SCALABLE AND DISTRIBUTED MMSE ALGORITHMS FOR UPLINK RECEIVE COMBINING IN CELL-FREE MASSIVE MIMO SYSTEMS

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ABSTRACT

In cell-free Massive MIMO systems, a large number of distributed wireless access points (AP) are simultaneously serving a number of user equipments (UEs). This setup has the ability to offer a good quality of service, be it that there is still a need for low-complexity signal processing algorithms. In this paper, the problem of optimal uplink receive combining is tackled by providing an efficient distributed MMSE algorithm, with a minimal number of exchanged parameters between the APs and the network center. Scalable versions of this distributed MMSE algorithm are also proposed ensuring that the algorithm can be used in large networks with many UEs.

Index Terms— Cell-free Massive MIMO, uplink receive combining, distributed algorithms, user-centric networking

1. INTRODUCTION

Cell-free Massive MIMO systems have recently been introduced [1, 2] where a large number of access points (AP) jointly serve a smaller number of user equipments (UEs). The APs use channel estimates, possibly obtained from received uplink pilots, and apply receive combining and transmit beamforming to transfer data from and to the UEs. It has been shown that Massive MIMO systems provide better performance compared to small-cell systems, even with the simple local maximum-ratio (MR) combining scheme [2–4].

An improved performance is obtained when the simple MR combining scheme is replaced with minimum mean squared error (MMSE) combining schemes [5–7], where typically the channel state information (CSI) has to be transmitted to a network center (NC) in order to determine the receiver vectors. The NC can either be a physical processing unit that is responsible for processing the signals of all UEs, or can be seen as a virtual set of tasks that are performed somewhere in the network. Although a significant performance increase can be achieved, the drawbacks of network-wide MMSE combining schemes, namely the need for centralizing the CSI and the increased computational complexity when the number of UEs and APs grows large, make them not very practical.

In this paper, the problem of optimal uplink receive combining is tackled where these drawbacks are resolved. An efficient distributed MMSE algorithm is proposed where the CSI of an AP is required only locally and only a small number of parameters have to be exchanged between the APs and the NC. Scalable versions of this distributed MMSE algorithm based on a user-centric approach [4, 7]

ensure that the algorithm can also be used for large networks with many UEs. The paper also includes simulations to show the performance of the proposed algorithms.

2. SIGNAL MODEL

Consider a cell-free Massive MIMO system consisting of K single-antenna UEs and L APs randomly deployed over the considered area, with M_l antennas in the l-th AP and with local processing capabilities in each AP. The APs are connected to a NC via a physical network. This setup allows for coherent reception of data from the UEs. In the cell-free Massive MIMO literature [1, 8] it is often assumed that $M \gg K$ and that both M and K are large, with $M = \sum_{l=1}^{L} M_l$ the total number of antennas in the considered area.

The UEs use τ_u time slots for uplink data transmission and τ_p time slots are reserved for channel estimation[1]. The channel from UE k to AP l is denoted by $\mathbf{h}_{kl} \in \mathbb{C}^{M_l}$ such that the channel from UE k and all the APs is given by $\mathbf{h}_k = [\mathbf{h}_{k1}^T ... \mathbf{h}_{kL}^T]^T \in \mathbb{C}^M$. The channel $\mathbf{h}_{kl}$ is assumed to remain constant during a coherence block $\tau_c = \tau_p + \tau_u$ and can be approximated as being drawn from an independent correlated Rayleigh fading realization $\mathcal{NC}(0, \mathbf{R}_{kl})$. $\mathbf{R}_{kl} \in \mathbb{C}^{M_l \times M_l}$ is a positive semi-definite spatial correlation matrix describing the large-scale fading, including geometric pathloss, shadowing, antenna gains, and spatial channel correlation [9]. The Complex Gaussian distribution models the small-scale fading. Due to the spatial distribution of the APs in the network, the channel vectors of different APs are independently distributed, i.e. $E\{\mathbf{h}_{kl}\mathbf{h}_{kn}^H\} = \mathbf{0}_{M_l \times M_n}$ for $l \neq n$, such that the channel estimation can be performed independently at each AP.

