The impact of resonant additions' footprint on the stop band behavior of 1D locally resonant metamaterial realizations

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Abstract

Metamaterials have been proven to hold potential to enhance the Noise, Vibration and Harshness (NVH) performance in several applications, as they can create stop bands, which lead to frequency zones of pronounced noise and/or vibration attenuation. This paper investigates the influence of the footprint between resonant inclusions and host structure on the stop band behavior of metamaterials. The impact of this design feature is assessed both numerically and experimentally. Numerically, it is shown that by increasing the footprint, the stop bands (SB) become narrower and shift to higher frequencies. Applied to finite structures, it is shown that the zones of pronounced vibration attenuation tend to follow the changes in the stop band limits. The experimental validation is performed on metamaterial beams and the results comply with the numerical model. This investigation demonstrates that the footprint of resonators is a design parameter which can have a beneficial effect when taken into account during the design stage of metamaterials.

Keywords: Lightweight structures; Metamaterials; Stop bands; Footprint

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1. Introduction

Due to ecological and environmental demands [1], the use of lightweight materials and designs has become crucial to engineering applications [2]. However, lightweight designs tend to exhibit worse NVH behavior, which diverges from customers expectations as well as legal regulations regarding human exposure to noise and vibrations [3]. Classical NVH solutions are usually bulky and heavy, hence, novel low mass and compact countermeasures are required in order to tackle the challenges of not only achieving a good NVH performance but also ensuring a lightweight design.

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Metamaterials with locally resonant behavior have attracted the attention and proven
 20 to hold potential to enhance the vibro-acoustic response of several engineering applications
 [4, 5, 6, 7, 8, 9], coming to the fore as possible candidates to combine good NVH behavior
 and lightweight design in desired frequency regions, referred to as stop bands. These are
 known as frequency zones where there is no wave propagation [10] and can be achieved by
 adding resonant inclusions to a host structure on a subwavelength scale [11] due to a Fano-
 25 type interference [12, 13]. They can be predicted by making use of dispersion diagrams,
 which can be calculated by unit cell (UC) modeling utilizing a finite element (FE) model
 together with the application of the Bloch-Floquet theorem [14, 15].

Various design parameters of resonant metamaterials have been shown to have an influence
 on these stop bands e.g. geometrical features and material properties [16, 17, 18,
 30 19, 20, 21, 22, 23]. On the one hand, Refs. [19, 20] analyze the effect of damping in the
 metamaterial constituents on the dispersion curves as well as in the reduction performance.
 The latter concludes that damping can lead to a broader frequency range of attenuation at
 the expense of peak attenuation performance. In Refs. [21, 22], the influence of boundary
 conditions (BCs) and the way they affect the stop band effect has been investigated. The
 35 former investigated the effect of the BCs on the improvement of Sound Transmission Loss
 (STL), which showed that the improvement from the stop band effect can be retained for
 different boundary conditions. The latter detected the presence of edge modes on clamped
 beams; the way they affect stop bands is assessed numerically and experimentally. The study
 showed that edge modes can reduce the stop band widths, and positioning the resonant in-
 40 clusions in optimal locations can prevent this from happening. However, these works do not
 investigate the influence of the resonant inclusions geometrical properties on the stop bands.
 On the other hand, Ref. [23] investigates the influence of the shape and size of metamaterial
 inclusions which reveals the existence of optimal designs in order to achieve the widest band
 gap in a low-frequency range. Ref. [24] performs a topology optimization of locally resonant
 45 acoustic metamaterials for maximizing the first bandgap, based on effective material proper-
 ties i.e. effective mass density [25]. Ref. [26] develops a technique for optimizing the design
 of metamaterials depending on their mechanical parameters such as the number, placement
 and properties of local resonators in order to obtain wider band gaps. Nevertheless, these
 works do not perform any experimental validation on the optimized designs

50 This work proposes to investigate numerically and experimentally the influence of the
 footprint (or contact area) between resonant additions and the host structure on the stop
 band behavior. This parameter is mostly overlooked when designing metamaterial realiza-
 tions and taking it into account during the design process can be beneficial since it could
 lead to an increase in the obtained stop band widths.

55 First, some insights are obtained by using tuned vibration absorbers (TVAs). In a second
 phase, realizable designs are studied and the numerical findings are evaluated experimen-
 tally. This investigation departs from the principle that the contact area between idealized
 resonators and host structure is infinitesimally small while realizable resonators occupy a
 certain area in the system.

