Color appearance model incorporating contrast adaptation - implications for individual differences in color vision

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Color appearance models are usually based on equations with fixed numerical coefficients that were predetermined to yield optimal agreement for a selected standard observer. Here we address the well-known fact that, among color-normal observers, there are significant differences of cone fundamentals, which often cause observer metamerism. This is not fully addressed by many appearance models. It might seem that observer metamerism could be accurately predicted by substituting an individual's color matching functions into an otherwise-unchanged color appearance model, but this is not assured because the model's coefficients were not designed for the new observer. Here we explore a solution related to how the visual system tends to compensate for changes in color information arising from variations cone fundamentals. For this purpose, we developed a simple color appearance model that uses only a few numerical coefficients, yet accurately predicts the perceptual attributes of Munsell samples under a selected standard lighting condition. We then added a feedback loop to automatically adjust the model coefficients, in response to switching between the cone fundamentals for different observers. This feature is intended to model long term contrast adaptation in the vision system by maintaining the average long-term levels of color contrast. Incorporating this adaptation principle into color appearance models could allow better assessments of displays and illumination systems, to help improve color appearances for most observers.

I. INTRODUCTION

Current color appearance models convert a scene's colorimetric data, based on the CIE standard observer, into predicted perceptual attributes such as hue, chroma and lightness¹. Color perceptions cannot be measured directly, but they are amenable to quantitative estimation through experiments involving color matching, just noticeable color differences, and subjects' estimations of the sizes of larger color differences. The net result is that color perceptions can be quantified and meaningfully compared to the corresponding radiative stimuli.

The accuracy of color appearance models is important, for both scientific and practical reasons. In particular, these models help guide the design of the spectral power distributions of light used for illumination and in image displays. In an era of increased focus on energy savings and human well-being, these design issues have become increasingly important². As an example, there is a need to balance the conflicting goals of energy efficiency and color perception excellence, which, needless to say, are significant societal issues.

We consider here a critical factor that is important to address from the perspective of the quality/efficiency trade-off in displays and lights – human diversity. Specifically, it is well established that observers with normal color vision can have cone fundamentals that differ significantly from those of an average observer^{3,4}. Consequently, it is important to understand how this diversity in cone fundamentals influences color appearance, and how this should be modeled. Importantly, the impact of these individual differences cannot be reliably predicted by substituting an individual's cone

functions in color appearance model formulas that were designed for an average, or standard, observer. The reason for this is that each individual's visual system is calibrated over time, according to the range of stimuli to which the observer is exposed⁵. This calibration, or adaptation, occurs over multiple timescales and depends on the properties of both the environment and the observer⁶. The net effect is that individuals that have the same cone fundamentals but have adapted to different environments may have different perceptual experiences of the same stimuli. Conversely, individuals with different cone fundamentals who have adapted to the same environment should have more similar perceptual experiences than their physiological differences might suggest⁷⁻¹². In the present paper we focus on the latter case – in principle, adaptation in color coding will tend to compensate perception for an individual's variations from the average cone fundamentals, thereby (partially) discounting the impact of those variations on their perception of color appearances. This idea reflects a well-known general property of all sensory systems, though its implications for modeling color appearance has not been fully explored. This may be because standard models of color appearance include comprehensive adjustments for changes in the stimulus, but do not correspondingly include comprehensive predictions for how color coding adapts to changes in the observer.

This paper begins with a summary of experimental observations of adaptive phenomena in human color vision. We then present a brief summary of the mathematics of color appearance modeling, starting with the retinal cone cells and extending to higher level stages that presume to yield the familiar attributes of color percepts: hue, chroma, and lightness. We then consider how a primary aim of this processing is to transform the pattern of cone excitations so that the density

distribution of color percepts becomes more uniform¹³⁻¹⁶. We show that for a descriptive model of color appearance this is easily achieved by including automatic gain adjustments aimed at achieving the desired uniform distribution of output values. The model is derived from one we recently developed to derive uniform color metrics from principles of efficient coding¹⁷. Here we show that if the numerical coefficients of this simple model self-adjust in response to long-term past stimuli, this predicts color perceptual attributes that agree well with the perceptually uniform Munsell system. We then illustrate the consequences of substituting a new set of cone fundamentals into the model. Without adaptation, this change of observer would predict that many colors would be noticeably shifted. However, by allowing the model parameters to automatically adjust to the resulting changes in long-term stimulation, most of the changes that would otherwise be predicted are greatly reduced, consistent with actual experimental observations. Finally, we argue that by incorporating long term adaptation in this way, current color appearance models could be elaborated to better assess the effects on color appearance metrics from the combination of switching (a) from the standard cone fundamentals to those of a specific observer, and (b) from a reference environment to an environment with a different illuminant or reflectance distribution. Thus, the model provides a framework for optimizing spectral power distributions for illumination and displays to account for variations in human color vision.

