Associations between repeating patterning, growing patterning and numerical ability: A
longitudinal panel study in four- to six-year olds

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Mathematical competencies in preschool (4- to 6-year-olds) are an important predictor for academic achievement in primary and secondary education (Claessens & Engel, 2013; Duncan et al., 2007; Nguyen et al., 2016; Rittle-Johnson et al., 2017), and mathematical achievement at the start of formal education has been associated with socio-economic status in adulthood (Ritchie & Bates, 2013). Overall, these findings point towards the importance of increasing our insight into the development of mathematical competencies in young children. Several researchers have investigated early mathematical competencies over the past decades (e.g., De Smedt et al., 2013; Purpura & Lonigan, 2013; Torbeyns et al., 2015), thereby mainly focusing on numerical abilities such as counting, magnitude comparison, or basic arithmetic (Clements & Sarama, 2007; Verschaffel, Torbeyns, & De Smedt, 2017). The narrow focus on young children's numerical abilities was recently complemented with studies addressing other aspects of early mathematical ability, such as patterning. Patterning covers children's understanding of regular configurations of elements (e.g., repeating patterns such as ABAB, growing patterns such as ABAABAAAB; Rittle-Johnson et al., 2017; Wijns, Torbeyns, Bakker, et al., 2019). Patterning performance in preschool (i.e., 4- to 6-year-olds) has been shown to contribute to children's mathematical performance up to fifth grade (e.g., Nguyen et al., 2016; Rittle-Johnson et al., 2017).

Research has shown that patterning and numerical ability are both predictors of later mathematical performance (Lüken, 2012; Nguyen et al., 2016; Rittle-Johnson et al., 2017). This raises the question whether and how both abilities are related throughout development. There are theoretical reasons for assuming an association between patterning and numerical ability, which have been supported by a few recent studies (Lüken, 2012; Rittle-Johnson et al., 2017; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019). Little is however known about the direction of the association between these two abilities (i.e., unidirectional or bidirectional) in early development, because studies so far were either cross-sectional or did

not include multiple measurements of both abilities. Moreover, the existing studies in early childhood have largely focused on repeating patterns, whereas growing patterns received little or no attention. Both pattern types have been related to mathematical development (e.g., Kidd et al., 2014; Rittle-Johnson et al., 2017), but it remains unclear how performance on different pattern types is related throughout development, and whether the association with numerical ability is similar for different pattern types. These questions are important not only to increase our knowledge on children's early mathematical development, but also to guide early childhood interventions. To address these questions, the present study investigated the direction of the association between repeating patterning, growing patterning, and numerical ability in a large sample of children by means of a longitudinal panel design in which children were tested annually from age 4 to age 6.

Patterning, Numerical Ability, and their Association

Patterning is an aspect of early mathematical ability that covers children's performance on a wide set of tasks, such as extending (i.e., What comes next?), translating (i.e., Create the same pattern using different materials) or identifying the pattern's structure (i.e., What is the smallest part that is repeated?), that can be done with regular configurations of elements in the environment (Wijns, Torbeyns, De Smedt, et al., 2019). These regular configurations are called patterns. There are different types of patterns, and a distinction is often made between repeating (e.g., $\Delta \square \square \Delta \square \square \Delta \square \square$), growing (e.g., 2-4-6, $\Delta \square \Delta \square \square \Delta \square \square \square$), and spatial structure patterns (e.g., ::; Mulligan & Mitchelmore, 2009). The present study focused on repeating and growing patterns, which are both sequences that can be continued infinitely. Spatial structure patterns, in contrast, represent two-dimensional configurations of elements. Repeating and growing patterns each rely on a different structure or rule that describes how the sequence continues: Repeating patterns consist of a unit (e.g., $\Delta \square \square$) that is reiterated, whereas growing patterns increase or decrease systematically (e.g., +2, + $\square$). The structure of

growing patterns is considered to be more complex than the structure of repeating patterns (Pasnak et al., 2016). The systematic repetition in repeating patterns is a mathematical understanding that is more basic than the systematic change in growing patterns. The structure of a repeating pattern is also more salient than the one of growing patterns: Whereas the systematic repetition in repeating patterns is visible, the systematic change in growing patterns has to be deduced by relating two consecutive units. Empirical studies support the idea that repeating patterns are easier than growing ones (Gadzichowski, 2012; Warren, 2005). Related to this difference in complexity, one might argue that understanding repeating patterns supports understanding of growing patterns, but, to the best of our knowledge, no research so far has evaluated this claim.

Numerical ability refers to knowledge and skills regarding numbers and arithmetic (Andrews & Sayers, 2015; Purpura & Lonigan, 2013), and has been the focus of most research on mathematics in early childhood (Mulligan et al., 2018; Starkey et al., 2004; Verschaffel et al., 2017). It covers several activities with symbolic and non-symbolic numerosities, including counting, arithmetic, number recognition, and comparison (Andrews & Sayers, 2015; Jordan et al., 2006; Purpura & Lonigan, 2013). There are several regularities in numbers, such as the successor principle stating that the next number in a counting sequence always represents one more (Rittle-Johnson et al., 2019; Zippert et al., 2019), the base-ten structure of our number words, or the alternation between even and odd numbers in the counting sequence.