60 The paper is organized as follows. In section 2 the concept of resonant additions with dif-
 ferent footprints is explained and the methodology for the stop band prediction is described

by making use of UC modeling. Section 3 analyses the impact of the resonators' footprint on stop bands for infinite and finite systems. In section 4, an experimental validation is performed of the influence of the footprint on the predicted stop bands as well as on the vibrational response of a 1D finite metamaterial realization. Lastly, section 5 summarizes the main findings of this paper. 65

2. Methodology

This section introduces the concept of resonant structures with different footprints by means of a theoretical approach and the modeling technique used to address these systems.

2.1. The footprint of resonant additions 70

In order to acquire insights on the influence of the resonant addition footprint on the stop bands, a theoretical approach is used by utilizing TVAs, which are single degree of freedom (SDOF) mass-spring systems [27]. Due to their resonance, they can create a stop band effect when added on a subwavelength scale onto the host structure.

By changing the number of springs of a TVA and distributing their counterparts over the UC, different footprints can be achieved, as shown in Figure 1. For each TVA, the resonance frequency and added mass are kept the same, such that for single and modified TVAs, the total spring stiffness is given by k and, for a modified TVA, the stiffness of each spring is calculated as $k_s = k/n$, where n is the number of springs. 75

Figure 1: SDOF mass-spring system on a UC of length L (a) single TVA (b) modified TVA occupying a square area.

Percentage ratios are used according to the area the addition occupies in the UC. For instance, a 1-spring TVA as in Figure 1 is evaluated as having 0% of footprint while a modified resonator has a footprint value according to the net area it occupies w.r.t. the entire area of the UC, i.e. a^2/L^2 . 80

For each case, the interaction area between resonator and host structure will change, hence the impedance perceived by the TVA will change. In order to investigate this difference in impedance, a modal analysis is performed to investigate the resonance frequency of TVAs with different footprints. 85

The plate has dimensions of 500 x 500 mm and a thickness of 5 mm. The material properties of the plate are given in Table 1, and the dimensions are chosen such that the influence of the simply supported boundaries is negligible on the resonance frequency of the TVA when connected in the middle of the plate. The analysis is performed for a TVA which is connected in one point, and a TVA which occupies an area of 40 x 40 mm. In both cases, the added mass is the same and the TVA is tuned to 2000 Hz for the case that the TVA is attached to a rigid surface, as shown in Figure 2.

Young's Modulus	Density	Poisson's Ratio
210 GPa	7800 kg/m ³	0.3

Table 1: Material properties of steel.

Figure 2: Illustration of the resonance frequency of a (a) single TVA (b) TVA with larger footprint.

When the TVA with the smallest footprint is added, the corresponding resonance frequency lies at 1997 Hz and, for the TVA with a larger footprint, this frequency is at 2044 Hz. The fact that this mode for the resonator with the biggest footprint is at a higher frequency indicates that the stiffness of the host structure perceived by the TVA is increased.

2.2. Unit cell modeling

Stop bands are typically predicted for the often periodic arrangement of resonant meta-materials by UC modeling. This modeling technique is performed by using Bloch-Floquet periodic boundary conditions together with the finite element method (FEM), leading to dispersion diagrams. These are a representation of the wave propagation as a function of the wavenumber and frequency of infinite periodic structures. In these dispersion diagrams, stop bands can be identified as frequency wave gaps. This work evaluates the wave propagation of undamped infinite periodic structures by calculating dispersion curves for the undamped case. The reader is referred to [28, 29, 30] for more information about the formulation behind this methodology.

Typically, the dispersion curves related to the undamped case are retrieved by solving the reduced eigenvalue problem to real ω for imposed real propagation constants, along the irreducible Brillouin contour (IBC) [10, 31, 32], where ω is the angular frequency.

3. Stop band identification for resonators with different footprints

This section performs an analysis of the impact of the resonators' footprint on the stop bands and expands the investigation to a finite numerical system.