II. ADAPTIVE PHENOMENA IN HUMAN COLOR VISION

Two primary processes of adaptation that have been identified are adjustments to the mean color signal and adjustments to the variance of the color gamut⁵. The former involves sensitivity changes that occur as early as the cone receptors and corrects for biases in the average cone responses. For example, if the cones are more strongly or weakly stimulated by the scene then their sensitivity or response gain is adjusted roughly reciprocally to help maintain a standard average response (light or dark adaptation)¹⁸. If the cones are differentially stimulated by a color bias in the scene then the same gain changes will help restore the average responses across the cones to remove the bias in their responses (chromatic adaptation)¹⁹. At short timescales these sensitivity adjustments are known as von Kries adaptation²⁰⁻²², though similar processes may operate over a range of timescales²³, reflecting for example, a long-term “memory” for the history of stimulation^{7,11,24-26}. In color appearance models, adjustments for the mean are often instantiated as a white balance for the reference chromaticity²⁷, and thus these models incorporate light and chromatic adaptation as an important and necessary component of the prediction.

However, the visual system also adjusts to higher-order statistics of stimulus distributions, and specifically their variance. For example, if a color gamut has more variation in blue and yellow than in other colors (typical of natural color distributions), then, over time, the sensitivity to the blue-yellow variance may again be reciprocally decreased relative to other color directions, again tending to remove biases in the responses to the gamut²⁸. These adjustments have been widely studied in the context of “contrast” adaptation, and primarily reflect sensitivity changes at later stages of the visual system - in the visual cortex - rather than the receptors and early retina. (Evidence for this includes the findings that unlike chromatic adaptation, contrast adaptation is selective for spatial

properties of the color distribution, like orientation; transfers between the eyes; and for color is much weaker in the retina than cortex when probed with physiological recordings⁵.) Chromatic and contrast adaptation operate largely independently, with chromatic adaptation adjusting to the mean and then contrast adaptation adjusting to the resultant responses to the adjusted mean²⁹. At short timescales, contrast adaptation tends to offset biases in the neural responses to color, and at long timescales may effectively discount these biases in order to “circularize” or “sphere” the response distribution so that the set of responses occur with equal probability even if the stimulus distribution is biased^{15,30}. These longer-term consequences may be particularly relevant for modeling the impact of individual differences in color appearance metrics, because the sources of these differences are themselves stable (e.g. variations in photopigment opsins) or change very slowly with time (e.g. changes in lens pigment density with aging).

Importantly, individual differences in cone sensitivities can impact both aspects of the neural response to color – the mean response and the variance of responses – and thus are likely to induce both forms of sensitivity regulation (i.e. both chromatic adaptation and contrast adaptation) to varying degrees. For example, filtering the spectrum by a yellower lens will primarily alter the average cone responses and could be primarily (though not completely) compensated by cone-specific gain changes or white-balancing for the mean³¹. Alternatively, changes in the spectral peaks of the cones (which can be substantial in color deficiencies) will strongly alter the effective chromatic contrasts and would require substantial changes in contrast sensitivity to correct for it³². Moreover, there are likely many sources of individual differences in neural sensitivity beyond the cone sensitivities that are not well characterized and would also require adaptive adjustments. However, whether the sources of the inter-observer differences are well-characterized or not, there are many lines of evidence that color perception does tend to be compensated for the specific spectral sensitivity of the observers. For example, color appearance remains similar among old and young adults despite the large age-related changes in spectral sensitivity^{9,10,33}, and changes less across the visual field than predicted by changes in macular pigment screening^{7,34,35}. Moreover, recent work suggests that even in individuals with color deficiencies, color percepts are more like those of color-normal individuals than would be predicted based on their cone fundamentals^{36,37}.

Here, we explore how color appearance models could be expanded to include procedures for automatically adapting the coefficients of their equations when the cone fundamentals are changed to those of a non-standard observer. The proposed adjustments are based on principles of information theory and coding efficiency^{13,14,30,38,39}. Color differences are expected to reflect the scaling that yields, over time, a uniform response distribution to very long term environmental stimuli (constrained by which aspects of the stimulus distribution are amenable to adjustment by the visual system)¹⁷. This has the predicted consequence that adaptation tends to make color differences, on average, more similar for different observers, if they have adapted for a long time to the same environment. This is because their visual systems are each adapted to compensate for biases introduced by their own visual sensitivity in order to regularize the neural responses for that environment⁶. Individuals with different cone fundamentals will still disagree about color matches for specific objects – an

effect known as observer metamerism. Indeed, it is very important for displays and lighting systems to minimize observer metamerism. However, our model assumes that the suprathreshold scaling of color appearance will nevertheless be more similar across observers than the uncompensated color responses would predict.

Within this context we develop a system for automatically taking these individual long-term calibration affects into account, under a standard lighting condition, in order to assess the impact of color vision diversity on models of color appearance. We propose that these types of adjustments could be incorporated within existing color appearance models to better predict the consequences of observer metamerism for appearance metrics. In turn, these ideas could help guide development of light sources and image displays to perform better over the diversity of most human observers.

