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Mapping between number notations in kindergarten and the role of home numeracy

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Abstract

Two recent studies investigated how children learn to map between digits, number words, and dots (Hurst, Anderson, & Cordes, 2017; Jiménez-Lira, Carver, Douglas, & LeFevre, 2017). In the current study we aimed to replicate these previous findings by examining a much larger sample (*N* = 195 kindergarteners, aged 2 years 6 months to 5 years 2 months) and taking into account home numeracy activities, that is, daily parent-child interactions with numerical content. In line with previous studies, the results showed that children first learn to map number words onto dots, and number words onto digits, and only afterwards – to map digits onto dots. Furthermore, number words ↔ digits mapping was a better mediator of the relation between digits ↔ dots and the dots ↔ number words mapping tasks, than the dots ↔ number words, suggesting that children rely on their symbolic number knowledge to learn the relation between digits and dots. Finally, both basic and advanced home numeracy activities were positively related to children's mappings skills. Furthermore, we observed that with increasing the children's age a shift from basic to advanced activities was present. These results emphasize the importance of tailoring the home numeracy activities according to children's age.

Keywords: early numerical development, symbolic number acquisition, numerical mapping, home numeracy

Introduction

Numerical information can be represented in a non-symbolic format (i.e., sets of items such as dot configurations), or in a symbolic format (i.e., as written digits or verbal number words). While adults can translate between dots, digits, and numbers words effortlessly, learning how to map between these different numerical formats is quite challenging for young children. So far, studies on numerical development have suggested that when learning the correspondence between the numerical formats, as a first step, children acquire the relation between small (up to 4-5) dots and number words (e.g., Benoit, Lehalle, & Jouen, 2004; Benoit, Lehalle, Molina, Tijus, & Jouen, 2013; Le Corre & Carey, 2007; Odic, Le Corre, & Halberda, 2015). How children learn the correspondence between dots and digits and number words and digits, however, is still debated. With respect to this topic, two main developmental accounts have been proposed – the "quantity account" and the "symbolic account" (see Hurst, Anderson, & Cordes, 2017). The "quantity account" assumes that children first learn to map number words to dots and digits to dots. Then, based on their knowledge of how symbolic numbers relate to non-symbolic ones, children also learn the associations between the two symbolic numerical formats, i.e., number words and digits (i.e., if "four" = "●●●●"and "4" = "●●●●", only then "four" = "4"; Benoit et al. 2013). The "symbolic account", on the contrary, assumes that children make the reversed inference. According to this account, children first learn mapping number words and dots, then number words and digits, and then use the latter associations to relate digits to their corresponding non-symbolic representation (i.e., if "four" = "●●●●", and "four" = "4", only then "4" = "●●●●"; Hurst et al., 2017).

Recently, two independent studies (Hurst et al., 2017; Jiménez Lira, Carver, Douglas, & LeFevre, 2017) contrasted both accounts and found evidence for the "symbolic account" (but see Benoit et al., 2013). However, due to their relatively small sample sizes (*N* = 48, and *N*= 62, respectively), these studies might have been underpowered. In addition, these studies did not address environmental factors such as parent–child interactions with numerical content, whereas it has already been demonstrated that these so-called home numeracy experiences (Blevins-Knabe & Austin, 2016) can influence children's mapping abilities (e.g., Mutaf-Yildiz, Sasanguie, De Smedt, & Reynvoet, 2018a). Therefore, in the current study, our aim was to replicate the findings of Hurst et al. (2017) and Jiménez-Lira et al. (2017) by examining a much larger sample (*N* = 195) and taking the role of home numeracy into account.

About the acquisition of digits and number words.

In numerical cognition research, there are at least two competing accounts of how children acquire the links between non-symbolic quantities and symbols in general. According to the traditional view, the acquisition of digits and number words is deeply rooted in the Object Tracking System (i.e., OTS) and the Approximate Number System (i.e., ANS) – two evolutionarily ancient core systems for numbers (e.g., Feigenson et al., 2004; Piazza, 2010). It has been suggested that in the OTS, numbers are represented precisely, as individual elements in a set. The representations of these individual elements are stored as long-term memory models and can be applied to a novel set of objects (e.g., [●●●] = [i, j, k]). However, because the OTS has a limited representational capacity for items up to 4-5, it has been argued that this core system only supports the learning of small symbolic numbers up to 4 or 5 (e.g., Carey, 2009; Carey, Shusterman, Haward, & Distefano, 2017; Feigenson et al., 2004; Piazza, 2010). On the other hand, in the ANS numbers are represented as an imprecise sum for a set of objects (e.g., Dehaene & Changeux, 1993; Nieder & Dehaene, 2009). These mental representations are assumed to be in the form of Gaussian distributions, organized on a left-to-right oriented Mental Number Line (Dehaene, 2001). Moreover, because the ANS has an unlimited representational capacity for numbers (i.e., any set of items can be represented in the ANS, irrespectively of its size), it has frequently been proposed as *the* essential building block of all number knowledge (e.g., Feigenson et al., 2004).

Indeed, developmental models of number acquisition have consistently attributed the ANS a crucial role in the acquisition of both number words and digits (Dehaene, 2007; Von Aster & Shalev, 2007). For example, in their four-step developmental model, Von Aster and Shalev (2007) pointed to the ANS as a first step and a "[…] necessary precondition for children to learn to associate a perceived number of objects or events with spoken or, later, written and Arabic symbols […]" (Von Aster & Shalev, 2007, p.870). In a second step, children acquire the number words, which enables them to learn the counting procedure, counting strategies, etc. In a third step, children acquire the digits, which enables the development of written calculation skills. Finally, children acquire more complex knowledge, such as ordinality (i.e., the knowledge of the relative position of an item in a set), which enables them to perform approximate calculations and arithmetic operations.

Alternatively, however, it has been suggested that children may not necessarily learn the meaning of digits by associating them with their corresponding non-symbolic representations (e.g., Carey, 2009; Carey & Barner, 2019; Reynvoet & Sasanguie, 2016). According to this alternative view, children acquire the meaning of digits via their already established knowledge of the corresponding number words. In line with this suggestion, Bialystok (1992, 2000; see Bialystock & Codd, 1996) proposed a three-stage developmental model for symbolic number knowledge. In this model, the first stage is *conceptual*. At this stage, children learn to recite the counting sequences, which enables to form a holistic representation (i.e., the counting sequence as a whole string). Using this holistic representation, children learn to identify individual members of the set and to categorize them (i.e., number words go to the number class and are different than letters or other words). In the second, *formal* stage, children use analogies to learn the relation between number words and digits, similarly to the way they learn that "bau bau" goes with a label for "dog". In the final, *symbolic* stage, the written and verbal numerals are organized in one system. Only then children are able to associate symbolic numbers with non-symbolic quantities. Indirect support for this model comes from studies using the moving word paradigm (Bialystock, 2000). In the numerical version of this task (see Bialystock, 2000) children are presented with two piles of objects (e.g., four Lego pieces vs six Lego pieces). Then, a card with a printed numeral (e.g., a card with printed "4") is put under the corresponding pile of objects, and the experimenter tells the child "This card says "four". Then, the experimenter moves the same card under the other pile of objects (i.e., the pile with six Lego pieces) and asks the child to say what is on the card. Results showed that for values above their current counting range, children often respond that the digit on the card indicates the number of pieces in the new pile (i.e., "six" in the given example). This shows that children do not yet understand that each digit represents a unique set of items. Interestingly, children do not make this mistake if the numerals and the objects are within their counting range. These results suggest that number words to digits mappings are necessary to understand the stable relation between digits and non-symbolic numbers (i.e., sets of items).

Meanwhile, a few studies explicitly compared both accounts (e.g., Benoit et al., 2013; Hurst et al., 2017; Jiménez-Lira et al., 2017; see also Hutchison, Ansari, Zheng, De Jesus, & Lyons, 2019; Malone, Heron-Delaney, Burgoyne, & Hulme, 2019). As the first, Benoit et al. (2013) tested children's mapping skills for small numbers (1 to 6). In this between-subject study, a total of 144 3-, 4-, and 5-year-old children were tested on the following mapping tasks: digits → dots, dots → digits, number words → dots, dots → number words, digits → number words, number words → digits. Results showed that all mappings were bidirectional, that is equally good in both directions of the mapping pair (e.g., digits → dot vs dots → digits). Moreover, the mappings between number words and dots yielded higher accuracies, followed by the mapping between dots and digits, and, finally, the mapping between the numbers words and digits. According to the authors, the mapping between number words and digits develops last, because children first need to be able to map both number words and digits onto the corresponding non-symbolic numerosity before these symbolic notations can be associated with each other. Therefore, these results clearly suggest that "[…] non-symbolic code used to represent arrays may have a privileged status in the acquisition of digit mappings" (Benoit et al., 2013, p.100). However, in this study, the dot configurations were presented as familiar dice-like patterns, which might have improved the performance of the children confronted with this notation. In addition, the design used in this study was not well balanced because the response alternatives presented to the children were not equivalent across the mapping tasks. That is, when mapping dots onto number words and digits onto number words, children were asked to provide their answer by reproducing the number word corresponding to the visually presented digit or dot. In the remaining tasks, however, children were asked to provide their answer by choosing one of the six presented response alternatives. This possibly made some mapping tasks more difficult than others (for similar claim see Jiménez-Lira et al., 2017).