3.1. Effect of the footprint on stop bands

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In order to investigate the influence of the resonators' footprint on the stop bands, two test cases with TVAs are used. The first case considers a TVA with 0% of footprint, which serves as a reference, and a TVA with 64% of footprint, which are added to a host structure made of steel with a square UC of length $L = 50$ mm and thickness $t = 5$ mm. The spacing between the TVAs is such that they are on a subwavelength scale for frequencies below 2000 Hz. Two mass ratio (m_{ratio}) values, ratio between the resonator's mass and the mass of the host structure in the UC, are considered, namely, 20% and 40%. The stop band identification is performed by means of dispersion diagrams. 120

Figure 3 illustrates the dispersion curves for out-of-plane motion for the considered test cases. The points O-A-B-O represent the IBC and they consist of propagation vectors in the dimensionless wave domain, that is $(0,0) - (\pi,0) - (\pi,\pi) - (0,0)$, respectively. 125

For the case of 0% footprint with 20% of mass addition, a stop band from 1956 Hz to 2172 Hz is found. For the case of 64% footprint, even though the TVA is tuned to the same frequency as before, the stop band region is slightly shifted to higher frequencies, ranging from 1991 Hz - 2191 Hz. This can be explained by the fact that as the footprint increases, the stiffness of the host structure perceived by the TVA is increased, which is in line with what is discussed in Section 2. In this case, the change in footprint also causes the stop band to reduce 16 Hz in width. Similarly, for the case with 40% of added mass, the predicted stop band for a 0% footprint TVA ranges from 1914 Hz to 2326 Hz whereas for a 64% footprint resonator, a stop band is predicted to open from 1982 Hz to 2362 Hz. This causes the stop band to reduce 32 Hz. Furthermore, this study indicates that the footprint of resonators can directly influence the stop bands, with more pronounced effects when heavier resonant additions are applied. 130 135

Figure 3: (a) Irreducible Brillouin contour. Comparison between dispersion diagrams of a bare UC and a UC with 0% and 64% footprint resonators (b) 20% and (c) 40% of added mass.

The present analysis was performed considering only one frequency ratio i.e. the ratio between the tuned frequency of the TVA and the frequency for which the subwavelength limit is reached, and with two mass addition values. It is indeed relevant to verify if the footprint effect varies for different frequency ratios. This ratio can be varied by changing the UC length. This investigation will be performed in the following section 3.1.1.

3.1.1. Effect of the footprint for different UC lengths

The analysis performed in the previous section is repeated here for different UC lengths L i.e. different frequency ratios. In view of practical applications, this analysis considers only TVAs with 20% of added mass to keep the lightweight solution premise of metamaterials. The TVAs are tuned to 2000 Hz. Figure 4 shows the comparison of stop bands versus footprint ratio for different UC lengths.

The figure in the left-hand side indicates that the lower limits of the stop bands are the most affected by changes in the footprint as the upper bounds undergo smaller variations. When the widths of these stop bands are analyzed, it can be seen that there is a trend towards wider stop bands as L increases and this is due to the fact that the value of the frequency ratio increases as the unit cell length becomes larger, resulting in a combination of interference based and resonance based stop bands [33]. Another perceived trend is that, again, the stop band widths decrease as the footprint ratio increases. This trend can be clearly seen for the case where $L = 70$ mm, with a high frequency ratio. Hence, in order for having wider stop bands, resonant additions need to be tuned to high frequency ratios and with designed small footprints.

In conclusion, this analysis shows that indeed the resonant additions' footprint has an influence on the stop band effect and needs to be further explored in order to understand its impact when finite structures are considered.

Figure 4: Comparison of stop bands as a function of the footprint for different UC lengths (a) Stop band lower and upper limits (b) Stop band widths.

3.2. Effect of the footprint on finite structures

In order to gain insights in the relationship between predicted stop bands for resonators with different footprints and their behavior in finite structures, the 2 types of TVAs with 0% and 64% of footprint addressed in earlier for $m_{ratio} = 20\%$ are used, and frequency response analyses are performed utilizing three different structures via NX Nastran, solver SOL 108 Direct Frequency Response: (i) a bare structure, (ii) a plate with 0% footprint resonators and (iii) a plate with 64% footprint resonators. This finite plate is obtained by combining 6 x 8 unit cells of $L = 50$ mm, previously used, therefore, having total dimensions of 300 x 400 x 5 mm. In addition, this study considers 1% of structural damping for the plate's material and 1% of viscous damping for the TVAs. Simply supported BCs are applied along the boundaries of the plate. A harmonic point force of 1 N of magnitude is applied at a point with coordinates (115mm,105mm) indicated with a red arrow in Figure 5.