Little is known about the mechanisms that might explain the association between patterning and numerical ability, but most researchers point towards the idea that patterns and numbers both provide opportunities to explore regularities and grasp structures (Rittle-Johnson et al., 2019; Zippert et al., 2019). On the one hand, understanding the regularities and structures in repeating and growing patterns might support children's numerical development.

For example, the child's understanding of the idea of a unit that repeats, which forms the structure of repeating patterns, may support the understanding of the base-ten structure of our number words in which the units (0-9) repeat for every ten (Zippert et al., 2019). Children who understand that the number words rely on a base-ten structure (i.e., understand the regularity that defines the number word system) will be able to recite the number sequence or to recognize number symbols more easily, at least for numbers above 20. Similarly, the idea of units that repeat could lead to more efficient counting strategies (e.g., counting pairs of objects) and forms the basis of multiplication as repeated addition (Lüken, 2018; Papic et al., 2011; Rittle-Johnson et al., 2019). We argue that also a good understanding of a systematic change, which forms the structure of growing patterns, might support number knowledge. Understanding that the difference between any two consecutive numbers in the sequence of natural numbers is always 1, and thus represents a systematic change, forms the basis of understanding the quantities that are represented by the natural numbers. It supports object counting ability, ordinality, and understanding of basic arithmetic principles, such as the successor principle stating that the next number in the sequence is $+ 1$. As such, both repeating and growing patterning ability can support understanding of numbers and numerical concepts, which form the basis for all kinds of numerical tasks that tap into children's numerical ability.

On the other hand, well-developed numerical ability might support the development of patterning ability. For example, being able to count a small set of objects fluently might make it easier to identify or construct repeating patterns, especially more complex ones such as ABB or ABCCC. Supporting this idea, a recent intervention study with 4- to 6-year-olds showed that labelling an ABB-pattern based on information on the quantities (i.e., "one-two") supported children's translating performance (Flynn et al., 2020). Likewise, performance on growing patterns might benefit from numerical ability. Children need to recognize the

quantities that are represented in the growing patterns, which requires object counting ability in case of growing patterns without number symbols (e.g., $OO\Delta OOOO\Delta OOOOOO\Delta$), and symbolic number identification in case of growing patterns with number symbols (e.g., 2-4-6). Moreover, basic addition and subtraction skills are needed to find the systematic change (i.e., $+OO$, $+2$), as well as the next element in a growing pattern.

Empirical Support for the Association Between Patterning and Numerical Ability

Recent studies provide empirical support for the association between patterning and numerical ability (e.g., Nguyen et al., 2016; Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019). To begin with, several cross-sectional studies showed an association between patterning and numerical ability. In preschool, children's (i.e., 3- to 5-year-olds) performance on repeating patterns was related to performance on a measure covering a range of numerical tasks (Nguyen et al., 2016), even when taking into account spatial skills (Zippert et al., 2019). In a study including both repeating and growing patterns, 4-year-olds' performance on both pattern types were uniquely, i.e., controlling for spatial skills and performance on the other pattern type, related to performance on a measure covering a range of numerical tasks (Wijns, Torbeyns, Bakker, et al., 2019). In a study with 6- to 9-year-olds, performance on growing patterns with numbers and clock faces were both correlated with symbolic number comparison, and performance on growing patterns with numbers, clock faces, and letters, were all uniquely related to calculation ability while controlling for number comparison, visuospatial working memory, and spatial ability (MacKay & De Smedt, 2019). Fyfe and colleagues (2017) found unique associations between repeating patterning ability and calculation ability while controlling for age and verbal working memory in 5- to 13-year-old children. In sum, current evidence suggests that both pattern types have a unique contribution to numerical ability and this contribution cannot be

explained by other cognitive skills such as spatial ability, verbal or visuospatial working memory.

There is also longitudinal evidence for an association between patterning and numerical ability. Four-year-olds' repeating patterning ability was a unique predictor of their numerical ability seven months later while controlling for verbal skills (i.e., verbal ability, verbal working memory), spatial skills (i.e., spatial ability, visuospatial working memory, and form perception), and prior numerical ability (Rittle-Johnson et al., 2019). In a longitudinal study focusing on repeating and spatial patterns, numerical ability at age 6 was associated with patterning performance one year later (Lüken, 2012). In line with these studies showing evidence for patterning as a predictor of later numerical ability (Rittle-Johnson et al., 2019), and numerical ability as a predictor of later patterning ability (Lüken, 2012), Rittle-Johnson and colleagues (2017) found support for a bidirectional association between patterning and numerical ability. In their study, 3- to 5-year-old preschoolers' performance on non-symbolic quantity and counting predicted repeating patterning ability in early primary school (i.e., 7- to 8-year-olds), and repeating patterning ability in preschool predicted symbolic mapping and calculation in early primary school.

Finally, some intervention studies on patterns suggest potential causal associations between numerical and patterning ability. In one study with 4- to 5-year-olds, the patterning intervention group outperformed a business-as-usual control group on a measure covering a range of numerical tasks post-intervention (Papic et al., 2011), but the researchers did not provide inferential statistics to support their findings. In a study with first-graders, Lüken and Kampmann (2018) found significant differences in post-test numerical ability between the intervention group that worked with repeating, spatial and growing patterns, and a business-as-usual control group. Both studies suggest that stimulating patterning ability might lead to

better numerical ability, but the reverse association has not been investigated yet through intervention research.