Taking into account these limitations, a bit later, Hurst et al. (2017) conducted a detailed investigation of children's mapping skills (and also Jiménez-Lira et al., 2017 did so, see later in the text). In Hurst et al.'s study, using a within-subject design, the performance of 24 3-year-old and 24 4-year-old children was examined on six mapping tasks between digits, numbers and dots (ranging from 1 to 5) using random dot patterns and five-alterative forced-choice paradigm. In contrast to the study by Benoit et al. (2013), children here performed worse on the digits ↔ dots mappings. Children performed better on number words ↔ digits, and dots ↔ number words mappings for which the performance did not differ. Mediation analyses were performed to investigate which mapping knowledge is required to map successfully between digits and dots. Results showed that number words ↔ digits mapping mediated the relation between dots ↔ number words and digits ↔ dots mapping. In contrast, children's performance in the number words ↔ dots mappings did not mediate the relations between number words ↔ digits and digits ↔ dots. According to the authors, these findings suggest that once children have learned the association between number words and dots number words becomes the preferred representation for learning digits. Hurst et al. (2017) described this developmental process as follows: "Once children have one mapping between a symbolic representation (i.e., number words) and quantities, they integrate the second representation (i.e., Arabic numeral) by mapping it to the existing symbolic structure and use the existing quantity-word mapping to understand the quantity-numeral mapping" (Hurst et al., 2017, p. 58). In an independent study published at about the same time, Jiménez-Lira et al. (2017) observed a very similar finding. In this study 62 2-to-4 years-old children were tested and here too it was observed that digits ↔ dots mapping developed last and depended on children's ability to map dots onto number words, and number words onto digits. Taken together, these two studies provide considerable support for the symbolic account, suggesting that children first learn the number words ↔ dots, and number words ↔ digits relation, and on the basis of this knowledge, children infer that the digits and dots are also related (see also Bialystock, 1992).

The current study

Despite the convergent findings of the studies by Hurst et al. (2017) and Jiménez-Lira et al. (2017), both studies have some limitations. First, both studies had relatively small sample sizes (N = 48 3-to-4-year olds, and N = 62 2-to-4-year-olds, respectively) and thus possibly have been underpowered. Second, both Hurst et al. (2017) and Jiménez-Lira et al. (2017) acknowledged the possibility that cultural and/or educational factors can affect children's mapping abilities but did not include these factors in their studies. As argued by Hurst et al. (2017), one important factor that influences early mapping abilities is home numeracy, that is, daily parent-child interactions that include experiences with numerical content such as singing counting songs, naming numerals, playing dice games, cooking, etc. (Blevins-Knabe & Austin, 2016; LeFevre, Skwarchuk, Smith-Chat, Kamawar, & Bisanz, 2009). One way to assess home numeracy is through parental self-reports on the frequency of various activities they conducted with their child (e.g., Huntsinger, Jose, & Luo, 2016; Kleemans, Peeters, Segers, & Verhoeven, 2012; LeFevre et al., 2009; Mutaf-Yildiz, Sasanguie, De Smedt, & Reynvoet, 2018a,b; Skwarchuk, Sowinski, & LeFevre, 2014). Many studies have now shown that the more parents engage in these activities with their child, the better the child's early number processing skills (e.g., Susperreguy, Burr, Xu, Douglas, & LeFevre, 2020; Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre,2020; Thompson, Napoli, & Purpura, 2017; for a review see Mutaf-Yildiz, De Smedt, Sasanguie, & Reynvoet, 2020). For example, Mutaf-Yildiz et al. (2018a), explicitly investigated the relation between home numeracy and children's early number processing skills (e.g., mapping, enumeration, etc.) in 5-to-6-year-old children. The results showed that children's mapping skills between digits and dots are positively related to the frequency of home numeracy activities (see also Benevides-Varela et al., 2016). Home numeracy activities are also typically split up into basic (e.g., reciting numbers) and advanced (e.g., learning simple sums) activities (e.g., Skwarchuk et al., 2014), typically reported more frequently in respectively young (e.g., 3-year-olds) and slightly older children (e.g., above the age of 4; see LeFevre et al., 2009; Thompson et al., 2017). Basic and advanced activities may be related to the mapping skills differently, and as a consequence, the relation between home numeracy and the children's skills to map between the three notations might be age-dependent. To address this latter question, again a much larger sample size and wider age range are needed than those in the previous studies examining mapping between the three number notations.

Against this background, it becomes apparent that a more powerful replication study would enable us to confirm the developmental pattern reported in previous studies, and in addition, to address the relation between basic and advanced home numeracy activities and mapping skills at different ages. To this end, we presented a total of 195 kindergartners (age range: 2 years 6 months to 5 years 2 months), divided into three age groups (see Hurst et al. 2017, p.47): 3-year-olds (M_{age} = 37 months), 4-year-olds (M_{age} = 48 months), and 5-year-olds (M_{age} = 57 months) with six numerical mapping tasks, between digits, number words and dots for numbers ranging from 1 to 4 (see Method section). In addition, a home numeracy questionnaire (Skwarchuk et al., 2014) was administered from the parents. Following Hurst et al. (2017) and Jiménez-Lira et al. (2017), we made the following predictions. *First*, we expected that children of all three age groups would perform worse on the digits ↔ dots mappings and would perform better on the number words ↔ digits and dots ↔ number words mappings. *Second,* if children learn the relation between digits and dots on the basis of their knowledge about how number words relate to dots and to digits, we expected the relation between children's performance on digits ↔ dots and dots ↔ number words mapping to be mediated by their performance on number words ↔ digits (see also Hurst et al., 2017). *Third*, we hypothesized that both basic and advanced home numeracy activities would be positively related to children's mapping skills (see LeFevre et al., 2009; Skwarchuk et al., 2014; Mutaf-Yildiz et al. 2018a). However, it is quite likely that the frequency of basic and advanced home numeracy activities (as reported by the parents) would be related to the mapping tasks differently, depending on children's age. Specifically, we expected that in the 3-year-olds, basic home numeracy activities would be related to the children's performance in the mapping tasks, while in the 4- and 5-year-olds, advanced home numeracy activities would be related to their performance on the mapping tasks (see for example Thompson et al., 2017; Mutaf-Yildiz et al. 2018a).

Method

Participants

A total of 195 Flemish kindergartners[1] were tested. In exchange for their participation, children were allowed to choose a small toy out of a surprise box. The experimental protocol was approved by the university's ethical committee (file number G–20160679). Two children were excluded because they failed on all mapping tasks (mean total accuracy score of 0.00 and 0.01 respectively). Consequently, the final sample consisted of 193 children, aged between 2 years 7 months and 5 years 2 months (M_{age} = 47 months, *SD* = 8 months, 112 girls). To make our sample as comparable to the one in the study by Hurst et al. (2017) we further divided the children into three age groups in correspondence to the sample characteristics in the study (see Hurst et al., 2017; p.47): 1) a group of 3-year-olds consisting of 62 children aged between 2 years 7 months and 3 years 6 months (M_{age} = 37, months, *SD* = 3 months; 31 girls), 2) a group of 4-year-olds consisting of 84 children aged between 3 years 7 months and 4 years 6 months (M_{age} = 48, months, *SD* = 3 months, 51 girls), and 3) a group of 5-year-olds consisting of 47 children aged between 4 years 7 months and 5 years 2 months (M_{age} = 57, months, *SD* = 2 months, 30 girls). Children were tested at their school in the second part of the academic year from the end of January until the end of March. Only children who were native Dutch speakers and spoke Dutch at home took part in the study. Prior to the testing, permission to conduct the study was obtained from the kindergarten school's principals, and written informed consent was obtained from the parents.