2.1. Channel estimation

It is assumed that AP l can obtain a local estimate $\hat{\mathbf{h}}_{kl}$ of $\mathbf{h}_{kl} = \hat{\mathbf{h}}_{kl} + \tilde{\mathbf{h}}_{kl}$ for UE k in each coherence block. Furthermore, the estimation is assumed to be unbiased with an estimation error $\tilde{\mathbf{h}}_{kl}$ that is uncorrelated with the estimation $\hat{\mathbf{h}}_{kl}$ and with known variance $\mathbf{C}_{kl}$:

$$\tilde{\mathbf{h}}_{kl} \sim \mathcal{NC}(0, \mathbf{C}_{kl}). \tag{1}$$

There exist multiple channel estimation techniques, that provide these quantities for example based on training sequences [7, 10] or Bayesian learning [11], where often an estimate of the spatial correlation matrix $\mathbf{R}_{kl}$ is required.

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[1]The uplink receive combining schemes considered in this paper can also be used for downlink transmit beamforming when the APs and UEs operate using a TDD protocol exploiting the duality between uplink and downlink [9].

2.2. Uplink signal model

During uplink data transmission, the received signal $\mathbf{y}_l \in \mathbb{C}^{M_l}$ at AP l is given by

$$\mathbf{y}_l = \sum_{k=1}^{K} \mathbf{h}_{kl} s_k + \mathbf{n}_l = \mathbf{H}_l \mathbf{s} + \mathbf{n}_l \tag{2}$$

where $s_k \in \mathbb{C}$ is the signal transmitted by UE k with transmit power $p_k = E\{s_k s_k^H\}$ and $\mathbf{n}_l \sim \mathcal{NC}(0, \mathbf{R}_{\mathbf{n}_l \mathbf{n}_l})$ is an additive Gaussian noise component, modeling antenna noise and quantization noise. The noise components of the different antennas of an AP are often assumed to be independent, i.e. $\mathbf{R}_{\mathbf{n}_l \mathbf{n}_l} = \sigma^2 \mathbf{I}_{M_l}$, but here a more general case is considered with a general $\mathbf{R}_{\mathbf{n}_l \mathbf{n}_l}$. Furthermore, $\mathbf{H}_l = [\mathbf{h}_{1l} \; ... \; \mathbf{h}_{Kl}]$ is the concatenation of the channels from all the UEs to AP l and $\mathbf{s} = [s_1 \; ... \; s_K]^T$. Stacking the received signals of all APs in $\mathbf{y} = [\mathbf{y}_1^T \; ... \; \mathbf{y}_L^T]^T \in \mathbb{C}^M$ as well as the noise components in $\mathbf{n} = [\mathbf{n}_1^T \; ... \; \mathbf{n}_L^T]^T \in \mathbb{C}^M \sim \mathcal{NC}(0, \mathbf{R}_{\mathbf{nn}})$ where $\mathbf{R}_{\mathbf{nn}} = \text{Blkdiag}\{\mathbf{R}_{\mathbf{n}_1 \mathbf{n}_1}, ..., \mathbf{R}_{\mathbf{n}_L \mathbf{n}_L}\}$, results in the network-wide signal model:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{3}$$

with $\mathbf{H} = [\mathbf{H}_1^T \; ... \; \mathbf{H}_L^T]^T = [\mathbf{h}_1 \; ... \; \mathbf{h}_K]$.

2.3. Uplink receive combining

In network-wide receive combining the signals $\mathbf{s}$ are estimated by linearly combining the received signals $\mathbf{y}$ by means of a receiver matrix $\mathbf{V} \in \mathbb{C}^{M \times K}$. Note that this linear combining can be performed in the network if AP l selects the local receiver matrix $\mathbf{V}_l = [\mathbf{v}_{1l} \; ... \; \mathbf{v}_{Kl}] \in \mathbb{C}^{M_l \times K}$ in $\mathbf{V} = [\mathbf{V}_1^T \; ... \; \mathbf{V}_L^T]^T$ and computes the local estimate $\mathbf{z}_l = \mathbf{V}_l^H \mathbf{y}_l$. The NC then estimates $\mathbf{s}$ by combining the local estimates as

$$\hat{\mathbf{s}} = \sum_{l=1}^{L} \mathbf{z}_l = \sum_{l=1}^{L} \mathbf{V}_l^H \mathbf{y}_l = \mathbf{V}^H \mathbf{y}. \tag{4}$$