Although the overview above clearly supports an association between patterning ability and numerical ability in early childhood, there are at least two issues that require further research. First, little is known about the direction of the association between patterning and numerical ability (Zippert et al., 2019). This would require a panel study in which both abilities are repeatedly measured on several time points within the same sample. Such a design allows one to control for auto-regressive effects, i.e., an effect of a measure on the same measure administered later. Researchers thus far only investigated the association in one direction, with both abilities being measured at different time points, e.g., patterning in preschool and numerical ability in first grade (Lüken, 2012; Rittle-Johnson et al., 2019). Some support for a bidirectional association between repeating patterning and numerical ability was found (Rittle-Johnson et al., 2017), but the autoregressive effect of numerical ability was not taken into account, because different numerical tasks were used at both time points.

Intervention studies found support for the impact of a patterning intervention on numerical ability, but this does not exclude the possibility that improved numerical ability might also lead to improved patterning ability. There is one study that suggests a unidirectional association from patterning towards general mathematical abilities (Pasnak et al., 2016).

These researchers found a significant correlation between growing patterning in the first semester and mathematical performance in the second semester of first grade, whereas the reverse association was not found. Importantly, the direction of the association between patterning and numerical ability might change over time. Multiple measurement points within a critical developmental period of patterning and numerical abilities are required to get a clearer view on the development of the direction of this association.

Second, little is known about the impact of pattern type on the association between patterning and numerical ability. Most research has either focused on repeating patterns or growing patterns only, or included performance on a range of patterns as one ability. The question remains whether the association with numerical ability is different for distinct pattern types. Some researchers have recently argued that the empirical evidence for a contribution to mathematical development is stronger for complex patterns, such as growing patterns, than for simple patterns, such as repeating patterns (Burgoyne et al., 2017; Pasnak, 2017). So far, the association between patterning and numerical ability has been explained by the many regularities that can be found in both patterns and numbers. Various types of patterns, however, have different underlying structures. It remains unclear whether the specific structure of different pattern types matters, or whether a more general understanding of regularity and structure fully explains the association between patterning and numerical ability. Several numerical concepts and skills can be related to the specific structure of both repeating and growing patterns. For example, the sequence of natural numbers is a growing pattern (i.e., systematic increase between consecutive numbers, $+ 1$) in which the units repeat from 0 to 9 for every ten. Counting objects in groups (e.g., in groups of 2) is based on the concept of equal units as in repeating patterns, but involves a counting sequence that forms a growing pattern (e.g., 2-4-6-8). To get a better view on the differential contribution of different pattern types to numerical ability, studies are required that include a comparable measure for each pattern type. To the best of our knowledge, only one study explored the contribution of performance on different pattern types to numerical ability separately. In a cross-sectional study, Wijns and associates (2019) found that 4-year-olds' ability in repeating and growing patterns each uniquely contributed to numerical ability. This suggests that different pattern types play a different role within early mathematical development. It remains to be determined whether the direction of the association between patterning and numerical

ability is also different for repeating as compared to growing patterns, and whether this changes throughout development.

The Present Study

The present study investigated the association between patterning and numerical ability by addressing the two above mentioned research gaps. First, we used a cross-lagged panel model with measurement points at age 4, 5, and 6, in order to evaluate the direction of the association between patterning and numerical ability and the development of the direction of this association in early childhood. The age range of 4 to 6 covers a critical developmental period in which several aspects of patterning and numerical ability are known to be acquired rapidly (Clements, 2014; Rittle-Johnson, Fyfe, Loehr, & Miller, 2015). Second, we simultaneously measured performance regarding both repeating and growing patterns with two different but comparable assessments. This allowed us to investigate the development of the direction of the association between patterning and numerical ability for both pattern types. In addition, this gave us the opportunity to explore the direction of the association between both patterning abilities, and the development of this association in early childhood. We did not put forward any hypotheses about the uniqueness and/or direction of the associations between repeating patterning ability, growing patterning ability, and numerical ability, given the limited theoretical and empirical evidence in the available research literature regarding this matter.

Previous studies have provided evidence that, although spatial skills can be differentiated from patterning and numerical ability, there are associations between spatial skills (such as visuospatial working memory and spatial ability) on the one hand, and numerical (Hawes et al., 2019) or patterning ability (Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019), on the other hand. Spatial skills might thus explain the association between patterning and numerical ability. Some of the abovementioned studies

showed that the association between patterning and numerical ability remained significant when controlling for spatial skills, indicating that the association is not fully accounted for by spatial skills (MacKay & De Smedt, 2019; Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019). We therefore also took spatial skills into account in this study. Given the visuospatial nature of the patterns that we used, we included both visuospatial working memory and spatial ability, two spatial skills that have been associated with both patterning and numerical ability (Hawes et al., 2019; Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019).

Taken together, the present study looked into the association between repeating patterning, growing patterning, and numerical ability in a critical developmental period, while controlling for spatial skills. The main aim was to analyze the direction of the association between repeating and growing patterning ability, on the one hand, and numerical ability, on the other hand. In addition, we analyzed the direction of the association between repeating and growing patterning ability.