To estimate the required sample size, we used G*power statistical software v.3.1.9.7 (Faul, Erdfelder, Lang, & Buchner, 2007). Prior studies have documented rather large effect sizes for the mapping tasks (e.g., $\eta_p^2 = 0.4$ in Hurst et al., 2017). However, given that only a handful of studies have been conducted explicitly investigating this issue, a possibility exists the true effect sizes are overstated (see Brysbaert, 2019). Consequently, assuming smaller to moderate effect sizes, such as $\eta_p^2 = 0.06$ is recommended (Cohen, 1988; Miles & Shevlin, 2001). With a power of 95%, $\alpha = 0.05$, and a repeated-measures ANOVA per age group with 3 levels, the minimum recommended sample

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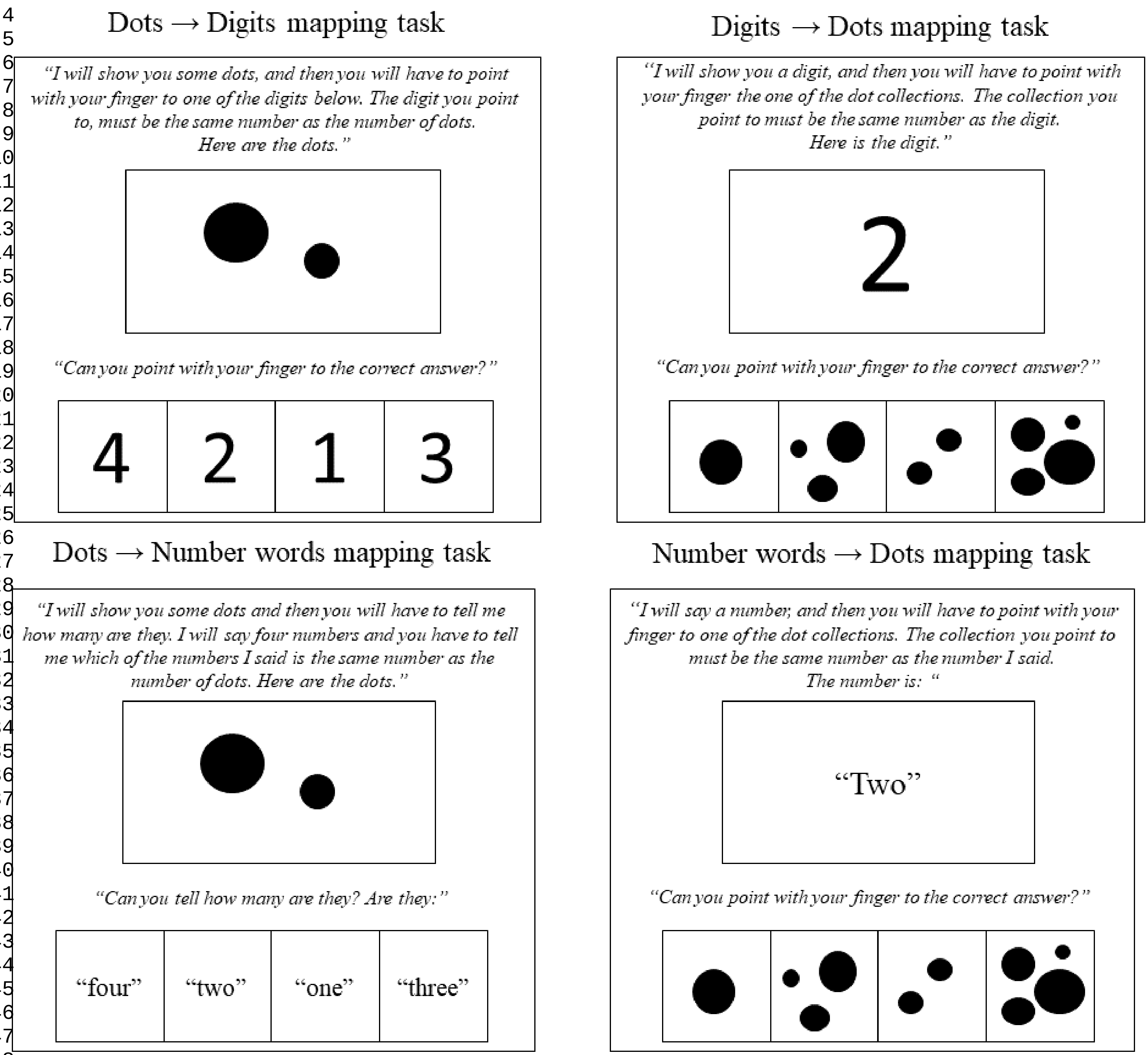
[1].In Belgium, children start formal schooling (i.e., 1st grade) around the age of 6 years. Before, most children attend about 3 years of kindergarten education, starting when they are between 2 years and 6 months and 3-year-old.

size to obtain this effect size is 42. Similarly, to obtain a moderate correlation of $r = 0.30$ (two-tailed) between the home numeracy activities and the mapping tasks, the recommended sample size is 134. It is worth noting, however, that although the total sample size is sufficient to investigate the role of home numeracy, we also report the correlations between the home numeracy and the mapping tasks per age group. Consequently, these analyses should be interpreted with caution.

Procedure, tasks, and stimuli

Numerical mapping tasks. All children were tested individually in a quiet, separate room at their own school. Children were presented with six four-alternative forced-choice (4AFC) mapping tasks (digits → dots; dots → digits; number words → dots; dots → number words; digits → number words; number words → digits) in a within-participant design. The tasks contained either a verbal (i.e., number words) or visual (i.e., digits, dots) target number stimulus, and a set of four randomly presented verbal (i.e., number words) or visual (i.e., digits, dots) response alternatives (see Figure 1). Both the target stimuli and the response items were varied in the number range between 1 and 4. The visual stimuli and the visual response items were printed in black font (Calibri, size 100, ≅ 4 cm in high) on a white card (standard A4 format 21.0 cm × 29.7 cm; see Figure 1). The size of the individual dots in a set was varied, and the configuration of the set changed on every trial. The six mapping tasks were presented in separate booklets. The order of the booklets was counterbalanced across participants. Each trial started with the experimenter announcing the instructions to the child, as shown in Figure 1. In the number words ↔ dots and number words ↔ digits mapping tasks, number words were (only) auditorily presented to the children, verbalized (in Dutch) by the experimenter. The experimenter wrote down whether the child responded correctly or not. Children scored “1” if they matched the target and the response item correctly, and scored “0”, if they made an error or did not respond at all. The children did not receive feedback on the accuracy level of their answers. Each task started with a practice trial. If the child failed to give a response, the instructions were mentioned again step by step by the experimenter until the child knew how to respond. Next, twelve experimental trials (3 times the numbers 1-4) were administered. After a small break, the next task was introduced. The six mapping tasks took about twenty minutes.


Dots → Digits mapping task
"I will show you some dots, and then you will have to point with your finger to one of the digits below. The digit you point to, must be the same number as the number of dots. Here are the dots."
"Can you point with your finger to the correct answer?"
4
2
1
3
Digits → Dots mapping task
"I will show you a digit, and then you will have to point with your finger the one of the dot collections. The collection you point to must be the same number as the digit. Here is the digit."
2
"Can you point with your finger to the correct answer?"
Dots → Number words mapping task
"I will show you some dots and then you will have to tell me how many are they. I will say four numbers and you have to tell me which of the numbers I said is the same number as the number of dots. Here are the dots."
"Can you tell how many are they? Are they:"
"four"
"two"
"one"
"three"
Number words → Dots mapping task
"I will say a number, and then you will have to point with your finger to one of the dot collections. The collection you point to must be the same number as the number I said. The number is: "
"Two"
"Can you point with your finger to the correct answer?"