The goal is then to choose a local receiver matrix $\mathbf{V}_l$ that provides a good estimate $\hat{\mathbf{s}}$, but where the CSI of an AP is required only locally. In cell-free Massive MIMO literature a MR combing scheme is often used with $\mathbf{V}_l = \hat{\mathbf{H}}_l$ [2–4]. Other heuristic schemes that perform generally better, but require more processing power of the AP are local MMSE combining schemes [12]. In this paper, network-wide MMSE receive combining schemes [7] will be considered, requiring typically network-wide CSI. However, in Section 3 it is shown that if a small number of parameters can be exchanged between the NC and the APs, this network-wide MMSE receive combining can still be obtained efficiently at the NC where the CSI is used only locally leading to an efficient distributed MMSE algorithm. Since the number of combining vectors that an AP has to compute, grows with the number of UEs in the network, inspired by [7], scalable versions of this distributed MMSE algorithm are also derived, resulting in combining schemes that scale independently of the number of UEs in the network presented in Section 4.

3. DISTRIBUTED MMSE RECEIVE COMBINING

The network-wide MMSE receiver matrix $\mathbf{V}^{\text{N-RC}} = [\mathbf{v}_1^{\text{N-RC}} ... \mathbf{v}_K^{\text{N-RC}}]$ is obtained by minimizing the mean squared error between the transmitted signal $\mathbf{s}$ and the estimate obtained by linearly combining the received signals $\mathbf{y}$

$$\mathbf{V}^{\text{N-RC}} = \arg\min_{\mathbf{V}} E\{||\mathbf{s} - \mathbf{V}^H \mathbf{y}||^2\} \tag{5}$$

where $E\{.\}$ is the expected value operator and $||.||$ is the Euclidean norm. The optimal solution of this convex optimization problem has a closed form and is given by

$$\mathbf{V}^{\text{N-RC}} = E\{\mathbf{y}\mathbf{y}^H\}^{-1} E\{\mathbf{y}\mathbf{s}^H\} \tag{6}$$

with the uplink correlation matrix $E\{\mathbf{y}\mathbf{y}^H\}$ given as

$$\begin{aligned} E\{\mathbf{y}\mathbf{y}^H\} &= E\{\mathbf{H}\mathbf{s}\mathbf{s}^H\mathbf{H}^H\} + E\{\mathbf{n}\mathbf{n}^H\} \\ &= \hat{\mathbf{H}}E\{\mathbf{s}\mathbf{s}^H\}\hat{\mathbf{H}}^H + E\{\tilde{\mathbf{H}}\mathbf{s}\mathbf{s}^H\tilde{\mathbf{H}}^H\} + E\{\mathbf{n}\mathbf{n}^H\} \\ &= \hat{\mathbf{H}}\mathbf{P}\hat{\mathbf{H}}^H + \sum_{k=1}^{K} p_k \mathbf{C}_k + \mathbf{R}_{\mathbf{nn}} \end{aligned} \tag{7}$$

where $\mathbf{C}_k = \text{Blkdiag}\{\mathbf{C}_{k1}, ..., \mathbf{C}_{kL}\}$ and $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{P} = \text{diag}\{p_1, ..., p_K\}$. In the second step, $\mathbf{H}$ is replaced by $\hat{\mathbf{H}} + \tilde{\mathbf{H}}$ and the fact that $\hat{\mathbf{H}}$ and $\tilde{\mathbf{H}}$ are uncorrelated is also used. In the last step, independence between the signals and the channel estimation error is used. Furthermore, the cross-correlation matrix $E\{\mathbf{y}\mathbf{s}^H\}$ is given by

$$E\{\mathbf{y}\mathbf{s}^H\} = \hat{\mathbf{H}}\mathbf{P}. \tag{8}$$

The closed form expression for the network-wide MMSE receiver matrix $\mathbf{V}^{\text{N-RC}}$ is then obtained as

$$\mathbf{V}^{\text{N-RC}} = (\underbrace{\hat{\mathbf{H}}\mathbf{P}\hat{\mathbf{H}}^H + \sum_{k=1}^{K} p_k \mathbf{C}_k + \mathbf{R}_{\mathbf{nn}}}_{\mathbf{T}})^{-1}\hat{\mathbf{H}}\mathbf{P}. \tag{9}$$