III. COLOR APPEARANCE MODEL COMPONENTS

To help illustrate these ideas, we describe here a very basic model of color appearance, based on standard assumptions about stages and transformations of color coding in the human visual system. We first develop the model for the standard observer and then consider the impact of individual differences. We emphasize that this is not intended as a complete formal model of color appearance – like many that have been successfully developed to account for many parameters that affect appearance. Rather, our intent is to use a restricted set of conditions to explore the impact of observer variation. We also emphasize that, while we have tried to develop the model in the context of plausible assumptions about adaptation and visual coding, the analysis is not intended as an actual account of the mechanisms mediating color coding, but rather as a model of the perceptual consequences of the resulting processes.

Cone Receptor Coding

Typical trichromatic color vision depends on sampling the light spectrum with three classes of cone receptors maximally sensitive to long, medium and short (L, M, S) wavelengths. For the standard observer, the L, M and S cones have in situ peak sensitivities at approximately 569 nm, 541 nm, and 448 nm, respectively ⁴⁰. The sensitivity functions are symbolized as $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, $\bar{s}(\lambda)$, and when expressed as the cone fundamentals, include not only the effective absorption spectra of the receptor photopigment but also the spectral screening effects of lens, ocular media and macular screening pigments.

We wish to characterize the color appearance of numerous test surfaces, each having a spectral reflectance function $R_i(\lambda)$. All of these surfaces are illuminated with a spectral power distribution $E_{e,\Omega}(\lambda)$. For simplicity, and in analogy with the familiar white balance methodology employed in the CIELAB calculation, we can define three ratios L_i/L_w , M_i/M_w , and S_i/S_w , with the subscript i signifying the sample number. The formula for the first ratio is shown in equation (1) (with the analogous form for M and S used for the other two). An advantage of this approach is that it accounts for full chromatic adaptation to the illuminant while also eliminating any concern over the method of normalization used for the cone fundamentals and the illuminant SPD. Of course, a more complete model would not make this simplifying assumption, but we believe the current version is sufficient for the purposes of this analysis.

$$\frac{L_i}{L_w} = \frac{\int_0^\infty \bar{l}(\lambda) E_{e,\Omega}(\lambda) R_i(\lambda) d\lambda}{\int_0^\infty \bar{l}(\lambda) E_{e,\Omega}(\lambda) d\lambda} \quad (1)$$

The limited and nonlinear (sigmoidal) output of the cone is often approximated by the Naka-Rushton equation, expressed in the simple form shown in Eq. (2a), for L cones, and with the analogous expression used for M cones and S cones.

$$L = \frac{\left(\frac{L_i}{L_w}/K\right)^n}{1 + \left(\frac{L_i}{L_w}/K\right)^n} \quad (2a)$$

L ranges notionally from 0 to 1, and K is the steady state stimulation level (also called the half-saturation value) at which the cone output signal is at the center of the output range, 0.5. The parameter n is a contrast factor – larger values correspond to the cone output transitioning more rapidly across its output range as the stimulation is increased. In human cones, the value of n is roughly 0.5 ⁴¹. Here, these were optimized by matching to the achromatic perceptual response (Value) as modelled in the Munsell system, yielding a value for K of 1.20, and for n , 0.54. These values were then used for determining the cone output signals used in the color calculations described below.

Although the algebraic form of Eq. (2a) is simple, the nature of the input-output relationship is somewhat difficult to picture, because the input ratio L_i/L_w appears both in the numerator (where it increases L), and in the denominator (where it decreases L). For this reason, some might prefer the mathematically identical version in Eq. (2b), which has the logically simpler form of a sigmoidal function operating on the logarithm of the stimulus, with the analogous form again used for M and S .

$$L = \frac{1}{2} + \frac{1}{2} \tanh \left[\frac{n}{2} \ln \left(\frac{L_i}{L_w}/K \right) \right] \quad (2b)$$

Cone-opponent processing

The signals from the cones are combined in the retina to form three principal pathways or “cardinal mechanisms” for conveying color ⁴². These include a non-opponent achromatic signal A formed by summing the signals from the L and M cones and mediating luminance, and two cone-opponent signals, a and b , that represent, respectively, the signals in the L versus M cones or the S cones opposed by the sum of the L and M cones. Here, we represent them as:

$$A = L + M \quad (5)$$

$$a = L - M \quad (6)$$

$$b = L + M - 2S \quad (7)$$

The achromatic and two opponent pathways are named for the magnocellular, parvocellular and koniocellular layers in the lateral geniculate nucleus (LGN) to which they project ^{43,44}. It should be pointed out however that other weightings have been used and can achieve very similar results by suitable selection of subsequent parameters in the calculation. The key characteristics are: A is a weighted additive blend of L , M , and S , while a and b are substantially different color contrast signals, both of which are zero when $L=M=S$. As an example, the well-known color appearance model CIECAM02 ^{45,46} uses the following:

$$A = N_{bb} \left(2L + M + \frac{1}{20}S - 0.305 \right) \quad (8)$$

$$a = L - \frac{12}{11}M + \frac{1}{11}S \quad (9)$$

$$b = \frac{1}{9}(L + M - 2S) \quad (10)$$

where N_{bb} is a numerical adjustment factor for background-brightness induction. For the very limited purposes described here, we found the simple form of Eqs. (5, 6, 7) to be adequate.