The responses due to the point force excitation are evaluated in terms of velocity in 120 randomly chosen points on the plate. Figure 6 shows the example of a plate with resonators of 64% of footprint.

Figure 5: Structural set-up for a plate with modified TVAs.

Figure 6 illustrates the frequency response functions (FRFs) for the considered plates. The black, blue and orange dashed lines represent the FRFs for the bare case, plate with added 0% and 64% footprint resonators, respectively, in terms of root mean square (RMS) velocity of the 120 chosen points on the plate. The blue and orange solid vertical lines indicate the predicted stop bands for each resonant addition. Around the previously predicted stop bands, zones of pronounced vibration attenuation can be clearly seen for the two meta-material cases. It can be seen that the 0% footprint resonators produced a larger pronounced vibration reduction zone when compared to the structure treated with resonators of bigger footprints, which complies with the stop band predictions addressed earlier. This numerical study indicates that this parameter can affect the performance of the finite metamaterial solution, suggesting that, in terms of stop bands widening, resonators should be designed with their footprint as small as possible. In section 4, the numerical predictions will be validated experimentally.

Figure 6: FRFs of a finite plate with resonant additions of different footprints.

4. Experimental validation of the footprint influence

This section performs an experimental validation of the influence of the footprint on the resulting behavior in a finite system. A metamaterial realization is presented in order to verify the previously addressed numerical findings.

4.1. Host structure

In order to experimentally validate the influence of the footprint of resonators, metamaterial beams are made. They allow a 1D approach, bringing ease of analysis and interpretation of the system. The experiments are performed on aluminum beams with dimensions 800 x 30 x 2 mm. Figure 7 illustrates the FE model of the beams, modeled with 6400 linear shell elements. Clamped-free boundary conditions are applied to the structures. These BCs provide the clamped end with fixed translations and rotations while free translations and rotations to the free end. They are chosen due to their extreme kinematic characteristics when compared to other BCs e.g. fully free, simply supported edges or both edges clamped [21]. A modal analysis is performed on a realized clamped-free bare beam and model updating procedures are applied in order to retrieve the material properties of the aluminum. Results are shown in Table 2. The unit cell dimensions are 40 x 30 mm.

Figure 7: FE model of the aluminum beams.

Young's Modulus	Density	Poisson's Ratio	% Structural Damping
63.14 GPa	2640.80 kg/m ³	0.34	0.01%

Table 2: Material properties of the aluminum beams.

4.2. Designed resonators with different footprints

For this validation, resonators with different footprints are iteratively designed to have similar values of resonance frequency, added mass and modal effective mass [34]. In addition,

they are designed in a way so that their footprints can be easily changed by varying the length of their base in the y-direction i.e. the footprint of the resonators change only in one direction while their thickness remains the same, given the nature of the chosen type of the host structure. Figure 8 depicts the proposed designs, where resonator type 1 serves as reference. Similar designs have been derived in other studies e.g. [35], however, the base of the resonators in the present paper is changed such that different footprints can be achieved. In order to verify the perceived effects of the footprint for different mass ratios, as discussed in sections 2 and 3, a similar realizable design is used for two arbitrary mass addition cases: (i) $m_{ratio} = 25\%$ and (ii) $m_{ratio} = 41\%$.

All resonators are made of polymethyl methacrylate (PMMA). They are 6 mm and 8 mm thick w.r.t the out of plane direction of Figure 8 for the two different m_{ratio} cases (i) and (ii), respectively. The material properties of the PMMA and are shown in Table 3. Two footprint ratios are then chosen such that the effect of the footprint can be seen for the 2 realizable designs on both cases. The system is designed to be on a subwavelength scale at 1655 Hz. Table 4 shows the numerically acquired features of each type of resonator. For a same tuned frequency, heavier resonators can lead to a wider stop band [33] while, here, even though, resonators type 2 are slightly heavier than resonators type 1 in both cases, the latter still create the widest stop band.

Figure 8: Design of resonators with $m_{ratio} = 25\%$ (a) Type 1 (b) Type 2 and $m_{ratio} = 41\%$ (c) Type 1 (d) Type 2. All dimensions in mm.