Method

Participants

This study is part of a large-scale study on the development of young children's early mathematical competencies [blinded for review]. We attempted to include children from the whole range of socio-economic backgrounds by selecting schools based on government data on the relative amount of children that are identified as having low-SES. The 17 schools that were selected and agreed to take part in the study had different relative amounts of low-SES children (range 2%-78.9%). We invited all children in those schools that were born in 2013 to participate, and received an active informed consent for 410 children. A parent questionnaire of 361 participants (i.e., 88%) showed that approximately 95% of these children were White, which is representative for the whole population in [Blinded for review]. At the first time

point, children were between 4y2m and 5y7m old and they were in the final half of the second year of preschool (spring 2017). In the country where this study took place, three years of preschool are organized and almost all children (98%) attend preschool. At the following two time points, children were in their third year of preschool (spring 2018) and first year of primary school (spring 2019). Formal mathematical instruction only starts in first grade and mainly focuses on numerical ability, whereas little or no attention is given to patterning instruction. In preschool, numerical activities and some repeating patterning activities are provided in an informal manner.

Procedure

All children were individually tested in a quiet room at three time points, namely spring 2017 (T1), spring 2018 (T2), and spring 2019 (T3). At each time point, data were collected during three sessions of about 30 minutes, addressing repeating patterning ability, numerical ability, and growing patterning ability. Each session was administered on a different day. All children received the tasks in the same order. In fall 2017, when children were in the first semester of their third year of preschool, we administered two spatial measures, spatial ability and visuospatial working memory, on two different days, together with some other tasks that are not used in this study. These two spatial measures were only administered once because they were used as covariates and they were not the primary variables of interest in this study.

Measures

Repeating and growing patterning ability

We used two patterning measures with each 18 items, one focusing on repeating patterns and the other focusing on growing patterns. Both measures covered three patterning activities that reflect a range of patterning abilities (Lüken, 2016; Rittle-Johnson et al., 2013; Sarama & Clements, 2009), namely extending, translating, and identifying. There were six

different items for each activity (i.e., AB, AAB, ABC, AABB, AABC, and ABCD for repeating patterns; and ABAABAAAB, ABABBABBB, AAABAABAB, ABBBABBAB, ABABBBABBBBB, and AAAAABAAABAB for growing patterns). Instructions and examples for each activity for both pattern types are provided in Figure 1. The internal consistency of both measures was acceptable at the three time points (i.e., $\alpha = .79$, $\alpha = .82$, $\alpha = .71$, for repeating patterns, and $\alpha = .67$, $\alpha = .73$, $\alpha = .73$, for growing patterns). More detailed information on the measures can be found in a previous study (Wijns, Torbeyns, Bakker, et al., 2019).

Numerical ability

This measure covered several aspects of numerical ability that are commonly included in early childhood research (Andrews & Sayers, 2015; Jordan et al., 2006; Purpura & Lonigan, 2013) and that are part of many standardized early mathematics measures, such as TEMA (Ginsburg & Baroody, 2003). Similar to TEMA, this measure did not include any patterning tasks. There are small differences between the items at each time point in order to have sufficient variability at each time point, but in essence the same skills were measured three times. We created one overall score of numerical ability at each time point by calculating the average of the standardized scores on each aspect of numerical ability. At the first time point (i.e., in the second year of preschool), the measure consisted of 88 items addressing verbal counting (i.e., “Count as high as you can”, $n = 1$), dot enumeration (e.g., “Count the dots”, $n = 9$), object counting (e.g., “Give me N stones”, $n = 8$), symbolic ($n = 12$) and non-symbolic comparison (e.g., “Which number is the largest”, $n = 12$), number order (e.g., “Which number comes before or after N?” , $n = 8$), number recognition (e.g., “Which number is this?”, $n = 30$ with stopping rule), and verbal arithmetic (e.g., “I put N stones in a box and add or subtract M, how many stones are in my box?”, $n = 8$). More information on this measure can be found in a previous study (Bakker et al., 2019).

At the second time point (i.e., in the third year of preschool), we included extra items on the dot enumeration task ($n_{total} = 18$) and created new items addressing symbolic arithmetic (e.g., “I have N and I add/subtract M, how much do I have?”, while showing $N + M$, $n = 8$), leading to a measure of 105 items. At the third time point (i.e., in first grade), the measure consisted of 132 items. We used similar items as in the first two time points to address number recognition ($n = 30$ with stopping rule) and symbolic and non-symbolic comparison ($n = 24$), but extra items were included for the comparison tasks. The instruction was slightly changed for verbal counting (e.g., “Can you count forth/back from N to $N+10/N-10$?”, $n = 6$ ascending and 6 descending), number order (e.g., “Are these three numbers in the right order? Click blue if yes, orange if no.”, $n = 24$), and symbolic arithmetic (e.g., “How much is ...?”, without reading the numbers aloud, $n = 18$). The internal consistency of this measure was good at the three time points (i.e., $\alpha = .93$; $\alpha = .94$; $\alpha = .93$).

Spatial skills

The Block Design subtest of the WPPSI-III-NL (Hendriksen & Hurks, 2002) was used to assess spatial ability. Children had to copy a certain construction of red and white blocks and they received 2 points for every correct construction within the time limit. There were 20 items and the assessment was terminated after three consecutive wrong trials. This standardized task has a good internal consistency ($\alpha = .76$).