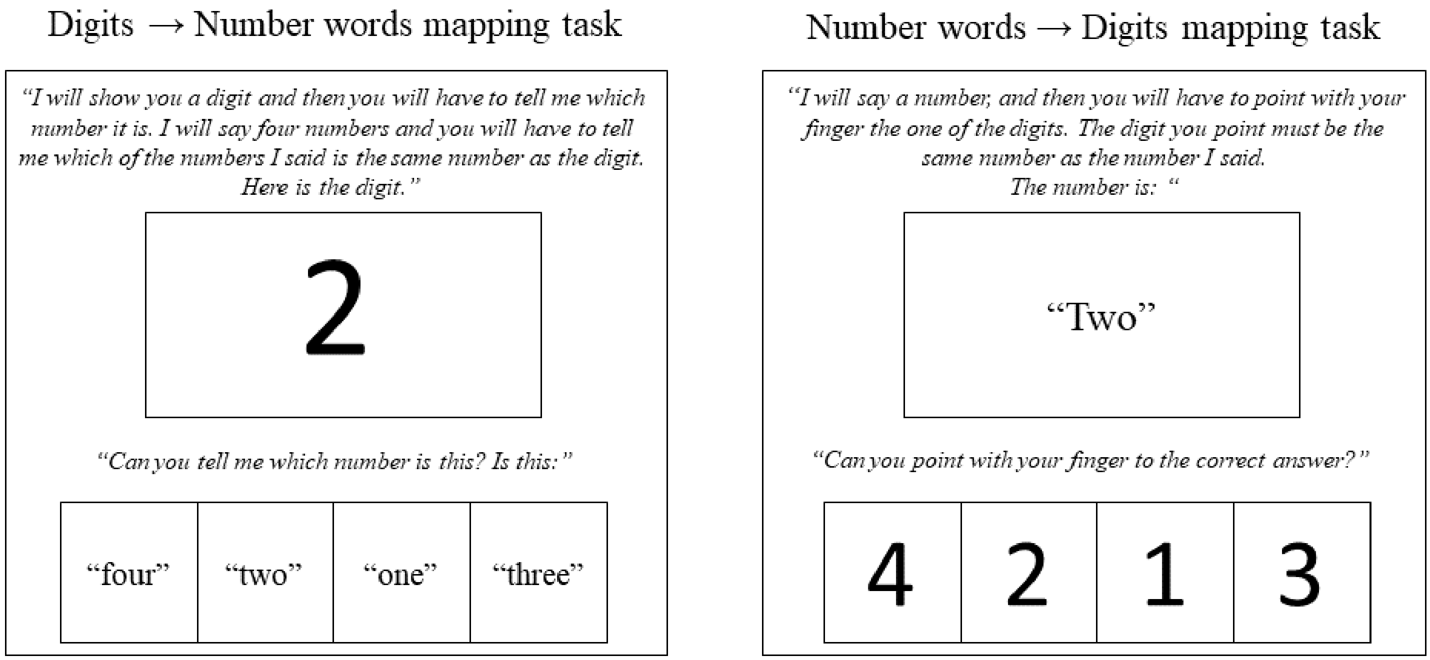


Figure 1.Visual representation of the six numerical mapping tasks with the corresponding task instructions. The number words were presented (only) auditorily, verbalized by the experimenter.

<u>Home numeracy questionnaire.</u> Parents completed a Dutch translation of the questionnaire used by Skwarchuk et al. (2014). The children took the questionnaire at home – where the parents completed it – and returned it to school. This questionnaire consisted of 13 questions about the frequency of number-related activities which parents do with their children at home. Parents indicated how frequently they engaged in these activities on a 5-point Likert scale (0 = "never", 4 = "everyday"). Given that previous research suggest that differences in children's number skills can also be explained by the socio-economic status (SES) of the family (e.g., Sarnecka, Negen, & Goldman, 2018) the questionnaire also measured the level of maternal education, as a proxy for the SES of the child's family (e.g., Davis-Kean, 2005; Mutaf-Yildiz et al. 2018a,b). Finally, the questionnaire also included items regarding the expectations parents have with respect to their children's academic achievements. Given that the focus of the current study is on the possible influence of the numerical home activities on children's mapping skills, we here only further considered the questions on home numeracy activities (n=13), and the level of maternal education as a control variable.

Data analysis plan

For replication purposes, we followed the analyses performed by Hurst et al. (2017) when analyzing the performance on the mapping tasks. To test our first hypothesis about the order in which children acquire the mappings between dots, digits, and number words, we performed two analyses. First, we performed paired sample *t*-tests per age group between each mapping pair (e.g., digits → dots vs dots → digits) to test whether children performed at the same levels in both direction of the numerical mapping pairs and thus acquired the mappings simultaneously. Then, we averaged across mapping pairs (e.g., digits → dots + dots → digits = digits ↔ dots) and performed repeated-measures ANOVAs with task (3 levels: dots ↔ number words, digits ↔ dots, number words ↔ digits) as a within-subject factor and age (3 levels: 3-year-olds, 4-year- olds, 5-year-olds) as between-subject factor.

To examine the second hypothesis that is whether children's ability to map between digits and dots depends on their knowledge of dots ↔ number words and number words ↔ digits mappings, we conducted mediation analyses with dots ↔ digits as an outcome variable, dots ↔ number words as a predictor and number words ↔ digits as the mediator. We also tested the reverse model with dots ↔ number words as mediator. This way we examined whether children acquire the digits ↔ dots relation by first mapping each symbolic number to its corresponding numerosity, or by relying on the knowledge of how number words relate to both digits and dots instead.

To examine the third hypothesis about the relation between home numeracy and children's mapping skills we first conducted a Principal Component Analysis (PCA) examining whether the questionnaire items can be divided into basic and advanced home numeracy activities as described in Skwarchuk et al., 2014. Second, to check whether the parents engaged their children in basic and advanced home numeracy activities with different frequency as the children got older , we performed repeated-measures ANOVA with type of activity (basic vs advanced) as a within-subject factor and age (3 levels) as a between-subject factor. Then, we performed partial correlations (controlling for SES and age) over all participants to examine whether basic and advanced home numeracy activities were related to the mapping tasks. Finally, to examine the pattern of these relations across the age groups, we performed partial correlations (controlling for SES) per age group.

Considering home numeracy, data from 180 children's parents was collected, because not all children's parents completed the questionnaire. Therefore, the analyses reported in the home numeracy section considers a sample of 180 (M_{age} = 48 months, *SD* = 8 months, range = 2;7 – 5;2, 105 girls) instead of 195 kindergartners. Of the 180 kindergartners, of which 52 were 3-year olds

(M_{age} = 37 months, *SD* = 3 months, range = 2;7 – 3;6, 26 girls), 81 were 4 -year-olds (M_{age} = 48 months, *SD* = 3 months, range = 3;7 – 4;6, 49 girls), and 47 were 5-year-olds (M_{age} = 58 months, *SD* = 2 months, range = 4;7-5;2, 30 girls). The socio-economic status of the family as indicated by the highest educational degree of the mother was middle to high: 2.78% reported having no degree, 2.22% reported having a degree of elementary school, 7.22% reported a degree of lower secondary education, 25.00% reported a degree of higher secondary education, 38.89% had a bachelor degree and, 21.68% had a master degree. For 2.22% of the parents, this information was missing.

Results

Numerical mapping tasks

Mean accuracies per mapping task, depicted in Table 1, were used for all further analyses (data freely available on the https://osf.io/ug8eb/). To make our data as informative as possible, in addition to the results obtained using the classical statistical approach, wherever it was possible, we also reported the corresponding Bayes factors (BF), or log(BF), if the BF value was too large to be interpreted (Jarosz & Wiley, 2014; Wagenmakers et al., 2018a,b) [2].. Whenever possible, to obtain both classical and Bayesian results, we used the JASP statistical package, version 0.13 (https://jasp-stats.org/) with a default Cauchy prior. For the mediation analysis, the principal component analysis (PCA), and the partial correlations, we used IBM SPSS Statistics version 24.Typically, the results of Bayesian and classical analyses converge. Nevertheless, discrepancies are possible, which can pose a challenge to the interpretation of the results (see Wagenmakers, 2020). Because the Bayesian approach allows us to evaluate both the alternative and the null hypotheses, in case of discrepancies between the classical and Bayesian results, we prefer to base the interpretations of our results on the Bayesian analyses.

[2] The BF_{10} is the ratio of the likelihood of the alternative hypothesis and the likelihood of the null hypothesis (while BF_{01} is simply the inverse ratio of these two likelihoods). For the more complicated models involving a larger number of factors (e.g., repeated measures ANOVA) we reported the $BF_{Inclusion}$ (see Wagenmakers et al., 2018b for the rationale). According to the interpretation of Jeffreys (1961), BF values between 1 and 3 are considered as anecdotal evidence (“not worth more than a bare mention”, Jeffreys, 1961) for the alternative hypothesis, BF values between 3 and 10 are considered as moderate evidence, BF values between 10 and 30 are considered as strong evidence, BF values between 30 and100 are considered very strong evidence, and BF values above 100 are considered as extreme evidence.

Table 1.Mean accuracies (proportion correct) with their corresponding (standard deviations), depicted per mappings task, mapping pair and age group.

Age group **Mapping task/pair**	**3-year-olds (*N*=62)**	**4-year-olds (*N*= 84)**	**5-year-olds (*N*= 47)**
Dots ↔ Number words	0.54 (0.28)	0.79 (0.22)	0.97 (0.12)
Number words → Dots	0.49 (0.29)	0.76 (0.25)	0.97 (0.11)
Dots → Number words	0.60 (0.33)	0.84 (0.25)	0.96 (0.14)
Digits ↔ Dots	0.21 (0.19)	0.48 (0.38)	0.87 (0.26)
Digits → Dots	0.22 (0.20)	0.49 (0.37)	0.80 (0.25)
Dots → Digits	0.20 (0.21)	0.48 (0.40)	0.85 (0.28)
Number words ↔ Digits	0.30 (0.20)	0.58 (0.34)	0.89 (0.23)
Digits → Number words	0.27 (0.21)	0.57 (0.36)	0.89 (0.25)
Number words → Digits	0.33 (0.26)	0.59 (0.36)	0.90 (0.23)

Direction and the relative difficulty of the mapping tasks. First, to assess whether children mapped equally well in both directions of the numerical mapping pairs (e.g., dots → digits vs digits → dots), we first performed a series of (Bayesian) two-tailed paired sample *t*-tests per age group.