Young's Modulus	Density	Poisson's Ratio	% Structural Damping
4850 MPa	1188.38 kg/m ³	0.31	5%

Table 3: Material properties of PMMA.

Features	$m_{ratio} = 25\%$		$m_{ratio} = 41\%$	
	Resonator 1	Resonator 2	Resonator 1	Resonator 2
Resonance frequency [Hz]	1654.87	1654.77	1654.88	1654.7
Static mass [g]	1.57	1.79	2.61	2.66
Added mass	25%	28%	41%	43%
Modal effective mass [g]	0.81	0.83	1.29	1.31
SB limits [Hz]	1401 - 1682	1421 - 1687	1254 - 1649	1361 - 1726
SB widths [Hz]	281	276	395	365
Footprint	21.50%	40.25%	21.50%	40.25%

Table 4: Resonators features numerically acquired.

The predicted stop bands of the derived designs for the two different mass addition cases are depicted in Figure 9. This study considers a wave propagation along the longitudinal direction of the metamaterial beams. The different curves in the dispersion diagrams are distinguished according to the ratio of out-of-plane versus in-plane motion of the host structure. The waves with an out-of-plane motion greater than 80% compared to the in-plane motion are considered bending waves and are indicated in red, whereas the curves in blue indicate in-plane, longitudinal and torsional waves, albeit the later also has an out-of-plane component, the ratio is low enough to easily be distinguished from the bending waves.

For the case when $m_{ratio} = 25\%$, the analysis shows that a stop band is found from 1401 Hz to 1682 Hz for resonator type 1. For resonators type 2, the stop band region ranges from 1421 Hz to 1697 Hz, being 5 Hz narrower than the former and with its limits shifted to higher frequencies. Similarly, when $m_{ratio} = 41\%$, the stop bands are created from 1254 Hz - 1649 Hz and from 1361 Hz - 1726 Hz for resonators type 1 and 2, respectively. In this case, the latter is 30 Hz, and hence 8%, narrower than the former. In addition, the change of footprint also caused the stop band limits to shift to higher frequencies.

These trends are in accordance with the investigation carried out in section 3 in the following aspects: (i) resonators with smaller footprints result in wider stop bands, (ii) the stop bands shift to higher frequencies as the footprint increases and (iii) the footprint of heavier additions can lead to more pronounced effects on the stop band widths. Thus, the proposed designs are valid and can be used for the experimental validation.

Figure 9: Dispersion diagrams for resonators with $m_{ratio} = 25\%$ (a) Type 1 (b) Type 2 and $m_{ratio} = 41\%$ (c) Type 1 (d) Type 2, with a sketch in their respective wave space. The gray shaded area represents the predicted stop bands for each type of resonant addition.

4.3. Realizable resonators with different footprints

In view of practical applications, the validation is performed by using the resonator designs for a $m_{ratio} = 25\%$, which is the most lightweight option among the proposed
 250 metamaterial solutions. The resonators are laser cut, as illustrated in Figure 10. In order to validate the resonance frequency of the resonators, several samples of each type are tested when glued to a metal block, rigidly connected to the stinger of a shaker, as shown in Figure 11. The velocity at the tip of the resonator's mass is measured using a Polytec Laser
 255 Vibrometer OFV353. An average value of resonance frequency $\bar{f}$ is calculated for each type of resonator by a linear average of measured resonance frequencies. The standard deviation

of resonance frequencies is calculated as

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (f_i - \bar{f})^2}, \quad (1)$$

where N and f_i are the number of tested resonators and measured resonance frequencies, respectively.

Table 5 shows the comparison between the numerical and experimental resonance frequencies for the 2 designs, which indicates that the measured and averaged values are relatively close to the design frequency with same standard deviation. 260

Figure 10: Samples of PMMA laser cut resonators (a) Type 1 (b) Type 2.

Figure 11: Test set-up to retrieve the resonance frequency of the resonant additions.

Resonator	Numerical (Hz)	Experimental (Hz)	% Standard Deviation
Type 1	1654.87	1655.20 ± 3.66	0.22
Type 2	1654.77	1654.30 ± 3.64	0.22

Table 5: Comparison between simulated and measured resonance frequencies for the 2 types of resonators.