We measured visuospatial working memory via a Corsi span task (De Smedt et al., 2009). Children received a score of 1 for every time they correctly reproduced a sequence of blocks that was tapped by the experimenter in a random order. There were three trials of each sequence length (2 – 9 blocks), but the assessment was terminated if two or more trials of one sequence length were wrong. The internal consistency of this task was good ($\alpha = .77$).

Analyses

We estimated two cross-lagged panel models with observed variables in Mplus version 8.4 (Muhtén & Muhtén, 2017). Cross-lagged panel models are a specific type of path models, which can be considered as a combination of regression analyses with multiple dependent variables that are all modeled at once. In cross-lagged panel models, an individual's relative position on a variable is modeled based on auto-regressive pathways between the same variable on two time points (e.g., repeating patterning ability T1 predicts repeating patterning ability T2), and cross-lagged pathways from one variable to another on two time points (e.g., repeating patterning ability T1 predicts growing patterning ability T2). Associations between two consecutive time points are referred to as lag-1 associations, associations between T1 and T3 are referred to as lag-2 associations, and so on. Cross-lagged panel models thus provide insight into associations between individual differences in variables across time points, but not into children's growth on a certain variable across time.

In the two models, the three variables of interest (i.e., repeating patterning, growing patterning, and numerical ability) at T3 were predicted by the three variables at T2, and these were in turn predicted by the three variables at T1. The only difference between the two models that were estimated in this study was the inclusion of spatial skills as a covariate in the second model. More specifically, two spatial measures (i.e., spatial ability and visuospatial working memory) were correlated with the main variables at T1 and used as predictors for the main variables at T2 and T3 in the second model.

We used the robust maximum likelihood estimator (MLR) in order to account for missing data. The MLR estimator allows to include all available data, without estimating missing data, and it has been shown to be a more robust method to handle missing data as compared to exclusion of data or imputational methods (Collins et al., 2001; Enders & Bandalos, 2001). Overall, less than 7% of all our data was missing (see Table 1 for an overview of the number of observations for each variable) and 56 children (14%) completely

left the study over the different time points due to moving to a different school. A one-way ANOVA showed that children with missing data at the third measurement point had significantly lower performance on repeating patterning ability, growing patterning ability, and numerical ability at the first time point, i.e., $F(1,402) = 21.11, p < .001$, $F(1,399) = 21.22, p < .001$ and $F(1,400) = 19.69, p < .001$, respectively. Model fit was assessed by the following fit indices: Root Mean Square Error of Approximation (RMSEA) $< .06$, Comparative Fit Index (CFI) $> .95$, Tucker-Lewis Index (TLI) $> .95$, and Standardized Root Mean Square Residual (SRMR) $\leq .08$.

Results

Descriptive Statistics and Correlations

Table 1 presents descriptive statistics for the main variables on the three time points. We first explored the association between the three main variables (i.e., repeating patterns, growing patterns, numerical ability) on the different time points by looking at the correlations (Table 2). Correlations between the three main variables within one time point were medium to large ($r_s > .40$), confirming that children's scores regarding repeating patterns, growing patterns, and numerical ability, were associated at the age of 4, 5, and 6. Correlations within one variable over time points were large, both between two consecutive time points (r_s between .54 and .69) and between T1 and T3 (r_s between .49 and .54). This suggests that individual differences in children's repeating patterning, growing patterning and numerical ability were relatively stable between the age of 4 and 6. Both spatial ability and visuospatial working memory were also moderately correlated with our main variables on the three time points (r_s between .35 and .50). Partial correlations controlling for these spatial measures remained significant (r_s between .19 and .59; Table 2).

Cross-Lagged Panel Models

We first specified a cross-lagged panel model without spatial skills as a covariate and this model resulted in a relatively good fit, RMSEA = 0.10 (CI: 0.07-0.13), CFI = 0.95, TLI = 0.83, SRMR = 0.03. We further explored whether there were lag-2 autoregressive associations. The inclusion of lag-2 autoregressive pathways between our three main variables resulted in a significantly better model $\Delta\chi^2(3) = 32.03, p < .001$. This final model had a good fit, RMSEA = 0.05 (CI: 0.00-0.09), CFI = 0.99, TLI = 0.95, SRMR = 0.02. The final model with standardized coefficients is shown in Figure 2.

All lag-1 autoregressive pathways were significant, indicating that individual differences were relatively stable for all variables. Lag-2 autoregressive pathways were significant for both patterning abilities, but not for numerical ability. From T1 to T2, all cross-lagged pathways were significant, suggesting bidirectional relations between repeating patterns, growing patterns, and numerical ability. This changed from T2 to T3, such that both patterning skills predicted numerical ability, but not the reverse. Performance on repeating patterns at T2 was also related to performance on growing patterns at T3, whereas the reverse association was not found.

We next specified a cross-lagged panel model with both spatial skills as covariates. Again, the inclusion of lag-2 autoregressive associations resulted in a better fit of the model $\Delta\chi^2(3) = 20.17, p < .001$. This final model had a good fit, RMSEA = 0.02 (CI: 0.00-0.07), CFI = 0.99, TLI = 0.99, SRMR = 0.01. The final model with standardized coefficients is shown in Figure 3.

Similar to the model without spatial skills, we found that all lag-1 autoregressive pathways were significant, and lag-2 autoregressive pathways were significant for the two patterning abilities. Spatial skills did not have an impact on the associations between numerical ability and the two patterning abilities: All cross-lagged pathways were still significant from T1 to T2, and both patterning abilities at T2 predicted numerical ability at T3.