It is shown [9] that the receiver vector $\mathbf{v}_k^{\text{N-RC}}$ maximizes the achievable spectral efficiency (SE) of UE k given by

$$\text{SE}_k = \frac{\tau_u}{\tau_c} E\{\log_2(1 + \text{SINR}_k)\} \tag{10}$$

where the expectation is with respect to the different channel realizations and where SINR_k is given by the ratio

$$\frac{p_k |\mathbf{v}_k^H \hat{\mathbf{h}}_k|^2}{\sum_{i=1, i\neq k}^{K} p_i |\mathbf{v}_k^H \hat{\mathbf{h}}_i|^2 + \mathbf{v}_k^H \mathbf{T} \mathbf{v}_k} \tag{11}$$

which will be used as a performance measure in the simulations.

To obtain this filter, all the APs have to send their local estimate $\hat{\mathbf{H}}_l \in \mathbb{C}^{M_l \times K}$, estimation error variance $\sum_{k=1}^{K} p_k \mathbf{C}_{kl} \in \mathbb{C}^{M_l \times M_l}$ and $\mathbf{R}_{\mathbf{n}_l \mathbf{n}_l}$ to the NC, which leads to a significant communication cost, especially when the number of antennas M_l of an AP l is large. The NC then has to invert an $M \times M$ matrix to obtain $\mathbf{V}^{\text{N-RC}}$. During receive combining, the NC needs to have access to all M received signals $\mathbf{y}$, which requires a larger network communication than when the local estimates can be combined in the network as in (4).

However the expression for the network-wide MMSE receiver matrix $\mathbf{V}^{\text{N-RC}}$ can be rewritten as

$$\begin{aligned} \mathbf{V}^{\text{N-RC}} &= \left[\mathbf{T}^{-1} - \mathbf{T}^{-1}\hat{\mathbf{H}}\left(\mathbf{P}^{-1} + \hat{\mathbf{H}}^H\mathbf{T}^{-1}\hat{\mathbf{H}}\right)^{-1}\hat{\mathbf{H}}^H\mathbf{T}^{-1}\right]\hat{\mathbf{H}}\mathbf{P} \\ &= \mathbf{T}^{-1}\hat{\mathbf{H}}\left(\mathbf{P}^{-1} + \hat{\mathbf{H}}^H\mathbf{T}^{-1}\hat{\mathbf{H}}\right)^{-1} \\ &= \begin{bmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_L \end{bmatrix}\left(\mathbf{P}^{-1} + \mathbf{X}\right)^{-1} \end{aligned}$$
$$\tag{12}$$

with

$$\mathbf{W}_l = \left(\sum_{k=1}^{K} p_k \mathbf{C}_{kl} + \mathbf{R}_{\mathbf{n}_l \mathbf{n}_l} \right)^{-1} \hat{\mathbf{H}}_l \qquad (13)$$

and

$$\mathbf{X} = \sum_{l=1}^{L} \hat{\mathbf{H}}_l^H \mathbf{W}_l. \qquad (14)$$

The Sherman-Morrison-Woodbury formula and the fact that $\mathbf{T}$ is a block-diagonal matrices are used in (12).

Based on this equivalence, an efficient way of obtaining the network-wide MMSE estimate is presented in Algorithm 1 as the network-wide distributed MMSE receive combining (N-DRC) algorithm. Here the CSI is only used locally to construct $\mathbf{W}_l$ and $\hat{\mathbf{H}}_l^H \mathbf{W}_l$, but does not need to be transmitted to the NC.

A simple procedure to obtain the in-network sum in step 2 of Algorithm 1 is based on the formation of a tree topology using the available physical links between the APs [13] with the NC as root node. A leaf node AP l with only one neighbor starts with transmitting its transformed signals to its neighbor. An AP l with more than one neighbor waits until it has received signals from all its neighbors, except one denoted by n and transmits $\mathbf{w}_l + \sum_{\bar{l} \in \{\mathcal{N}_l \setminus n\}} \mathbf{w}_{\bar{l}}$ to AP n, where $\mathcal{N}_l$ denotes the set of neighbors of node l. This continues until the root node NC has received signals from all its neighbors. The root node NC can then compute $\mathbf{w}$ straightforwardly. A similar procedure can be followed to construct $\mathbf{X}$, but since $\mathbf{X}$ is Hermitian symmetric, the transmission of only $\frac{K^2+K}{2}$ i.s.o. K^2 parameters is required.