Cortical color processing

The nature of higher stages of color processing in the cortex are less clearly known and understood⁴⁷, but can be readily modeled in terms of their net effect on color appearance. We assume this net effect is to further convert the long-term distribution of a and b values, which are typically distributed rather asymmetrically, into an approximately uniform symmetrical distribution. Throughout the remainder of this discussion, we will focus only on the a , b signals, and we will use the term “circularizing” of the perceptual distribution in the color plane as presumably yielding roughly equal perceptual weight to different dimensions of the stimulus distribution, e.g. equating the relative strength of different hues and chroma variations. We note that there is nothing new about this idea¹⁵; here we merely introduce an automatic way to model it.

Efficient coding of color

Another way to think about these transformations is to consider them as optimal coding schemes for representing color. Numerous previous studies have discussed these in terms of information theory and coding efficiency^{14,48}. For example, the nonlinear responses within the receptors and subsequent neurons may be designed to match the expected stimulus distribution such that the probability of each output level is approximately equivalent^{30,49}. For an approximately Gaussian input distribution (with most input levels near the average with progressively fewer levels above or below the average), the optimal transducer response is approximately the cumulative normal distribution or sigmoidal function, which is represented by Eq. 2b. (A sigmoidal, as in Eq. (2b), has the steepest response at the input levels that are most frequent to allocate more information resources to coding levels that are more common).

Similarly, a common rationale for the cone-opponent responses is to de-correlate the signals carried by the cones, so that each channel carries independent information^{13,39}. This is not possible at the level of the receptors, because their overlapping sensitivities result in very high correlations (e.g. the correlation in the responses in the L and M cones is ~ 0.98 for natural spectra)⁵⁰. By recoding the cone responses into their sums (L+M) and differences (L-M; L+M-2S) in the luminance and chromatic pathways, this redundancy is largely removed so that luminance and color carry separate, independent information about the stimuli. Finally, to maximize channel capacity, each of these post-receptor channels should similarly adjust its transduction characteristics so that the full available range of outputs is optimally used to represent the range of inputs³². This predicts that the average response level within the opponent mechanisms should be the same even if the range of their inputs is very different. In fact, these input differences are enormous, again because of the overlapping sensitivities of the cones. This overlap causes the root-mean-square (RMS) value of L-M to be significantly less than that for

L+M-2S. However, many lines of evidence suggest that the gain levels in neural processing adjust to offset this difference³².

The net effect of these optimizations is that for a given stimulus distribution, the output of the system should have approximately equal variance and constant density of the responses along each of the cone opponent dimensions. However, this alone is not enough to make the distribution completely uniform (or optimal), because there may also be correlations between the cone-opponent responses. A common way that this is now recognized to arise is because natural color distributions and natural lighting variations tend to vary predominantly along a blue-yellow axis of color space⁵¹⁻⁵³. This is intermediate to the chromatic dimensions coded by the cardinal chromatic a and b mechanisms (which instead vary from reddish to cyan and from purple to yellow-green). The blue-yellow bias in natural scenes thus produces co-varying responses within the early cone-opponent mechanisms. To remove these, additional transformations would be required to again give equal weight to different hue directions. There is now substantial evidence that this happens. Visual sensitivity and responses are weaker for blue-yellow chromatic variations than for, e.g. reddish-greenish differences, consistent with discounting the higher blue-yellow contrasts to give uniform weight to different hues⁵⁴⁻⁵⁶. How this happens is less certain – because the adjustments could reflect further rotations and scaling of the response axes^{15,57}, or a re-coding of color into multiple hue-selective channels with each adjusted in gain to match their input range. The latter alternative is consistent with a range of psychophysical and physiological studies demonstrating multiple, “higher-order” color channels beginning at the level of the early visual cortex⁵⁸. However, here we adopt the computationally simpler approach of sequentially applying simple, physiologically plausible operations to the color information, which has essentially the same effect of normalizing the distribution of the color percepts.

An illustration of uniform perceptual sampling

Since we wish to model a system that reproduces the spacing of perceptually uniformly distributed stimuli, it is helpful to begin with a well-known standard set that has that characteristic. For this purpose, we selected a subset of the Munsell set of standard colors⁵⁹.

Specifically, we selected the 17 combinations of Value and Chroma for which all 40 Munsell Hues are available. Table (1) lists these, (corresponding to a total of 680 samples). Ideally, for each such set of 40 spectral reflectance factors, their spectral average should be achromatic, which was nearly, but not perfectly, the case. As a minor improvement, we added to each the small spectral adjustment that achieves this. Here, we will refer to this resultant sample set as the Munsell Circular Subset.

Value		Chroma		
3	2			
4	2	4		
5	2	4	6	
6	2	4	6	8
7	2	4	6	8
8	2	4		
9	2			

Table 1: The 17 Value Chroma combinations for which all 40 Hues are available, corresponding to 680 samples.