The only difference with the model without spatial skills was found regarding the association between repeating and growing patterns. From T1 to T2, both cross-lagged pathways were still significant, but from T2 to T3 the cross-lagged pathway from repeating to growing patterning ability was no longer significant.

Discussion and Conclusion

The present study investigated the direction of the associations between repeating patterning, growing patterning, and numerical ability by means of a cross-lagged panel design in which abilities were assessed at age 4, 5, and 6. Results showed bidirectional associations between repeating patterning, growing patterning, and numerical ability from age 4 to 5, even while controlling for spatial skills. From age 5 to 6, these associations became unidirectional such that both patterning abilities predicted numerical ability one year later, but the reverse was no longer true. In addition, the associations between repeating and growing patterning ability disappeared from age 5 to 6 once spatial skills were taken into account. We discuss our findings in light of current theories on patterning and numerical ability, and present some directions for further research.

Associations Between Repeating Patterning, Growing Patterning, and Numerical Ability

Previous studies showed that patterning and numerical ability in early childhood are associated (Rittle-Johnson et al., 2017; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019). The present study supports these findings, but extends them by investigating the direction of the associations and their development from age 4 to 6 for both repeating and growing patterns. To the best of our knowledge, the present study was the first to explore the direction of the association between patterning and numerical ability, and did so by means of a cross-lagged panel design with three measurement points. Previous longitudinal studies looked at this association from one direction only and thereby found evidence for patterning ability as a predictor of later numerical ability (Rittle-Johnson et al., 2019), as well as for

numerical ability as a predictor of later patterning ability (Lüken, 2012). In one study, evidence for a bidirectional association was found (Rittle-Johnson et al., 2017), but there were only two measurement points with a 2-year-time interval, and, critically, the auto-regressive effect of the numerical tasks was not taken into account. In a study with 6-year-olds on the association between patterning and mathematical ability, evidence for a unidirectional association from growing patterning to mathematical ability was found (Pasnak et al., 2016), but again the auto-regressive effect of mathematical ability was not taken into account.

We found bidirectional associations between both patterning abilities and numerical ability from age 4 to 5. Repeating and growing patterning ability at the age of 4 thus predicted numerical ability one year later, and, vice versa, numerical ability at the age of 4 predicted performance on repeating and growing patterns one year later. A different pattern of findings was, however, found from age 5 to 6. For both pattern types, patterning performance at age 5 was still predictive of numerical ability at age 6, but the reverse was no longer true. Our findings are in line with those of Rittle-Johnson et al. (2017), who found a bidirectional association between repeating patterning and numerical ability in preschool, and with the ones of Pasnak et al. (2016), who reported a unidirectional association between growing patterning and mathematical ability in first grade.

The above results indicate that both patterning abilities are predictors of numerical ability between the age of 4 and 6, and thus suggest that understanding both repeating and growing patterns supports the development of numerical ability throughout this critical developmental period. One explanation for this finding might be the many regularities that can be found in numbers. Other researchers have already pointed to the successor principle (i.e., the next number in the sequence of natural numbers is +1), which can support object counting and the mapping between symbol and quantity (Rittle-Johnson et al., 2019; Zippert et al., 2019). Likewise, the discovery of regularities in the base-ten structure and the place-

value system can scaffold the development of symbolic number identification (e.g., knowing that 46 is “forty-six” because you know 36 and 40) and counting above 20 (e.g., twenty-one, twenty-two, ..., thirty, thirty-one, thirty-two, etc.). For example, Yuan et al. (2019) found that preschoolers were able to map number words to written Arabic multi-digit numbers and to compare their magnitudes, without formal instruction. It is plausible that early patterning ability contributes to this understanding of multi-digit numbers, a possibility that should be tested in future studies.

This association between patterning and numerical ability was similar for repeating and growing patterns at ages 4, 5, and 6. Importantly, for both pattern types individual differences in performance uniquely contributed to numerical ability, while controlling for prior numerical ability and performance on the other pattern type. This indicates that abilities regarding each of these pattern types support later numerical ability in a unique way, and that, therefore, the association between patterning and numerical ability is not fully accounted for by a general ability to detect regularity and structure. Rather, understanding the specific structure of repeating patterns and the one of growing patterns are each uniquely important for later numerical ability. There are several regularities in numbers that can be related to both repeating and growing patterns. For example, the sequence of natural numbers is a growing pattern (i.e., systematic increase between two consecutive numbers, +1) in which the units in every ten repeat from 0 to 9 (i.e., the base-ten principle). The structure of repeating patterns could also support the acquisition of more efficient counting strategies (e.g., counting in pairs), which lead to a counting sequence that represents a growing pattern (e.g., 2-4-6-8). These insights and skills are key in many of the numerical tasks that were addressed in this study.