Algorithm 1: Network-Wide Distributed MMSE Receive Combining (N-DRC)

Perform the following steps in each coherence block:

1. - Each AP l obtains a local estimate of $\hat{\mathbf{H}}_l$ and $\mathbf{R}_{\mathbf{n}_l \mathbf{n}_l}$ and computes $\mathbf{W}_l$ using (13).
   - Each AP l transmits the parameter $\hat{\mathbf{H}}_l^H \mathbf{W}_l \in \mathbb{C}^{K \times K}$ and the transformed signals $\mathbf{w}_l = \mathbf{W}_l^H \mathbf{y}_l \in \mathbb{C}^K$ for all received signals in the coherence block to the NC.

2. The network is used to perform an in-network sum to obtain

$$\mathbf{w} = \sum_{l=1}^{L} \mathbf{w}_l, \quad \mathbf{X} = \sum_{l=1}^{L} \hat{\mathbf{H}}_l^H \mathbf{W}_l. \qquad (15)$$

3. The NC then computes the network-wide MMSE estimate as

$$\hat{\mathbf{s}} = \left(\mathbf{P}^{-1} + \mathbf{X} \right)^{-H} \mathbf{w}. \qquad (16)$$

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4. SCALABLE DISTRIBUTED MMSE RECEIVE COMBINING

4.1. Scalability issue and solution

The N-DRC algorithm presented in the previous section scales with the number of UEs K in the network. Each AP needs to compute $\mathbf{W}_l$ for all UEs in the network. Therefore an AP has to estimate all channels $\hat{\mathbf{H}}_l$ and transmits a $K \times K$ matrix in each coherence block. Since the received signal $\mathbf{h}_{kl} s_k$ at AP l becomes weaker when the distance between AP l and UE k increases, the estimate $\hat{\mathbf{h}}_{kl}$ will be

worse due to background noise and interference from other UEs that are in the proximity of AP l. Also the number of parameters that need to be transmitted and received in each iteration, may become too large for the obtained benefit in performance.

As proposed in [7], this issue can be solved by moving to a user-centric approach, where a UE k is only served by a subset of APs for which a good channel estimate $\hat{\mathbf{h}}_{kl}$ can be obtained. This will be represented by defining the binary serving matrix $\mathbf{D}$ as

$$[\mathbf{D}]_{kl} = \begin{cases} 1 & \text{if AP } l \text{ is serving UE } k \\ 0 & \text{else.} \end{cases} \qquad (17)$$

Defining the set of UEs that are served by AP l as $\mathcal{D}_l = \{k | D_{kl} = 1\}$, each AP l only needs to compute a local receiver vector $\mathbf{v}_{kl} \; \forall k \in \mathcal{D}_l$ instead of for all UEs in the network. Heuristic approaches to obtain $\mathbf{D}$ such that $|\mathcal{D}_l|$ (where $|.|$ denotes the cardinality of a set) is constant or independent of the total number of UEs K, are presented in [7] and it is assumed that the NC knows the UE-assignment. By also bounding the number of interfering UEs in the MMSE estimation, fully scalable MMSE receive combining objectives can be proposed for which a distributed algorithm can be derived. Two scalable objectives are presented in the next subsections.

4.2. Scalable network-wide distributed MMSE receive combining

In this scalable version of N-DRC, each AP l only estimates $\mathbf{h}_{kl}$ if $k \in \mathcal{D}_l$ and ignores the effect of the other channels by setting them to 0, i.e. $\hat{\mathbf{h}}_{kl} = \mathbf{0}$ and $\mathbf{C}_{kl} = \mathbf{0}$ if $[\mathbf{D}]_{kl} = 0$. If these modifications are used in (5), a similar expression for the scalable network-wide MMSE receiver matrix $\mathbf{V}^{\text{SN-RC}}$ as (12) is obtained, but with a different expression for $\mathbf{W}_l$ and $\mathbf{X}$ given by

$$\mathbf{W}_l^{\text{S}} = \left(\sum_{k \in \mathcal{D}_l} p_k \mathbf{C}_{kl} + \mathbf{R}_{\mathbf{n}_l \mathbf{n}_l} \right)^{-1} \hat{\mathbf{H}}_l \mathbf{D}_l \qquad (18)$$