For the *3-year-olds*, there was strong evidence for a difference between dots → number words vs number words → dots mappings, $t(61) = 3.\ 28$, $p = 0.002$, $d = 0.32$, $BF_{10} = 16.17$, with the accuracy for dots → number words being slightly higher than for the reverse mapping. There were no significant differences for digits → dot vs dots → digits pairs, $t(61) = 1.01$, $p = 0.32$, $d = 0.13$, $BF_{10} = 0.22$. Finally, the evidence for a difference between digits → number words vs number words → digits was very weak, $t(61) = -2.09$, $p = 0.04$, $d = -0.27$, $BF_{10} = 1.05$. For the *4-year-olds*, the results were largely the same. The dots → number words mapping differed significantly from the number words → dots mapping, $t(83) = 3.19$ $p = 0.002$, $d = 0.35$, $BF_{10} = 12.63$. There were no differences for digits → dots vs dots → digits pairs, $t(83) = 0.66$, $p = 0.51$, $d = 0.07$, $BF_{10} = 0.14$, and neither for digits → number words vs number words → digits, $t(83) = -0.81$, $p = 0.42$, $d = -0.09$, $BF_{10} = 0.17$. For the *5-year-olds*, there were no significant differences between the mapping directions for any of the pairs: dots → number words vs number words → dots mapping, $t(46) = -$

1.05, $p = 0.30$, $d = -0.15$, $BF_{10} = 0.26$, digits → dots vs dots → digits, $t(46) = 1.35$, $p = 0.18$, $d = 0.20$, $BF_{10} = 0.37$, digits → number words vs number words → digits, $t(46) = -0.54$, $p = 0.59$, $d = -0.08$, $BF_{10} = 0.18$. Overall, these results replicated the findings of Hurst et al. (2017), showing that the mappings between digits, number words, and dots, are acquired simultaneously. Also similar to Hurst et al. (2017) (and also Benoit et al., 2013), 3- and 4-year-old children performed slightly better when mapping dots to number words, than number words to dots.

Second, we compared the relative difficulty of the mapping tasks. Following Hurst et al. (2017) and given that children did not show strong performance differences between any of the mapping directions, we reduced the number of experimental conditions from 6 to 3 by averaging across mapping tasks in the following way: dots ↔ number words (= dots → number words + number words → dots), digits ↔ dots (= dots → digits + digits → dots), and number words ↔ digits (= number words → digits + digits → number words). These averaged scores were submitted to a (Bayesian) repeated measures ANOVA with mapping task (3 levels) as a within-subject factor and age as a between-subject factor. Whenever the assumption of sphericity was violated, the Greenhouse–Geisser correction was applied.

Results showed a main effect of task, $F(1.48, 281.38) = 104.51$, $pGG < 0.001$, $\eta_p^2 = 0.08$, $\log(BF_{Incl}) = 32.46$, a main effect of age, $F(2,190) = 81.26$, $pGG < 0.001$, $\eta_p^2 = 0.35$, $\log(BF_{Incl}) = 32.46$, and an interaction between mapping task and age, $F(2.96, 281.38) = 7.72$, $pGG < 0.001$, $\eta_p^2 = 0.012$, $\log(BF_{Incl}) = 9.15$. To disentangle this interaction, we performed post hoc ANOVA per age group with mapping task (3 levels) as a within-subject factor.

For the *3-year-olds*, there was a significant main effect of the mapping task, $F(1.47, 89.67,) = 64.90$, $pGG < 0.001$, $\eta_p^2 = 0.52$, $BF_{Incl} = 39.34$. The dots ↔ number words task yielded higher accuracies than both digits ↔ dots, and number words ↔ digits mappings, Cohen's $d = 1.40$, $p_{bonf} < 0.001$, $\log(BF_{10}) = 26.35$, and Cohen's $d = 1.01$, $p_{bonf} < 0.001$, $\log(BF_{10}) = 14.85$, respectively. Number words ↔ digits mappings were also more accurate than the digits ↔ dots, Cohen's $d = 0.39$, $p_{bonf} = 0.008$, $\log(BF_{10}) = 7.51$. For the *4-year-olds*, the results were the same. There was a main effect of task, $F(1.49,123.98) = 58.24$, $pGG < 0.001$, $\eta_p^2 = 0.41$, $BF_{Incl} = 39.34$.The dots ↔ number words task yielded higher accuracies than both digits ↔ dots and number words ↔ digits mappings, Cohen's $d = 1.15$, $p_{bonf} < 0.001$, $\log(BF_{10}) = 27.31$, and Cohen's $d = 0.79$, $p_{bonf} < 0.001$, $\log(BF_{10}) = 13.67$, respectively. Number words ↔ digits mappings were also more accurate than the digits ↔ dots, Cohen's d = 0.36, p_{bonf} = 0.004, log(BF_{10}) = 8.81. For the *5-year-olds,* there was a main effect of task, F(1.39,64.33) = 9.14, pGG = 0.001, η_p^2 = 0.17, log(BF_{Incl}) = 4.64. The dots ↔ number words task yielded higher accuracies than digits ↔ dots and number words ↔ digits mappings, Cohen's d = 0.61, p_{bonf} < 0.001, log(BF_{10}) = 3.11, and Cohen's d = 0.43, p_{bonf} = 0.012, log(BF_{10}) = 1.50, respectively. Number words ↔ digits mappings were not significantly more accurate than the digits ↔ dots, Cohen's d = 0.18, p_{bonf} = 0.703, log(BF_{10}) = -0.09. 1.. In sum, these results showed that the children 3- and 4-year-olds performed best on dots ↔ number words mappings, followed by the number words ↔ digits mappings, and performed worse on the digits ↔ dots mappings. This latter observation replicates the findings of both Hurst et al. (2017) and Jiménez-Lira et al. (2017), where children of the same age groups performed worse on the digits ↔ dots mappings. It is worth noting, however, that in the study by Hurst et al. (2017) number words ↔ digits mappings yielded higher accuracy scores (e.g., 0.69, for 3-year-olds and 0.90 for 4-year-olds), which were at the same level as the performance on the dots ↔ number words mappings (e.g., 0.67 for 3-year-olds, and 0.91 for 4-year-olds; Table 1 in Hurst et al., 2017; see also Table 1 in Jiménez-Lira et al. 2017). In the current study, the number words ↔ digits mappings yielded much lower accuracies (i.e., 0.30 and 0.58) which were also lower than the accuracies observed in the dots ↔ number words task (0.54 and 0.79, see Table 1). Finally, for the 5-year-olds, whose performance was not examined by either Hurst et al. (2017) or Jiménez-Lira et al. (2017), we found that the accuracy scores on the dots ↔ number words mappings were again better. However, the performance on number words ↔ digits and digits ↔ dots mappings was very similar.

Mediation analyses. Following Hurst et al. (2017), we examined which of the two developmental paths predicted children's skills to map between digits and dots best. Do children learn this digits ↔ dots mappings by relying only on their already acquired skill to map dots ↔ number words mappings, or do they learn the digits ↔ dots mapping by relying on their symbolic number knowledge, i.e., the number words ↔ digits mappings (see Hurst et al., 2017)? In the case of the first hypothesis, it is expected that the dots ↔ number words mapping would meditate the relation between the number words ↔ digits and the digits ↔ dots mappings. Alternatively, if children learn the digits ↔ dots mapping via their symbolic knowledge, it is expected that the number words ↔ digits mapping would be a significant mediator. To address this question, we performed a bootstrapped (10,000 samples) mediation analysis in SPSS using PROCESS v.3.1 (Hayes, 2018) with number words ↔ digits task as a predictor, digits ↔ dots pair as an outcome variable, and dots

↔ number words as a mediator (controlling for age as a continuous variable)[3]. The significance of the mediation effects is based on the confidence interval (CI): If the CI includes zero, there is no mediation effect

Figure 2A shows that there was a very small, but positive indirect effect *ab* = .05, 95% CI [.02, .09], indicating that the dots ↔ number words mapping task mediated the relation between number words ↔ digits and digits ↔ dots mapping. However, this mediation was only partial, given that the total effect c = .91, $p < .001$, 95% CI [.83, .99] was not completely accounted for by the mediator and the direct effect c' remained significant, c' = .86, $p < .001$, 95% CI [.78, .94].

We also tested the reversed model, switching the predictor and mediator, to check whether the children's understanding of how digits and dots are related, is mediated through their (symbolic) knowledge of number words ↔ digits mapping (controlling for age). Again, Figure 2B shows that there was only a partial mediation, because the total effect c = .56, $p< .001$, 95% CI [.38, .81] was not completely accounted for by the mediator and the direct effect c' = .20, $p = .003$, 95% CI [.09, .31] remained significant. However, in this case, the indirect (mediation) effect *ab* = .36, 95% CI [.20, .55], of number words ↔ digits mapping on the relation between dots ↔ number words and digit ↔ dots mapping was much larger than in the reversed mediation model (i.e. Figure 2A, were *ab* = .05).

