2419, (2014).
- 396 [36] H. Cui, W. Lin, H. Zhang, X. Wang, J. Trevelyan. J. Acoust. Soc. Am. **135**(6), 3398,
 397 (2014).
- 398 [37] R. F. S. Hearmon, A. Maradudin. A. Physics Today, **14**, 48, (1961).
- 400
- 401 [38] R. Wasley. Stress wave propagation in solids: an introduction, Monographs and
 402 textbooks in material science, **7**, (1973).
- 403 [39] W. P. Mason. J. Acoust. Soc. Am. **28**(6), 1197-1206, (1956).
- 404 [40] G. Bohme. North-Holland Series in Applied Mathematics and Mechanics, (1987).
- 405 [41] A. H. Nayfeh, D. E. Chimenti. J. Appl. Mech. **56**(12), 181, (1989).
- 406 [42] D. Royer, E. Dieulesaint. Masson, **2**, (1999).
- 407 [43] I. A. Viktorov. Plenum Press, New York **2013**, 1-9, (1967).
- 408 [44] B. A. Auld. Wiley-Interscience publication. (1973).
- 409 [45] S. Dahmen, H. Ketata, M. H. Ben Ghazlen, B. Hosten. Ultrasonics, **50**(4-5), 502,
 410 (2010).
- 411 [46] M. Musgrave. Holden-Day, (1970).
- 412 [47] C. C. Habeger, R. W. Mann, G. A. Baum. Ultrasonics, **17**(2), 57-62, (1979).
- 413 [48] S. A. Markus, M. D. Kaplan, S. V. Veremeenko. SOVIET JOURNAL OF
 414 NONDESTRUCTIVE TESTING-USSR, **21**(11), 739-744, (1985).
- 415 [49] S. Dahmen, M. Ben Amor, M. H. Ben Ghazlen. Compos. Struct. **153**, 557, (2016).
- 416 [50] E. L. Adler, J. K. Slaboszewicz, G. W. Farnell, C. K. Jen. IEEE Trans. Ultrason.
 417 Ferroelectr. Freq. Control. **37**(3), 215, (1990).

418 [51] J. Laurent, D. Royer, T. Hussain, F. Ahmad, C. Prada. J. Acoust. Soc. Am. **137**(6),
419 3325, (2015).

420