Highlights

Online crop height and density estimation in grain fields using LiDAR

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- LiDAR allows to estimate crop height and density in small grain crop.
- Crop height cannot be estimated properly at angles to the horizontal below 70°
- Variance based crop density estimation is robust against small viewing angle variations.
- LiDAR data correlate better to ears dry mass than to number of tillers.

Online crop height and density estimation in grain fields using LiDAR

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Abstract

Crop height and density estimation ahead of the combine harvester were investigated over the last 30 years, but remain a challenge. LiDAR technology is increasingly being investigated for phenotyping and monitoring of cereals. However, so far, little has been published about the influence of mounting position and robust online measurement of height and density from a single LiDAR scan. Therefore, the influence of the angle and height of the LiDAR mounting on crop height and density estimation in wheat and barley has been investigated in this study. Tests were conducted in different crop heights, densities, moisture levels and varieties. The crop height could be estimated with a root mean squared error of 0.082 m at an angle of 90° between the LiDAR scanning plane and the horizontal. Two methods were compared for crop density estimation. The inter-percentile and the transversal variance

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method both successfully predicted the crop density, but the variance based method performed better with a coefficient of determination (R^2) of 0.77 and a root mean squared error of 82 g of dry ears per m^2 . When considering only one variety, the performance improved to reach an R^2) of 0.8 and a root mean squared error of 44 g of dry ears per m^2 . Variation in mounting angle of the sensor had less effect on the prediction accuracy than the mounting height.

Keywords:

LiDAR; Wheat; Barley; Crop height; Crop density; Mounting position

1 Nomenclature

2	α	Angle between the top of the crop profile and the horizontal	°
3	a	Slope of the linear regression	
4	b	Intercept of the linear regression	
5	n_{wind}	Number of moving windows	
6	n	Number of samples	
7	p_{gnd}	Percentile value for the ground position estimation	%
8	p_{max}	Percentile value for the upper inter-percentile distance	%
9	p_{min}	Percentile value for the lower inter-percentile distance	%
10	p_{toc}	Percentile value for the top of the crop position estimation	%
11	p	Percentile value to estimate the distance travelled into the crop	%

12	Std	Standard deviation	
13	Var_x	Average value of the variance	m
14	Var	Variance	m
15	x	Position in the transversal direction	m
16	y	Position in the range direction	m
17	θ	Angle between the ground surface and the scanning plane	°
18	φ_{lim}	Half field of view restriction for the LiDAR	°
19	CD	Estimated crop density	g m^{-2}
20	CD	Estimated crop height	m
21	$LiDAR$	Light Detection And Ranging	
22	$Perc_{p_{\text{gnd}}}$	Position of the p_{gnd} percentile	m
23	$Perc_{p_{\text{max}}}$	Position of the p_{max} percentile	m
24	$Perc_{p_{\text{min}}}$	Position of the p_{min} percentile	m
25	$Perc_{p_{\text{toc}}}$	Position of the p_{toc} percentile	m
26	$Perc_p$	Position of the p percentile	m
27	R^2	Coefficient of determination	
28	$RMSE$	Root Mean Square Error	
29	x_{horz}	X coordinates of the detections corrected for terrain slope	m

30	y_{adj}	Distance travelled through the crop	m
31	y_{horz}	Y coordinates of the detections corrected for terrain slope	m
32	y_{offset}	Y coordinates with estimated top of the crop as reference	m

33 1. Introduction

34 As the combine is harvesting a field, the crop density can vary locally,
35 requiring the driver to constantly adjust the driving speed of the machine to
36 cope with these variable conditions and avoid blockage of the machine. In
37 order to support the driver, automatic feed-rate control systems have been
38 implemented (Coen, 2009; K. Taylor et al., 2005). These systems rely on a
39 sensor measuring the crop intake of the machine. As the crop is cut and trans-
40 ported to the threshing and cleaning section of the machine, it passes through
41 different conveyors like the auger of the header and the feeder. Measuring
42 the drive torque of these conveyors provides information on the throughput
43 of biomass of the machine (Missotten, 1998). As the crop is already in the
44 machine when the biomass is measured, there is no time for anticipation and
45 little for reaction to an incoming sudden crop biomass change. Therefore,
46 these systems require to incorporate some safety margin which results in
47 an under utilization of the machine capacity. To reduce the under-capacity
48 margin and operate as close as possible to full load, several researchers have
49 tried to move the biomass estimation as much to the front of the machine
50 as possible. Klassen et al. (1994) proposed to measure the capacitance dis-
51 turbance between parallel plates on the table of the header. This method
52 showed good performance over a wide range of crop moisture and density.

53 Ehlert and Schmidt (1996) developed a pendulum-based method where a
54 horizontal metal bar makes contact with the crop and the moment that the
55 crop exerts on the metal bar is measured. The higher the crop density, the
56 higher the moment. Correlation with the crop density was good (R^2 of 0.89),
57 but the driving speed and crop condition had a lot of influence on the results.
58 A speed difference of 0.5 kph influenced the measurement by up to 2%. To
59 reduce this dependency, De Clippel (2000) replaced the metal bar by a plate.
60 The magnitude of the normal force on the plate as well as friction forces were
61 measured. Correction for terrain slope and driving speed was applied. This
62 method showed good correlation with a coefficient of determination (R^2) of
63 0.77.