Overall these mediation analyses showed that both dots ↔ number words and number words ↔ digits mappings were significant mediators (see also Jiménez-Lira et al. 2017). However, in line with the findings obtained by Hurst et al. (2017), the mediating effect for number words ↔ digits was much larger than the mediating effect for the dots ↔ number words. These results are in line with the symbolic account and indicate that the relation between digits and dots is primarily learned through the knowledge of number words ↔ digits mapping.

[3] Bayesian mediation analyses are available (Nuijten, Wetzels, Matzke, Dolan, & Wagenmakers, 2015), however they do not implement yet the possibility to include control variables. We therefore report only the classical mediation results.

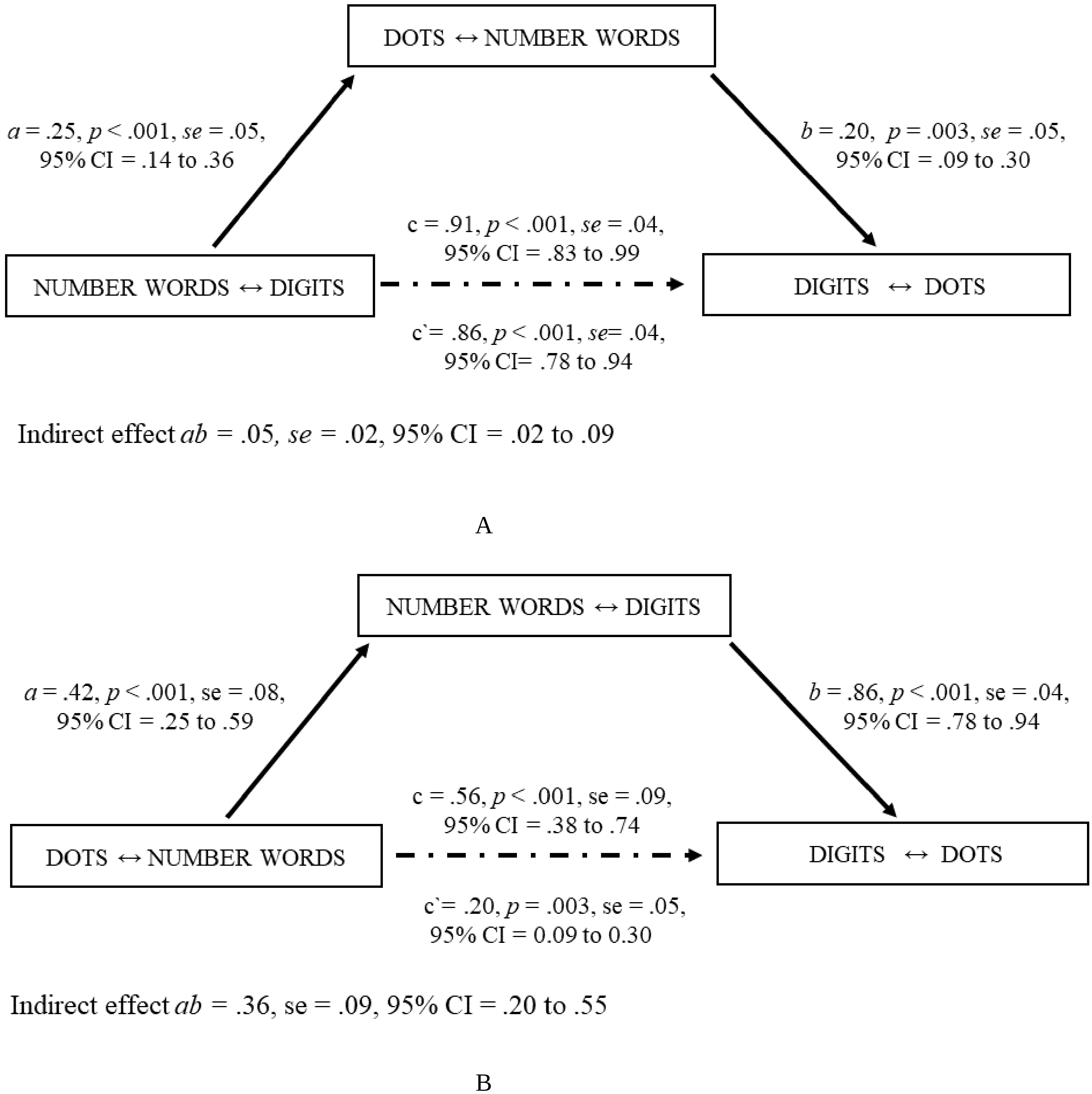


Figure 2.Mediation analysis testing the A) dots ↔ number words mappings and B) number words ↔ digits mappings as mediators

Home numeracy

Principal Component Analysis (PCA). Two out of the 13 items were excluded because ≥50% of the parents responded 'never' on these particular items: Item 2 "I encourage my child to do math in his/her head" (51%) and Item 10 "I encourage collecting" (63 %) were removed (see also Mutaf-Yildiz et al., 2018a,b; LeFevre et al., 2009). For the remaining 11 items, Cronbach's α = .80, 95% CI [.75, .84], indicated a high internal consistency (see Table 3). Then, a Principal Component Analysis (PCA) with orthogonal rotation (Varimax rotation; cases excluded listwise; Eigen values > 1) was performed on the overall scores to reduce further the number of items, which were highly related. The PCA provided a three-factor solution, accounting for 53.57% of the variance. However, two of the items cross-loaded on more than one factor (see Table 2). Because the difference between the cross-loadings was < 0.20, these items were excluded, and a second PCA was performed on the remaining nine items. The PCA provided a two-factor solution, accounting for 46.89% of the variance, Factor 1 was named *advanced* home numeracy, and Factor 2 was named *basic* home numeracy (Skwarchuk et al., 2014). Because one item cross-loaded on both factors, this item was further excluded from the factor scores. Then, we computed the factor scores for basic and advanced activities by averaging the raw item scores, loading on the same factors[4]. These scores were used to investigate further the relations between home numeracy activities and children's mapping skills.

It is worth noting, however, that with respect to the division of basic and advanced activities, some differences were observed when comparing the current study to the findings by Skwarchuk et al. (2014). Concretely, in the current study, the advanced home numeracy activities consisted of the items "I help my child learn simple sums (e.g., 2 + 2).", "We talk about time with clocks and calendars" , "I teach my child to recognize printed numbers", and "I encourage the use of fingers to indicate how many", while in the study by Skwarchuk et al. (2014) the advanced home numeracy factor consisted of the items "I help my child learn simple sums (e.g., 2 + 2).", "I encourage my child to do math in his/her head", "We talk about time with clocks and calendars", "I help my child weigh, measure, and compare quantities". With respect to the basic home numeracy activities, there was almost complete overlap. In the current study the basic home numeracy activities included the items "We sort and classify by colour, shape, and size", "We play board games or cards", "I help

[4] Raw factors scores were preferred here to keep the interpretation of the results as straightforward as possible. It is worth noting, however, that analyzing the data with weighted factor scores yielded similar results.

my child to recite numbers in order", "We sing counting songs", while in the study by Skwarchuk et al. (2014) the basic home numeracy activities included the items "We sort and classify by colour, shape, and size", "We play board games or cards", "I help my child to recite numbers in order", "I encourage collecting". Possibly the differences between the current study and the study by Skwarchuk et al. (2014) are due to the age of the participants. Concretely, the current study assessed children aged between 2;7 years and 5;2 years, while Skwarchuk et al. (2014) tested older children between 5;3 years and 6;5 years. Because of their young age, the children in the current study are likely to be engaged less frequently by their parents in home numeracy activities and especially in advanced home numeracy activities (e.g., Thompson et al., 2017). Consequently, this led to the omission of some items due to low frequency (e.g., "I encourage my child to do math in his/her head"), and to the cross-loading of other items (e.g., "I help my child weigh, measure, and compare quantities"), which would have otherwise be included in PCAs. Despite these subtle differences in their PCA results, concerning the division into basic and advanced activities (and the content of these activities; see also Mutaf-Yildiz et al., 2020) the findings of the current study and the study by Skwarchuk et al. (2014) are generally in line.*Repeated-measures ANOVA.* First, we wanted to assess whether the type of home numeracy activity differed across the age groups. Therefore, we conducted a repeated-measures ANOVA with type of activity (basic vs advanced) as a within-subject factor, and age group (3 levels: 3-year-olds, 4 -year-olds, and 5year-olds) as a between-subject factor. Results showed no main effect of type of activity, $F(1,177) = 2.60$, $p = 0.11$, $\eta_p^2 = 0.01$, and no main effect of age group, $F(2,177) = 1.33$, $p = 0.27$, $\eta_p^2 = 0.02$. Bayesian ANOVA, however, showed evidence for the presence of a main effect of type of activity, $BF_{Incl} = 126.73$, and a main effect of age, $BF_{Incl} = 96.70$. The interaction between type of activity and age group was significant, $F(2,177) = 12.27$, $p < 0.001$, $\eta_p^2 = 0.12$, $BF_{Incl} = 475.07$. Post hoc comparison (Bonferroni corrected; Bayesian paired samples *t*-test showed that the 3-year-old children engaged more frequently in basic than in advanced activities, $p_{bonf} < 0.001$, $BF_{10} = 27.65$. There were no significant differences between basic and advanced activities for the 4 -year-olds, $p_{bonf} = 1.00$, $BF_{10} = 0.57$. For the 5 -year-olds, the pattern was not significant, $p_{bonf} = 0.09$, but was moderately supported by the Bayesian analyses, $BF_{10} = 9.22$, suggesting that the parents of these children engaged them in advanced activities more frequently than in basic activities. Overall, these results are broadly in line with the developmental literature and mainly show that when children get older, they are more frequently engaged in advanced home numeracy activities on the initiative of their parents(e.g., Thompson et al., 2017).