and

$$\mathbf{X}^{\text{S}} = \sum_{l=1}^{L} \mathbf{D}_l \hat{\mathbf{H}}_l^H \mathbf{W}_l^{\text{S}} \qquad (19)$$

where the diagonal matrix $\mathbf{D}_l$ has 1 on its k'th diagonal element if $[\mathbf{D}]_{kl} = 1$ and zero otherwise. The N-DRC algorithm can be transformed to the scalable network-wide distributed MMSE receive combining (SN-DRC) algorithm by replacing the matrices $\mathbf{W}_l$ and $\mathbf{X}$ with the scalable versions defined above. Since here only $|\mathcal{D}_l|$ elements of $\mathbf{w}_l$ and $|\mathcal{D}_l| \times |\mathcal{D}_l|$ elements of $\mathbf{D}_l \hat{\mathbf{H}}_l^H \mathbf{W}_l^{\text{S}}$ are non-zero, this will strongly reduce the transmitted data of an AP l. However, care should be taken when the in-network sums are constructed using a tree topology, since the different signals need to added in a coherent way.

4.3. Scalable partial distributed MMSE receive combining

Even with the communication reduction proposed in the previous section, the NC still has to invert a $K \times K$ matrix to construct the estimate $\hat{\mathbf{s}}$ in (16), which still scales with the number of UEs K. In [5] it is stated that the interference affecting UE k is mainly generated by a small subset of other UEs. Therefore, the subset of UEs

$$\mathcal{P}_k = \{i | \exists l : D_{kl} D_{il} = 1\} \subset \{1, ..., K\} \qquad (20)$$

Table 1: Comparison of proposed algorithms.

Scheme	Parameters transmitted by each AP	Parameters received at NC	PC at each AP	PC at NC
N-RC	$M_l K + \frac{M_l^2 + M_l}{2}$	$MK + \sum_{l=1}^{L} \frac{M_l^2 + M_l}{2}$	-	$O(M^3)$
N-DRC	$\frac{K^2 + K}{2}$	$\frac{K^2 + K}{2}$	$O(M_l^3)$	$O(K^3)$
SN-DRC	$\frac{|\mathcal{D}_l|^2 + |\mathcal{D}_l|}{2}$	$\frac{K^2 + K}{2}$	$O(M_l^3)$	$O(K^3)$
SP-DRC	$\frac{|\mathcal{D}_l|^2 + |\mathcal{D}_l|}{2}$	$\frac{K^2 + K}{2}$	$O(M_l^3)$	$O(K|\mathcal{P}_k|^3)$

is assumed to have a significant effect on the received signals used to estimate $\hat{s}_k$. The subset considers all the UEs that have at least one AP in common with UE k.

As such, a heuristic partial MMSE receiver vector $\mathbf{v}_k^{\text{P-RC}}$ is proposed to estimate s_k:

$$\mathbf{v}_k^{\text{P-RC}} = \left(\hat{\mathbf{H}} \mathbf{Q}_k \mathbf{Q}_k^H \mathbf{P} \mathbf{Q}_k \mathbf{Q}_k^H \hat{\mathbf{H}}^H + \sum_{k=1}^{K} p_k \mathbf{C}_k + \mathbf{R}_{\mathbf{nn}} \right)^{-1} \hat{\mathbf{h}}_k p_k \tag{21}$$

with $\mathbf{Q}_k = \mathbf{I}_{:,\mathcal{P}_k}$, selecting the $|\mathcal{P}_k|$ columns of $\mathbf{I}_K$ with index $i \in \mathcal{P}_k$. This heuristic partial MMSE receiver vector $\mathbf{v}_k^{\text{P-RC}}$ can be rewritten as

$$\mathbf{v}_k^{\text{P-RC}} = \begin{bmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_L \end{bmatrix} \mathbf{Q}_k \left(\left(\mathbf{Q}_k^H \mathbf{P} \mathbf{Q}_k \right)^{-1} + \mathbf{Q}_k^H \mathbf{X}^S \mathbf{Q}_k \right)^{-1} \mathbf{q}_k \tag{22}$$