Fig. 1 displays a top down and 3D view of the 680 samples of the Munsell Circular Subset. Here we will primarily be concerned with variations in the chromaticity plane normally defined by Munsell Hue and Chroma. In this context, the aforementioned a and b are equivalent to the Munsell Chroma multiplied by, respectively, the cosine and sine of the Munsell Hue angle, with 90° assigned to Munsell Hue 5Y. In this space, a just noticeable difference (JND) is estimated, very roughly, as 0.5 units – the hue spacing of the Chroma 3 Munsell samples.

Figure 1. Left: A plot of the 160 pairs of color coordinates for the Munsell Circular Subset. Right: An illustration of the 3D structure of this set, showing every fifth color coordinate for 7 Munsell lightness values comprising this set of 680 sample

IV. Examples of Simple Models for Predicting Uniform Color Percepts

Because experimental data on the color appearance of surfaces has a fairly high degree of uncertainty, and due to various needs of complexity and comprehensiveness, numerous mathematical models of colour appearance co-exist¹. They make similar, but not identical predictions which are reasonably consistent with available data. With the goal of providing a reasonably accurate but theoretically motivated model, we recently¹⁷ described a model comprising four simple successive phases, which together achieve the required “circularizing” operation for a standard observer.

This model can be summarized as a succession of simple mathematical operations on the a , b cone opponent signals represented in Eqs. (6,7). It is helpful to define two vector operators. The first is a simple rotation and scaling operator $R(S, \theta)$ defined in Eq. (12).

$$R(S, \theta)(a, b) = S((\cos(\theta)a - \sin(\theta)b), (\cos(\theta)b + \sin(\theta)a)) \quad (12)$$

The second is a differential gain operator, defined in Eq. (13)

$$G(g_{a^+}, g_{a^-}, g_{b^+}, g_{b^-})(a, b) = \left((g_{a^+}) \frac{a+|a|}{2}, (g_{a^-}) \frac{a-|a|}{2}, (g_{b^+}) \frac{b+|b|}{2}, (g_{b^-}) \frac{b-|b|}{2} \right) \quad (13)$$

This operation has the effect of applying a multiplicative gain g_{a^+} if a is positive, and otherwise it applies the gain g_{a^-} , and equivalently for b . The effect of the adaptation is simply this: If the RMS values of positive instances of a are too large, g_{a^+} is adjusted downward, and if too small, upward. The same idea is applied to the other three gain parameters. An example of a resultant gain characteristic is depicted in Fig. 1. This automatic

The precise value of a JND in this space is not critical, but this estimate can provide some perspective on color shifts arising from changes of the observer’s cone fundamentals. Also, we note that this Munsell Circular Set is not intended to represent the history of color stimuli to which an individual has been exposed, and clearly it does not. It simply provides a practical gauge of color contrast sensitivity. In turn, this information determines the required changes of the model’s numerical coefficients in order to counteract the contrast sensitivity changes arising from a change of the cone fundamentals.

adjustment is carried out by a simple, non-iterative, invertible, calculation.

Figure 2. An example of the differential gain operator on a and b signals, yielding differential slopes.

Based on the individual operators described above, the overall operation is depicted in Eq. (14a), and the details of these four operations are described next.

$$(a_{out}, b_{out}) = R_M G_2 R_{\pi/4} G_1(a, b) \quad (14a)$$

We describe this color appearance model with the name “Adaptive Dual Differential Scaling” (ADDS), because it is a two-stage model in which each signal undergoes different scaling for positive and negative values. It will be helpful to illustrate how it operates in reference to the spectral reflectance factors of the Munsell Circular Subset, beginning with Fig. 3, which plots the result of Eqs. (6,7) for the 680 samples of the Circular Munsell Subset. The pattern is both non-uniform and non-circularly symmetric, bearing little resemblance to the perceived pattern shown in the left side of Fig. 1.

Figure 3 The 680 (a, b) input data points from Eqs. (6,7), for the Munsell Circular Subset, illuminated by CIE Illuminant C at 500 lux and using the CIE 2006 2° L,M,S cone fundamentals

To begin to address this, the four parameters in the first differential gain operator, $g_{a+}, g_{a-}, g_{b+}, g_{b-}$, are automatically adjusted so as to make the RMS value of a' unity, and the same for b' . Figure 4 shows the resultant plots for the (a', b') values, as defined in Eq. (14b), in reference to Eq.(14a).

$$(a', b') = G_1(a, b) \quad (14b)$$

Figure 4 The 680 (a', b') data points resulting from applying differential scaling to the (a, b) data points.

Although, the distribution is now more symmetric in both a' and b' , the data points are not yet fully de-correlated and the pattern is not yet circularly symmetric. This is solved in two simple remaining steps¹⁵. First, the pattern is rotated counterclockwise 45° by applying the differential scaling operation of Eq. 12, with S being 1 and θ being $\pi/4$ radians (45°). Figure 5 shows the result values (a'', b'') , as defined in Eq. (14c), in reference to Eq.(14a)

$$(a'', b'') = R_{\pi/4} G_1(a, b) \quad (14c)$$

These are basically sum and difference calculations, which are common in neural circuits. However, we note that the same result could arise if the hue circle were encoded by multiple chromatic mechanisms each sensitive to a different angle in the plane, with each adapted to the chromatic contrast it is exposed to. This may be more representative of cortical color coding in the human visual system^{28,58}.