Partial correlations. To confirm the previously observed relation between home numeracy and children's mapping skills (e.g., Mutaf-Yildiz et al., 2018a,b), partial correlations over the whole sample (controlling for age and SES) were computed between the performance on the three types of mapping tasks on the one hand and basic and advanced home numeracy scores on the other (see Table 3). Results showed that basic home numeracy activities, as reported by the parents, were positively correlated to all mapping tasks. Advanced home numeracy activities were positively correlated with number words ↔ digits and dots ↔ digits mapping tasks, but not with the dots ↔ number words mapping task.

Second, to examine whether basic and advanced home numeracy were differentially related to the performance in the mapping tasks in each age group, partial correlations (controlling for SES) were computed for each age group separately (see Table 3). For the *3-year-olds*, neither basic home numeracy activities, nor the advanced ones, were related to all mapping tasks. Although these correlations did not reach significance (possibly due to the relatively small sample size), it is worth noting that the size of the correlations with basic home numeracy activities tended to be higher than the size of the correlations with advanced home numeracy activities. For the *4-year-olds*, advanced home numeracy activities were significantly related to digits ↔ dots, and number words ↔ digits mappings, but both did not correlate with the dots ↔ number words. For the *5-year-olds*, both basic and advanced home numeracy activities were related only to the number words ↔ digits mappings, but both did not correlate with the dots ↔ number words and digits ↔ dots mapping. In sum, the results of the partial correlation analyses per age group showed an inconclusive developmental patter. That is, for the 3-year-olds, neither basic nor advanced activities were related to the mapping tasks. For the 4-year-olds, the advanced activities were related only to two of the mapping tasks (i.e., number words ↔ digits and digits ↔ dots mapping). Finally, in the 5-year-olds, both basic and advanced activities were again related only to two of the mapping tasks (i.e., number words ↔ digits and digits ↔ dots mapping). We discuss these findings further in the discussion section below.

Table 2.Descriptive statistics and the rotation factors loadings for the home numeracy activities depicted per age group.

Items description	Mean (SD)				Min	Max	PCA 1 factor loadings			PCA 2 factor loadings	
	3-year-olds	4-year-olds	5-year-olds	Overall			1	2	3	1	2
1. I help my child learn simple sums (e.g., 2 + 2).	1.08 (1.28)	1.30 (1.16)	2.11 (1.18)	1.46 (1.26)	0	4	0.77			0.82	
2. I encourage my child to do math in his/her head.*	0.80 (1.18)	0.89 (1.13)	1.68 (1.27)	1.07 (1.23)	0	4					
3. We talk about time with clocks and calendars.	1.86 (1.46)	2.10 (1.31)	2.32 (1.26)	2.09 (1.34)	0	4	0.68			0.66	
4. I help my child weigh, measure, and compare quantities.	1.12 (1.18)	1.10 (1.09)	1.41 (1.17)	1.19 (1.14)	0	4		0.56		0.43	0.59
5. We play games, involving counting, adding, subtracting.	1.53 (1.14)	1.78 (1.18)	1.78 (1.09)	1.69 (1.14)	0	4	0.53	0.48			
6. I teach my child to recognize printed numbers.	1.85 (1.30)	2.10 (1.66)	2.60 (1.04)	2.17 (1.19)	0	4	0.65			0.75	
7. We sort and classify by colour, shape, and size.	2.14 (0.92)	1.78 (1.02)	1.83 (1.03)	1.89 (1.04)	0	4		0.71			0.80
8. I ask about quantities (e.g., How many spoons?).	2.56 (1.04)	2.66 (1.02)	2.86 (0.86)	2.66 (1.00)	0	4	0.42	0.54			
9. We play board games or cards.	1.96 (0.87)	2.05 (0.88)	1.96 (1.00)	1.96 (0.94)	0	4		0.72			0.63
10. I encourage collecting (e.g. cards, stamps, rocks).*	0.75 (1.11)	0.67 (0.99)	0.66 (0.94)	0.68 (1.00)	0	4					
11. I help my child to recite numbers in order.	2.62 (1.32)	2.56 (1.25)	2.37(1.11)	2.53 (1.20)	0	4			0.65		0.43
12. We sing counting songs.	1.92 (1.30)	1.57 (1.06)	1.60 (1.28)	1.68 (1.19)	0	4			0.58		0.52

13. I encourage the use of fingers to indicate how many.	1.92 (1.36)	1.73 (1.24)	2.00 (1.16)	1.86 (1.26)	0	4	0.63	0.41
Basic home numeracy activities	2.17 (0.71)	1.97 (0.70)	1.94 (0.75)	2.02 (0.71)	0	4		
Advanced home numeracy activities	1.70 (0.94)	1.82 (0.83)	2.26 (0.82)	1.90 (0.88)	0	4		

Note.Factor loadings < .4 are not depicted. For the PCA two: 1 = advanced formal numeracy, 2= basic formal numeracy. not included in the PCAs.*

Table 3.Two-tailed partial correlations between the mapping tasks, basic and advanced home numeracy activities, depicted across the whole sample (controlling for SES and age) and per age group (controlling for SES).

All children (*N*=180)	1	2	3	4	5
1. Digits ↔ Dots	—				
2. Dots ↔ Number words	.41***	—			
3. Number words ↔ Digits	.84***	.33***	—		
4. Basic home numeracy	.18*	.18*	.19**	—	
5. Advanced home numeracy	.26**	.08	.27***	.51***	—
3 -year-olds (*N*=52)	**1**	**2**	**3**	**4**	**5**
1. Digits ↔ Dots	—				
2. Dots ↔ Number words	.24	—			
3. Number words ↔ Digits	.58***	.27	—		
4. Basic home numeracy	.23	.23	.15	—	
5. Advanced home numeracy	-.005	-.001	.01	.25	—
4 -year-olds (*N*=81)	**1**	**2**	**3**	**4**	**5**
1. Digits ↔ Dots	—				
2. Dots ↔ Number words	.59***	—			
3. Number words ↔ Digits	.87***	.42***	—		
4. Basic home numeracy	.13	.19	.14	—	
5. Advanced home numeracy	.31*	.19	.32*	.60***	—
5 -year-olds (*N*=47)	**1**	**2**	**3**	**4**	**5**
1. Digits ↔ Dots	—				
2. Dots ↔ Number words	.61***	—			
3. Number words ↔ Digits	.92***	.57***	—		
4. Basic home numeracy	.28	.10	.31*	—	
5. Advanced home numeracy	.29	.05	.30*	.58***	—

*Note. *p < .05, ** p < .01, ***, p < .001*

General Discussion

How children learn to map between different number notations, that is, digits, number words, and dots, is debated. On the one hand, it has been suggested that children learn to map number words and digits directly onto dots, and only then learn to map between number words onto digits (i.e., the 'quantity account'; e.g., Benoit et al., 2013; Von Aster & Shalev, 2007). Alternatively, it has been argued that children acquire the mapping between dots and digits based on their knowledge about how number words are related to both dots and digits (i.e., the 'symbolic account'; e.g., Bialystock, 1992; Hurst et al. 2017; Jiménez-Lira et al. 2017). In their recent studies, Hurst et al. (2017) and Jiménez-Lira et al. (2017) explicitly dissociated between these two approaches and provided evidence for the symbolic account. However, these studies were possibly underpowered due to their small sample sizes (i.e., N= 48, and N=62, respectively). In addition, environmental factors, such as home-numeracy already demonstrated to be related to children's mapping skills, were not addressed in these studies. Therefore, the current study was conducted to replicate the findings of Hurst et al. (2017) and Jiménez-Lira et al. (2017), using a larger sample (N = 195) of kindergartners and additionally examining the role of these kindergartners' home numeracy experiences in their mapping skills.