Numerical ability was a predictor of both patterning abilities from age 4 to age 5, but not from age 5 to age 6. Although researchers have claimed that repeating patterning activities

do not require numerical knowledge (Rittle-Johnson et al., 2019), it is plausible to assume that some very basic insight in small quantities might be helpful when solving repeating patterning tasks, especially in case a unit consists of multiple instances of the same element (e.g., AAB or AABB pattern). Some support for this idea can be found in the study of Flynn and colleagues (2020), who showed that labelling an ABB pattern with “one-two” is beneficial for children’s patterning performance. Also for growing patterns, as measured in the current study, children need to understand small quantities and be able to do some basic arithmetic (i.e., +1, +2, -1, -2) to complete the tasks. We therefore argue that performing well on some basic numerical activities can support performance on activities with repeating and growing patterns. At age 4, individual differences in numerical ability will probably reflect differences in such more basic numerical activities, such as object counting. At age 5, however, these more basic numerical abilities might have been already well-developed, resulting in little individual differences that might explain patterning performance one year later. It is likely that at age 5, there will still be meaningful individual differences in numerical ability between children, but on more complex numerical tasks, such as multi-digit number recognition or number order, which might be less relevant for the patterning activities that we investigated.

Although the present study extends current understanding of the associations between patterning, and numerical ability in early childhood, we must acknowledge that our results are limited in their generalizability. About 14% of the children dropped out due to changing to a different school over the course of two years and our analyses showed that these children were performing lower than the rest of our sample. This will remain a challenge for future longitudinal studies.

More work needs to be done in order to understand the mechanisms that underlie the associations between patterning and numerical ability. The present study showed that the observed associations between the two patterning abilities and numerical ability were not

merely explained by individual differences in spatial skills. Both patterning ability (Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019) and numerical ability (Hawes et al., 2019) have already been associated with spatial skills, as we also observed in the current study, and therefore spatial skills could mediate the association between patterning and numerical ability. The inclusion of spatial skills as a covariate, however, did not influence the association between the two patterning abilities and numerical ability. This is in line with previous studies that found a unique association between patterning and numerical ability while controlling for spatial skills (MacKay & De Smedt, 2019; Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019). Future investigations could evaluate the role of other cognitive measures in explaining the association between patterning and numerical ability. One interesting candidate would be a general measure of children's ability to detect regularity and structure, as in measures of analogical reasoning, wherein one has to identify a common relational structure between two situations and generate an inference driven by this structural commonality (Gentner & Smith, 2012). Analogical reasoning has already been associated with patterning (White et al., 1998) and could explain part of the association between patterning and numerical ability.

The fact that we found unique associations for repeating and growing patterning ability, suggests, however, that a general ability to detect structure will not fully explain the association between patterning and numerical ability. Some part of this association is unique for each pattern type. Future research on the mechanisms that underlie the association between patterning and numerical ability should take this into account. One important avenue for future studies is to investigate how different numerical abilities are associated with different patterning abilities. Zippert and colleagues (2019) found in their study that 4- to 5-year-olds' repeating patterning ability was a unique predictor of general numerical ability, but also of performance on specific numerical abilities, such as verbal calculation and magnitude

comparison. The question remains whether such associations could also be observed longitudinally, and whether the association with different patterning abilities is different for various numerical activities. This requires longitudinal studies that evaluate the association between different pattern types and a diverse range of numerical activities. A theoretical framework that links different numerical activities to different types of patterns is, however, necessary in order to make well-considered hypotheses.

Future research should also evaluate the predictive value of repeating patterning, growing patterning, and numerical ability for children's later mathematical development. Previous research has shown that patterning and numerical ability in preschool uniquely predict later mathematical performance (Lüken, 2012; Nguyen et al., 2016), but thus far performance on different pattern types was not taken into consideration. Moreover, the inclusion of the association between the predictor variables will shed a light on direct and indirect associations between repeating patterning, growing patterning, and numerical ability on the one hand, and mathematical ability on the other hand.

This study provides useful information for the development of guided intervention studies in early childhood. Children's repeating and growing patterning ability were important predictors of later numerical ability, an ability that is crucial in early mathematical development and which has a prominent place in early educational settings. In line with many other researchers (Papic, 2013; Rittle-Johnson et al., 2017), these results suggest that the inclusion of repeating and growing patterns in addition to numerical activities in early educational settings is warranted. We must however remain cautious, given that our study was only correlational and experimental evidence is necessary to make causal claims.

Associations Between Repeating and Growing Patterning Ability

The cross-lagged panel model that was set up to answer our main research question gave us also the opportunity to investigate the mutual association between repeating and

growing patterning ability. Individual differences in performance on one pattern type at age 4 were related to individual differences in performance on the other pattern type at age 5, even while controlling for prior performance on that pattern type, numerical ability, and spatial skills. Both pattern types were thus bidirectionally associated between age 4 and 5, and this association was not explained by spatial skills or numerical ability. The direction of the association however changed from age 5 to age 6, such that repeating patterning performance predicted growing patterning performance, but the reverse was no longer true. When including spatial skills as a covariate, this association was no longer significant, suggesting that spatial skills accounted for the predictive value of repeating patterning ability at age 5 to growing patterning ability at age 6.

The bidirectional association between age 4 and 5 could be explained by a general understanding of regularity and structure, leading children who understand one pattern type to perform better on another pattern type as well. This general understanding could represent the understanding that different elements in a sequence can form a regularity. The potential role of such a general understanding could be investigated further by evaluating the role of general cognitive competencies in the association between repeating patterning and growing patterning performance. If the bidirectional association between repeating and growing patterning from age 4 to 5 is indeed driven by a more general understanding of patterning, then general cognitive competencies might explain a large part of this bidirectional association.