Figure 5 The 680 (a'', b'') data points resulting from applying a 45 degree rotation to the (a', b') data points.

The net result is that the pattern shown in Fig. 5 is now de-correlated, and can be made circular by the application of a final differential scaling operation, with parameter self-adjusting to give output RMS values of unity, as depicted in Eq.(14d).

$$(a''', b''') = G_2 R_{\pi/4} G_1(a, b) \quad (14d)$$

Figure 6 The 680 (a''', b''') data points resulting from applying a differential scaling to the (a'', b'') data points.

As shown in Fig. 6, the desired result is a set that is approximately circular, de-correlated, and has a RMS value of unity for a''' and b''' . The only remaining step is to apply a final rotation and scaling operation to match the arbitrary size and orientation of the Munsell set itself. This is done by adjusting the parameters S and θ in Eq. 12 so as to match the average Chroma of the Circular Munsell Set, (which is 4.0) and to nullify the average of the hue differences between the two sets, as already depicted in Eq. (14a). Fig. 7 shows the results obtained using a 500 lux CIE illuminant C, and the CIE 2006 2° standard cone fundamentals.

Figure 7 The 680 a -out and b -out data points are obtained by applying a final rotation and scaling operation in order to match the arbitrary orientation and scale of the Munsell system depicted on the right as values M_a , M_b .

The RMS error between the a -out, b -out values thus obtained and those for the Munsell set are respectively 0.25 and 0.35 Munsell units. This is essentially an imperceptible error, which is remarkable considering that the color contrast parameters were not adjusted in order to minimize differences between individual sample predictions and their intended values. In other words, the “tuning” as described, was based only on the overall statistics of the optical information being processed.

An advantage of this representation is that the signal transforms as represented in Eqs. (12,13,14) can be emulated in neural circuits. It is common for neural signals to be combined in an additive or subtractive manner with gains that can adjust slowly over time. In particular, the differential gain operator in Eq. (13) can be achieved with components that react differently to positive differences than negative ones. These components are sometimes referred to as “half-wave rectifiers” since they break the opponent dimension into two separate processes controlling the opposite responses (e.g. “on” and “off” cells) ⁶⁰. In the field of neural networks for artificial intelligence, it has recently been shown that the inclusion of rectification elements can enhance performance ⁶¹, and this “split-coding” scheme has also been shown to enhance the capacity of the visual system to represent color ³⁰. Moreover, several studies have demonstrated that color coding in the cortex likely depends on chromatic mechanisms with rectified responses, and this may be because cortical cells have low spontaneous activity and thus they can only represent information by excitation and not inhibition ^{30,62}.

V. Application of the ADDS Color Appearance to Study Individual Differences in Color Vision

An important advantage of this adaptable color metric is that it can account for the large individual differences in the cone fundamentals of color-normal observers. The sources and range of these differences have been summarized elsewhere ⁴, and include:

Lens pigment density

The crystalline lens of the eye includes inert screening pigment that reduces short wavelength visible light and ultraviolet light. Variations in the density of the pigment thus alter sensitivity to short wavelengths. Importantly, lens pigment density increases steadily with aging and light exposure, and thus is a major source of age-related differences in normal color vision ⁶³.

Macular pigment density

The very center of the visual field is also screened by a second inert pigment in the retina that maximally absorbs light at 455 nm, again reducing the short wavelength light reaching the receptors. Because macular pigment density falls off rapidly with increasing eccentricity, this is a major source of cone fundamentals difference even within the individual observer ⁶⁴ and is the main source of difference between the CIE 2-degree and 10-degree observer ⁴⁰.

Peak wavelength of photopigment absorption

The individual cones (particularly L and M) can vary in their wavelength of maximum sensitivity over a range of a few nanometers. A major source of this variation comes from common polymorphisms in the genes coding the photopigments ⁶⁵.

Photopigment optical density

The cones can also vary in the density of the photopigment and variations with position on the retina. Higher densities result in higher probabilities of photon absorption (self-screening) that differentially increases sensitivity in the tails of the absorption spectra ⁶⁶.

Relative numbers of cone classes

On average, there are roughly twice as many L cones as M cones, while S cones make up less than 5% of the total cone population. However, the L and M cone ratios can vary dramatically across individuals from less than 1:1 to greater than 20:1 ^{67,68}. For an individual, these ratios do not affect color matching to stimuli with different spectral content (since the match is determined by the independent equivalence within each cone class), but they do strongly affect the individual’s luminance sensitivity function, since it depends on the overall summed responses of the L and M cones ⁶⁹.

Importantly, these factors are large enough that their effects on sensitivity can be readily observed. For example, cone fundamentals for normal trichromats differ significantly; two spectra that are metameric for one observer can often be easily discriminated by another ⁴. As a result, there is increasing interest in developing metrics for color vision that recognize this diversity and represent color probabilistically (e.g. in terms of how likely an observer is to exhibit the behavior) rather than canonically in terms of a single standard observer.