4.4. Experimental set-up and results

The experiments are performed on 3 aluminum beams: one serving as a bare case and two others with resonators of different footprints. The finite beams consist of 20 unit cells, thus a total of 20 resonators are added to each beam. The beams are then clamped and vertically suspended, as shown in the left-hand side of Figure 12. The first section is located close to the clamped end of the beams whilst the last one is located at the free end. Three pieces of reflective tape are added to each section to obtain the vibrational response through laser vibro-measurements upon hammer excitation, namely 1 at the center and 2 at the side boundaries of the UC, as illustrated in the right-hand side of Figure 12. The clamping is realized by using a heavy clamp of about 15 kg clamped to a rigid aluminum wall, as shown in Figure 13.

Figure 12: Sections in a clamped-free metamaterial beam.

Figure 13: Beam clamped by a clamp.

All of the resonators are attached to the host structure by using *Loctite*® 406TM contact adhesive, with the center of their base in the center of the UC, as shown in Figure 14, in order to avoid edge modes [22]. Here, the used glue is considered to have a negligible effect on the resonators performance, assuming that there is a quasi-perfect connection between the resonant additions and the host structure; other studies used the same assumption [20, 35]. In fact, this assumption can be reinforced by the results shown in Table 5 where the experimental and numerical resonance frequencies are within the same range of values.

The beams are excited by an automatic impact hammer Maul-Theet VLMPACT-60 at a point with coordinates (15mm,15mm), as illustrated in Figure 15. An excitation point in the center of the structures at the x direction is chosen such that the flexural modes are excited while avoiding exciting torsional modes.

Figure 14: Example of a resonator type 1 placed on the centre of the UC.

Figure 15: Impact hammer excitation set-up.

The response is acquired by a Polytec PSV-500 Scanning Laser Vibrometer and evaluated over 60 points along the 20 sections on the beams. The RMS values of FRFs of each measurement point are calculated per frequency, according to:

$$\text{RMS}_{\frac{v}{F}} = \sqrt{\frac{1}{N} \sum_{j=1}^N \left| \frac{v_j}{F}(\omega) \right|^2}, \quad (2)$$

where ω is the angular frequency, $\frac{v_j}{F}$ the FRF velocity/force per response point and N the number of points.

Figure 16 depicts the comparison between the experimental FRFs of the bare beam and the beams with added resonators. The blue and orange vertical lines represent the predicted stop band limits for resonators type 1 and type 2, respectively, previously shown in Figure 9.

An improvement in the vibration response can be seen at the predicted stop band regions for the 2 types of resonators. Furthermore, the region of improvement for the metamaterial beam type 1 is wider than metamaterial beam type 2. This complies with the numerical study carried out by using TVAs which showed that the SB of resonators with smaller footprints are wider than those with bigger footprints. Another trend which is confirmed experimentally is that the zones of pronounced vibration attenuation shift to higher frequencies, which corresponds to the findings of previous sections that the stop bands limits shift to higher frequencies as the footprint increases.

Figure 16: Comparison of experimental FRFs for the bare beam and the beams with resonators.

5. Conclusion

In this work, the footprint of resonators i.e. their contact area with the host structure has been studied in order to investigate its influence on the stop band behavior of locally resonant metamaterials. This was investigated numerically through an example of spring mass systems for 2D cases (plates), where UC models and finite system simulations were used, with an analysis for different frequency and mass ratios, and actual realizable designs for 1D cases (beam). Two of the realizable designs were also realized for experimental validation.

The numerical study showed that greater footprints causes the resonator stiffness to be distributed over a larger area on the host structure, which leads to an increased stiffness of the host structure perceived by the TVA. This design parameter has shown to affect not only the stop band widths but also their location in the frequency domain. Stop bands tend to become narrower and shift to higher frequencies as the footprint of the resonant additions becomes larger. The effect of the footprint is more pronounced depending on parameters such as the amount of added mass and frequency ratio. It has been shown that, for a constant frequency ratio, heavier resonators can lead to greater reductions in the stop band widths as the footprint increases. Similarly, when higher frequency ratios are considered for a constant mass ratio, this reduction is even more apparent.

The experimental results confirmed the expected trends. It is concluded that the footprint has indeed an effect on the stop band behavior. It was shown in the paper that by merely varying the footprint size can lead to a change of more than 8% of the stop band width. Thus, in a design stage, the footprint can be taken into account in order to maximize the size of the stop bands or, for finite cases, even adjust the pronounced attenuation zones to the most suitable position in the frequency domain.

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