The unidirectional association from repeating patterning towards growing patterning between age 5 and 6 could be driven by children's ability to construct a pattern based on units and deconstruct a pattern into units. Being able to flexibly switch between the pattern (e.g., ABABAB or ABAABAAAB) and its units (e.g., AB AB AB or AB AAB AAAB), will support children's patterning performance. Deconstructing a pattern into units is easier in

repeating patterns, where the unit remains the same, than in growing patterns, where the unit constantly changes. Children's ability to deconstruct repeating patterns into units might support later growing patterning ability, whereas their ability to deconstruct growing patterns into units might not yield additional support for later repeating patterning ability. The finding that this unidirectional association disappeared when controlling for spatial skills, supports this hypothesis. Our spatial tasks also required children to deconstruct spatial information into "units" or chunks.

More research is required to better understand how performance on one pattern type can support performance on another pattern type, and why this association changes throughout development. Our study was only correlational in nature, and intervention studies are needed to provide insight into potential causal associations between performance on both pattern types. More specifically, one could evaluate the impact of two patterning conditions (i.e., repeating vs. growing patterns) on both patterning abilities. The results of the present study suggest that whether or not a patterning intervention that focuses on one pattern type transfers towards another pattern type, might depend on children's age.

The complex role of spatial skills should also be investigated in more detail. The impact of spatial skills on the association between repeating and growing patterns differed over the course of two years: Spatial skills fully accounted for an association between both pattern types from age 5 to age 6, but not from age 4 to age 5. In line with other researchers (Rittle-Johnson et al., 2019), we therefore argue that it is crucial to take into account spatial skills when investigating the contribution of patterning abilities within early mathematical development.

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Tables and Figures

Table 1

Descriptive statistics for the main variables on three time points.

Variable	Second year of preschool (T1)		Third year of preschool (T2)		First grade (T3)	
	<i>N</i>	<i>M</i> (<i>SD</i>)	<i>N</i>	<i>M</i> (<i>SD</i>)	<i>N</i>	<i>M</i> (<i>SD</i>)
Age (in months)	407	58.14 (3.51)	382	70.09 (3.49)	354	82.25 (3.44)
Male [%]	407	213 [52%]	382	194 [51%]	354	179 [51%]
Repeating patterns	404	8.45 (3.93)	381	12.58 (3.86)	354	15.54 (2.35)
Growing patterns	401	4.35 (2.79)	381	7.63 (3.22)	354	10.32 (3.47)
Numerical ability	402	0.01 (0.71)	379	0.00 (0.55)	347	0.00 (0.66)

Note. Performance on the patterning measures are the number of correctly solved items. Numerical ability represents the average of the standardized score on each aspect of numerical ability that was addressed at that time point.

Table 2

Zero-order (below diagonal) and partial (above diagonal) correlations for the main variables on three time points, the two spatial measures and age.

Variable	1	2	3	4	5	6	7	8	9
1. Repeating patterns T1	-	.53	.50	.41	.40	.41	.31	.31	.30
2. Growing patterns T1	.67	-	.42	.35	.40	.38	.33	.36	.30
3. Numerical ability T1	.66	.58	-	.36	.37	.59	.24	.29	.41
4. Repeating patterns T2	.61	.52	.55	-	.46	.30	.45	.32	.36
5. Growing patterns T2	.58	.54	.55	.61	-	.37	.29	.48	.39
6. Numerical ability T2	.59	.52	.69	.52	.53	-	.19	.24	.46
7. Repeating patterns T3	.49	.45	.42	.62	.46	.40	-	.28	.23
8. Growing patterns T3	.50	.51	.47	.50	.59	.43	.47	-	.34
9. Numerical ability T3	.49	.46	.54	.52	.52	.59	.41	.51	-
10. Spatial ability	.49	.41	.43	.45	.38	.42	.36	.45	.45
11. Visuospatial working memory	.43	.41	.44	.42	.43	.40	.35	.38	.37
12. Age at T1	.24	.25	.31	.16	.17	.25	.18	.14	.15

Note. All correlations are significant at $p \leq .001$, except for correlations with age which were all significant at $p \leq .01$. Partial correlations control for visuospatial working memory and spatial ability.

Figure 1. Instruction and example of a repeating and growing pattern item for extending, translating, and identifying. Adapted from (Wijns, Torbeyns, Bakker, et al., 2019).

Activity	Repeating pattern	Growing pattern
Instruction		
Extending “Look carefully at this row. There is a pattern in it. At the end there is something missing. One of these figures has to be placed in the empty spot. Do you know which of these has to be on the empty spot?”
Translating “Look carefully at this row. There is a pattern in it. Please make the same pattern with your figures on the paper strip.”
Identifying “You will soon see a row with a pattern in it. You will have to look very closely at it and try to remember the pattern. After a short period I will hide the pattern and you will have to copy it.”

Figure 2. Cross-lagged panel model for our final model without spatial skills

Note. * $p < .05$, ** $p < .01$, *** $p < .001$. Coefficients are standardized. For clarity reasons, within-wave covariances and error terms are not shown.

Figure 3. Cross-lagged panel model for our final model with spatial skills

Note. * $p < .05$, ** $p < .01$, *** $p < .001$. Coefficients are standardized. For clarity reasons, within-wave covariances, error terms, and associations with spatial measures are not shown.