Figure 8 Plots depicting processing of the cone signals in five model stages in the first five columns (from left to right) as described, respectively, in Eqs. (6&7, 14b, 14c, 14d, 14a). The sixth column depicts the average vector difference, at 20 selected hues, relative to the 2° reference observer. Each row corresponding to a selected set of cone fundamentals and a selected degree of adaptation from the base case depicted in the top row. Row 1 depicts the result for that 2° reference observer. In Row 2, the observer has been switched to the 10° observer, and the internal coefficients are unchanged. In Row 4, full adaptation to the 10° observer has taken place. In Row 3, the model coefficients have been made the mean of those for Rows 2 and 4, with intermediate results. Rows 5, 6, 7 bear a similar relationship to Row 1, with the observer in this case having the L cone peak displaced to a lower wavelength so as to reduce the mean spacing of the L and M cones by 50%.

VI. Effects of individual cone fundamentals differences on color appearance.

Despite these pronounced sensitivity differences, and the limits they impose on color matching and metamerism, their impact on color appearance is surprisingly small. This has now been demonstrated in a wide range of studies. For example, color appearance changes much less with aging or between the fovea and near periphery than would be expected from the changes in the density of the lens and macular pigments^{9,10,34}. Similarly, hue percepts are uncorrelated with large individual differences in the relative number of cones⁷⁰⁻⁷². Recent work suggests that even with the more extreme variations in color deficiencies, color appearance may differ less than the altered sensitivities would imply. For example, anomalous trichromacy (the most common inherited color deficiency) results in a shift in the L or M peak sensitivity so that they are much closer together. This reduces the strength of the difference signals produced by the L-M channel, yet measurements of these observers' suprathreshold color percepts differ less than expected from those of standard trichromats^{73,74}.

These effects would be surprising if color coding was based on a fixed set of transforms and parameters. However, as we noted, the visual system – and indeed all sensory systems – are highly adaptable and constantly adjusting coding to match the prevailing stimulus context⁶. We do not yet know all the possible forms of these adaptations for color, but as described above, two well-documented ones are adapting to the mean of the stimulus (chromatic adaptation) (which the model described here assumes from the start) and to the variance or contrast of the stimulus (contrast adaptation)⁵. Crucially, the same effects will operate whether it is the stimulus that changes or whether it is the observer. If an anomalous trichromat has reduced sensitivity to the reddish greenish contrasts carried by the L-M signals, then this could again in theory be discounted simply by increasing the gain in the L-M channel^{31,32}. The point then, is that we should often expect color appearance to be relatively impervious to many sensitivity changes, because the very design of the visual system acts to remove these sensitivity differences from the percept. Again, it should be emphasized that long-term adaptation of an individual's visual system to their own unique cone fundamentals cannot undo the effects of observer metamerism, since once the signals are equated for the cones in an individual observer, any subsequent processing will maintain this equivalence. However, this adaptation does impact the scaling of the signals that are different for the observer, and the intent here is to show how this adaptation reduces differences in the scaling between observers that would be predicted if there were instead static transformations of the cone signals. Modelling these effects can thus augment assessments of the broader impacts of observer metamerism in the design of lighting systems and displays.

VII. Modelling the impact of individual differences on color appearance

With these ideas and the proposed algorithms in hand, we now assess the effect of changing from a standard observer, to a person who has a "non-standard" set of cone fundamental functions. Previously, others have approached this by using CAM02-UCS, and effectively changing the cone fundamentals from those of a standard observer to those of a selected individual⁷⁵. In some situations, that approach likely works well, but in others that is not the case. The reason again is that

this substitution alone does not incorporate the consequences of compensation for the subject's own unique cone fundamentals.

Fig. 8 provides a number of plots to help understand the key relationships in terms of a number of graphics comparisons. Row 1 of the figure shows, side by side, reduced versions of Figs. 3-7. It is labelled "2°-A" to indicate the calculation uses the standard 2° LMS cone fundamentals and that the adaptive calculations described above have been used to adjust the 10 coefficients involved in the four processing stages previously described.

It is instructive to first consider the consequences of swapping the cone-fundamentals without adapting. This can be illustrated, for a given model, by observing the shift in the calculated color percepts arising from keeping all of the model's parameters constant and simply switching the color matching functions that represent the observer's cone responses. In this case, any change of observer would have sufficed, and we chose to make the well-known switch between the CIE 2006 2-degree observer and the CIE 2006 10-degree observer. The primary difference between the two is in the impact of macular pigment, which, as noted, absorbs short wavelength light but is concentrated only in the fovea. Thus, the short-wavelength cone stimulation is substantially lower for the 2° observer than for the 10° observer. This alters the relative cone responses for the two field sizes. However, adaptation to the mean light levels sensed by each cone class at the two locations will again discount the mean differences in their responses, equating the white point. However, independently rescaling the cones does not completely compensate for spectral bias in the lighting or the observer, and it has been shown that the effect differs from what von Kries scaling or a general white balance would predict (though on the other hand the correction is not complete)^{34,35}. As a result, there are additional adjustments at play, and one of them may again be equivalent to making the color appearance output for the individual uniform. The magnitude of these effects is shown in row 2, 3, 4 of Fig. 8.