with $\mathbf{q}_k = \mathbf{Q}_k^H \mathbf{e}_k$. Substituting $\mathbf{W}_l$ and $\mathbf{X}$ with $\mathbf{W}_l^S$ and $\mathbf{X}^S$ from (18) and (19) respectively results in a fully scalable filter, denoted by $\mathbf{v}_k^{\text{SP-RC}}$. The N-DRC algorithm can again be adapted to provide the output of the scalable partial MMSE receiver vector $\mathbf{v}_k^{\text{SP-RC}}$ by changing the final combining method in step 3 of Algorithm 1 to

$$\hat{s}_k = \mathbf{q}_k^H \left(\left(\mathbf{Q}_k^H \mathbf{P} \mathbf{Q}_k \right)^{-1} + \mathbf{Q}_k^H \mathbf{X} \mathbf{Q}_k \right)^{-H} \mathbf{Q}_k^H \mathbf{w} \tag{23}$$

for each UE k and by replacing $\mathbf{W}_l$ and $\mathbf{X}$ with its scalable versions (18) and (19) respectively. The obtained algorithm will be denoted with the scalable partial distributed MMSE receive combining (SP-DRC) algorithm. A mayor advantage of the SP-DRC algorithm is that in (23) only a $|\mathcal{P}_k| \times |\mathcal{P}_k|$ matrix needs to be inverted i.s.o. the $K \times K$ matrix of (16).

As a summary, Table 1 gives a comparison of the different proposed algorithms in terms of the number of parameters that need to be exchanged in each coherence block as well as in terms of the processing complexity (PC) of performing the required inversion operation. The algorithms strongly reduce the communication requirement of the network and the PC at the NC, but require that each AP has local processing capabilities. The SP-DRC scales best when the number of UEs grows large (since $|\mathcal{D}_l|, |\mathcal{P}_k|$ are independent of K), but its performance will be shown to be suboptimal to the other algorithms.

5. SIMULATIONS

Numerical results are provided in this section to demonstrate the performance of the proposed distributed algorithms. A similar setup as [7] with the MMSE-channel estimator [12] is considered ($K = 100$
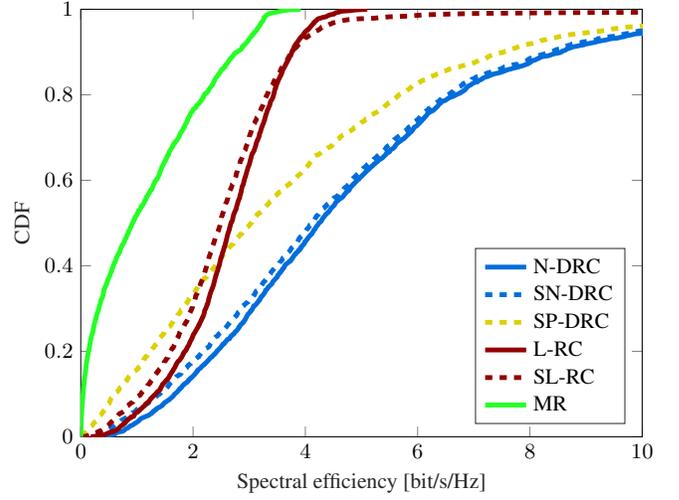

Fig. 1: Uplink SE per UE for the proposed algorithms.

UEs) but the number of APs is decreased to $L = 50$ with 16 antennas per AP . Figure 1 shows the cumulative distributed function (CDF) of the SE per UE, estimated using 25 network realizations and 1000 channel realizations for the different algorithms. Also the performance of the conventional MR combining scheme [2–4] and (scalable) local MMSE ((S)L-RC) combining schemes [7] are provided as benchmarks.

The results show that the proposed algorithms perform very well compared to the benchmarks. The performance decrease between the N-DRC algorithm and the scalable SN-DRC algorithm is very limited, while the reduction in channel estimations and in the transmission of parameters and transformed signals for the algorithms is significant since $|\mathcal{D}_l| = 10 << 100$. The SP-DRC algorithm performs better than the (scalable) local MMSE combining schemes for 60% of the UEs and the maximal value for $|\mathcal{P}_k|$ is 54 in all the simulations, making this an interesting substitute for local MMSE receive combining.

6. CONCLUSION

This paper presented different MMSE receive combining algorithms for cell-free Massive MIMO systems, that allow for an efficient distributed implementation when a small number of parameters can be exchanged between the NC and APs. To avoid scalability issues when the number of UEs grows large, scalable version are proposed and simulations confirm that their performance is very similar to the performance of its non-scalable version.

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