Considering the acquisition of the numerical mappings, the results of our study replicated the findings of Hurst et al. (2017) and Jiménez-Lira et al. (2017) and thus provided additional evidence in favour of the symbolic account (Bialystock, 1992). First, in line with Hurst et al. (2017) and Jiménez-Lira et al. (2017), we observed that 3- and 4-year olds performed worse on digits ↔ dots mapping task. The best performance was observed on the dots ↔ number words mapping task, followed by the number of words ↔ digits mapping. For the 5-year-olds again the dots ↔ number words mapping yielded the highest performance, however, in this group, children performed at the same level on the digits ↔ dots and number words ↔ digits mappings. Overall these results suggest that indeed children first learn to map between dots and number words (e.g., Benoit et al., 2004; Benoit et al., 2013; see also Hurst et al., 2017), then between digits and number words, and only after children learn how number words relate to dots and to digits, they infer that dots and digits are also related.

Second, the results from our mediation analyses were also in line with Hurst et al. (2017). Here, we observed that the number words ↔ digits mapping performance strongly mediated the relation between dots ↔ number words and digits ↔ dots mappings. Interestingly, however, the mediation analyses also showed that dots ↔ number words mediated the relation between number words ↔ digits and dots ↔ digits mapping, but this mediation effect was much smaller. Overall, these results suggest that the acquisition of the digits ↔ dots mappings depends on both number words ↔ digits and dots ↔ number words mappings (see also Jiménez-Lira et al. 2017) but to a different degree: Children rely on their ability to link number words with digits more than on their ability to link number words with dots.

Although the current results are generally in line with previous studies, there are certain limitations to be acknowledged. First, the children in our study (both overall and per age group) performed worse on the number words ↔ digits mapping compared to the performance of the children in the studies by Hurst et al. (2017) and Jiménez-Lira et al. (2017) on the same task. Considering the close similarities between the designs of these studies, we believe that this performance discrepancy might stem from sample characteristics. Although this is speculative, it is possible, for example, that the children we tested possibly had insufficient knowledge of Arabic numerals at the time of testing, leading to an overall worse performance in the symbolic mapping task.

Second, the results from our mediation analyses could also be influenced by the fact that in the current study, small numbers from 1 to 4 were used, which fall within the subitizing range. A separate cognitive system could be underlying the processing of numbers within this range (i.e., the Object Tracking System (OTS); Carey, 2009; see the elaboration on this topic further in the text). Nevertheless, our observation largely replicated the mediation results reported by Jiménez-Lira et al. (2017), who used numbers above the subitizing range (from 1 to 9) and also observed that both number words ↔ digits and dots ↔ number words mappings contributed to the performance in the digits ↔ dots task. Therefore, it could be argued that irrespectively of the number range, when children are learning the relation between dots and digits, their knowledge of how two symbolic notations relate to each other becomes more important than the knowledge how a symbolic number relates to non-symbolic numerosity. However, studies investigating this question with numbers outside the subitizing range are still sparse and is something to be considered for future studies.

In relation to the above interpretation, previous studies demonstrated that the link between symbolic and non-symbolic numbers depends on the number range. To begin with, several numerical cognition theories (e.g., Carey, 2009; Carey & Barner, 2019; Piazza, 2010;) have argued that small numbers (e.g., up to 4-5) rely on a different pre-verbal system, i.e., the object tracking system (OTS) or parallel individuation system (PI), while large numbers are grounded in the ANS. Both systems substantially differ in how number is represented (Carey, 2009; Feigenson et al., 2004; Hyde, 2011) and in the way they (possibly) support the symbolic number processing (Reynvoet & Sasanguie, 2016; VanMarle et al., 2018). For instance, Carey (2009) suggests that children learn the meaning of small numbers by associating them with small sets of items by relying on the OTS. According to Carey (2009), to acquire the meaning of larger numbers, children create associative relations between symbolic numbers themselves, without relying on pre-verbal representations anymore. This latter claim has been recently supported by a developmental study by Hutchison et al. (2019), who demonstrated that the strength of the relation between symbolic and non-symbolic numbers depends on the number range. Specifically, Hutchison et al. (2019) tested 540 5-year-old children on symbolic and non-symbolic number comparison tasks (ranging 1 to 9) and they observed that the correlation between symbolic and non-symbolic numbers was considerably stronger for numbers within the subitizing range (1-4) than for numbers outside the subitizing range (6-9). Furthermore, it was observed that within the subitizing range, the symbolic and non-symbolic tasks were mutually predictive for their growth. On the other hand, however, for the numbers outside the subitizing range, the growth in the non-symbolic comparison task performance was predicted by the performance in the symbolic comparison task only. According to the authors, these findings suggest that for small numbers (i.e., up to 4), a bidirectional relation is present (symbolic↔non-symbolic), while for numbers outside this range a unidirectional relation is present (symbolic→non-symbolic). In sum, the present study shows that the contribution of dots ↔ number words mapping to children's skill to map digits onto dots is also important, possibly because of the small numbers. Nevertheless, the present data demonstrate that number words ↔ digits mappings have a stronger contribution to the acquisition of the digits ↔ dots mapping than the dots ↔ number words mapping.

When it comes to home numeracy, our results provided evidence that younger children (i.e., 3-year-olds) were engaged by their parents more in basic than in advanced home numeracy activities. . In turn, there was also evidence that the parents of older children (i.e., 5-year-olds) engaged them more in advanced than in basic activities. These results are in line with previous studies (e.g., LeFevre et al., 2009; Skwarchuk et al., 2014; Thompson et al., 2017; Mutaf-Yildiz et al., 2020) and suggest that basic home numeracy activities, which typically involve practices about learning to recognize the numerical formats, are more appropriate for younger children and are thus practiced with a higher frequency. In turn, advanced home numeracy activities, which focus more on manipulating these numerical formats, are thus more suitable for older children and are reported with a higher frequency than the basic activities.

Concerning the relation between home numeracy practices and children's mapping skills, over the whole sample, we showed that both basic (e.g., reciting numbers) and advanced (e.g., learning simple sums) home numeracy activities were positively related to children's mapping skills (e.g., Mutaf-Yildiz et al., 2018a,b; 2020; Skwarchuk et al., 2014). However, the pattern of these relations across the age groups was less conclusive. For the 3-year-olds, neither basic nor advanced activities were related to children's mapping abilities. For the 4-year-olds advanced activities were related to number words ↔ digits and digits ↔ dots, but not to the dots ↔ number words mapping task. On the one hand, these results show that there is a tendency for advanced home numeracy activities to share more variance with the performance on mapping tasks with increasing the children's age. This tendency is also demonstrated in our ANOVA results (discussed above), and is also broadly in line with the literature on home numeracy (e.g., Thompson et al., 2017; Mutaf-Yildiz et al., 2018a; Skwarchuk et al., 2014; Susperreguy, Burr et al., 2020; Susperreguy, Douglas et al., 2020).. In addition, these findings suggest that depending on the children's age, parents engage them in basic and advanced activities with various frequencies (low or high). On the one hand, this indicates that the activities should be age-appropriate and focus on what children acquire at that time. On the other hand, it is worth noting that the frequency of these activities might also be influenced by the developmental needs of the child, their personal interests etc. Overall, these results suggest that home numeracy contributes to the development of numerical mapping skills

Nevertheless, we urge the reader to interpret these correlational results with caution. First, because the sample sizes are relatively small for correlational analyses, and second, because the pattern of results for the 5-year-olds was unexpected with respect to previous findings (e.g., Thompson et al., 2017). Following a visual inspection of the scatter plots in the 5-year-olds group, we observed that most children performed at the ceiling (i.e., depending on the mapping task between 70% and 90% of the 5-year-olds scored 100% correctly; see also Table 1). However, there was still a small number of children who performed less accurately on the mapping tasks and were also less frequently involved by their parents in both basic and advanced activities. Therefore, we believe that the observed correlation pattern is driven by this small group of children. Consequently, more demanding numerical tasks are needed to reliably investigate the contribution of basic and advanced home numeracy activities

In conclusion, the current study aimed to investigate how children learn to map between digits, number words and dots, by replicating the studies by Hurst et al. (2017) and Jiménez-Lira et al. (2017) using a larger sample and taking into account the role of home numeracy. Overall, our findings replicated these previous studies by demonstrating that children performed worse on mapping tasks between digits and dots, and performed better on mapping tasks between number words and dots, and number words and digits. The current results also demonstrated that, although a close link is present between digits, number words and dots, children's ability to link number words to digits, plays a crucial role when children learn to map between digits and dots. Together these findings provide evidence for the "symbolic account" and suggest that children do not learn the mapping between numerical formats by first associating digits and number words to dots. Rather, children use their knowledge of how number words relate to both dots and digits to learn the mapping between dots and digits. Finally, with regards to home numeracy, our results showed that overall, both basic and advanced home numeracy activities were positively related to children's mapping skills. Moreover, we observed the tendency that across the age groups, a shift from basic to advanced activities was present. These results underscore the importance of tailoring the home numeracy activities according to children's age.

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