Row 2 (10°-U) shows the results, without subsequent readjustment of the 10 gain parameters, which we label here "U" for unadapted. To clarify, in this context, this means that the parameters in the model are unchanged from those that arose from adaption to the 2° observer. As can be seen, there is considerable shifting of the color coordinates. The rightmost figure in Row 2 depicts the color differences, for 20 selected samples, all with Chroma 8 and Value 7, with equally spaced Hues including 5Y. For each, the arrow represents the color difference between this case and the Row 1 "base case". Some of these predicted differences are large enough that they should be readily observable, contrary to the observed similarity of actual color appearance observed in 2- and 10-degree fields.

In Row 4 (10°-A), the adaptation algorithms, as previously discussed, were allowed to operate for the 10° observer. As a result, the deviations depicted in the right-most plot have been made negligible. Thus, it seems plausible that adaptation could compensate for these particular shifts.

Row 3 (10°-M) is placed between Rows 2 and 4 to represent an intermediate level of adaptation. This is modelled by setting the gain parameters to be the arithmetic mean between those of Row 2 and Row 4. As can be seen, the result is a plausible, reasonable representation of partial adaptation.

Lastly, Rows 5-7 (S/2-U, S/2-M, S/2-A) of Fig. 8 show the same results for a significantly different set of cone fundamentals, in which the spectral peak of the L cone has been shifted downward by 14 nm, approximately halving the mean separation between the L and M

cones, (hence the label "/2"). Clearly, the unadapted case in Row 5 (S/2-U) depicts radically changed predicted color appearances, which might be representative of some anomalous trichromats. Interestingly, though, the previously described adaptation mechanisms, partially applied in Row 6 (S/2-M) and fully applied in Row 7 (S/2-A), can remove nearly all the error.

VIII What this adaptation model is not designed to address

It is important to note that there are many aspects of inter-observer differences in color appearance that this model does not address. The first is observer metamerism, which as noted above is a fundamental constraint imposed by differences in color matching that the rest of the visual system cannot undo⁴. The second is that there are large individual differences in observer's reports of their color percepts. For example, the stimulus that is identified as pure yellow for one observer may be quite different from that of another. The bases for these differences and the extent to which they reflect real perceptual differences versus how individuals have learned to categorize or interpret their color experiences remains unknown. Here we only note that in most cases these differences vary independently of differences in the observers' spectral sensitivities and therefore cannot be accounted for by the observer's cone fundamentals⁷⁶. Finally, we have assumed that these adjustments reflect very long-term adaptation to the specific optical and neural properties of the observer, and that these adjustments are largely complete. However, it is possible that the mechanisms or strength of adaptation can themselves differ across observers. Moreover, the model is likely to overcorrect for short-term adjustments. For example, these could occur for a sudden change in the observer (e.g. because of injury or disease), or in the frequent events when it is the stimulus that changes rather than the observer (e.g. with a change in illumination)⁷⁷. In such cases the compensation for the change is typically only partial, and could again differ among observers.

IX. Relation to previous work

A large number of studies have addressed color coding from the perspective of information theory^{13-15,30,39,49,50,78}. Again, these share the general premise that the neural representation of color should be optimized so that neurons adjust their operating range to best match their input range. In turn this predicts that the probability distribution of the responses across neurons is flat, and that the responses across neurons are independent or decorrelated^{13,15,39,57}. If color differences correspond to the resulting response differences, then this predicts uniform color metrics directly from the stimulus distributions. Our results build on these ideas to develop an

explicit model of uniform color differences that is easily amenable to predicting the impact on color appearance metrics of individual differences in cone fundamentals. We have done this by explicitly incorporating factors corresponding to both long-term chromatic adaptation and long-term contrast adaptation in response to the changes introduced by an observer's individual cone fundamentals. A related metric was developed by Ikeda⁷⁹. Though this model carried out compression on the XYZ tristimulus values, assumed different nonlinearities, and did not address adaptation, the two studies are similar in generating a uniform metric based on the Munsell system. Finally, we note again that while the current model encapsulates the consequences of these adaptation effects, this is not intended to imply that the computations are carried out in the same way by the visual system. In particular, models of long-term adaptation to color have also been developed based on contrast adaptation acting independently within multiple channels^{31,80} rather than interacting within two color-opponent channels as modeled here.

X Conclusion

Color metrics are increasingly being designed to recognize and account for individual differences in the observer. Here we have argued that the consequences of known forms of long-term adaptation to these observer differences are important for evaluating their practical impact on color appearance. We implemented these adaptation effects by automatic adjustment of the model's parameters in order to maintain constant levels of color contrast. This predicts significantly smaller color appearance differences than expected from a non-adaptive transformation of the cone signals, and better represents the reported percepts for subjects with differing cone fundamentals. Coupled with the many other phenomena addressed by current color appearance models, incorporating these observer-dependent adaptation effects offers additional ways to assess appearance and evaluate improvement of light sources and displays for diverse observers. Thus, we suggest that this approach also be considered in the more sophisticated color appearance models that address a wider range of illumination conditions and visual stimuli.

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