64 As the contact sensors need to be attached to the machine, the gain in an-
65 ticipation is limited. Forward-looking non-contact sensors could predict much
66 more in advance and do not mechanically interact with the crop, therefore
67 reducing the potential grain loss in mature crop. Homburg (1980) proposed
68 to measure the attenuation of a sound signal by the crop. He placed an
69 ultrasonic emitter and receiver on each side of the header facing each other.
70 One sensor transmitted an acoustic wave which travelled through the crop
71 to the other side where it was recorded by the other sensor. While travelling
72 through the crop, absorption of the wave would happen proportionally to the
73 crop density and travelling distance. As the noise of the knife extends far
74 above the ultrasound region, an additional microphone was added to remove
75 the ambient noise. Unfortunately, measured absorption of the ultrasonic
76 waves did not correlate enough with the crop density. Maertens et al. (2003)
77 reported that reflection of ultrasonic waves was not good enough, but trans-

mission over 3 rows of crop correlated exponentially with the crop density. Despite the cheap price of the sensor, the application in transmission mode remains impractical and as the mechanical waves are highly dispersive, they are very sensitive to wind, dust and crop arrangement.

Electromagnetic waves do not suffer from these disadvantages. Therefore, Taylor et al. (1986) placed a laser transmitter and receiver 0.80 m apart in front of the header. They showed that the frequency at which the laser beam is interrupted correlates non-linearly with the crop density. However, compensation for the driving speed had to be taken into account. Missotten (1998) placed an infrared sensor at the reel position to sense the ears. The relationship was nonlinear, because in the case of small grains, several plants can pass at the same time. Beck (2003) patented a laser scanning device mounted on a combine harvester capable of estimating the amount of biomass to be harvested in front of the header. By measuring the distance to the crop they were able to calculate the crop volume. By measuring the intensity, they deduced the crop density and could differentiate crop from soil. Ehlert et al. (2009) proposed a method based on triangulation laser and time of flight laser. They assumed that the amount of detection decreases with the range when the density increases. The average distance to the detected objects in the vertical direction was highly correlated with the crop density. They obtained a coefficient of determination (R^2) of 0.96 with the triangulation laser and 0.99 with the time of flight laser. In the meantime, Saeys et al. (2009) proposed a method based on statistical analysis of the LiDAR detection distribution over the range. Coefficients of determination (R^2) up to 0.96 were found for the crop density estimation. The high frequency laser

103 scanner coupled with the ratio of the peak for top of the crop and ground
104 in the vertical distribution of the detections showed the best performance.
105 A three-dimensional spline fitting method was also proposed, but was not
106 compatible with real time measurements. The proposed methods showed
107 robustness towards different driving speed and vibrations from the machine
108 when the 3 dimensional field is not reconstructed.

109 More recent studies have focused on using LiDAR for phenotyping pur-
110 poses. Deery et al. (2014) proposed to stack LiDAR measurements after each
111 other in order to reconstruct a raster image. An algorithm would then count
112 the ears. Unfortunately, the idea was not demonstrated, and the performance
113 was not quantified. Jimenez-Berni et al. (2018) proposed two methods to es-
114 timate the crop density. In the first method, the measurements were stacked
115 behind each other and the 3d space was divided into voxels of fixed dimen-
116 sion. The number of points in each voxel was then counted and assigned to
117 the voxel. Voxels with less than 10 points were eliminated. The ratio be-
118 tween the number of voxels containing points and the number of subdivisions
119 in the horizontal plane (width x length) was then calculated. Depending on
120 the voxel size, the coefficient of determination ranged from 0 to 0.64. In
121 the second method, the vertical plane was divided into successive horizontal
122 layers of 0.01 m. For each layer, the number of points in that layer was di-
123 vided by the total number of points. An extinction coefficient was applied to
124 each layer depending on their depth from the top surface and the number of
125 points above the layer. The sum of the corrected fraction of points for each
126 layer was then calculated. The highest coefficient of determination obtained
127 with this method was 0.76. The crop height was estimated by subtracting

128 the crop surface model from the digital terrain model, which was obtained
 129 by measuring the bare soil distance to the LiDAR. The percentile method
 130 similar to the one proposed by Hämmerle and Höfle (2014) and Friedli et al.
 131 (2016), was used for the profile estimation. The best quantile for crop height
 132 estimation was found to be 0.955 with an RMSE of 0.017 m and an R^2 of 0.99.
 133 Similarly, Pérez-Ruiz et al. (2020) calculated the distance from the ground
 134 peak in the detections distribution over the height and the 95th percentile to
 135 estimate the crop density on a reconstructed field for phenotyping purposes.
 136 They obtained an RMSE of 0.026 m for crop height estimation. Yuan et al.
 137 (2018) used the driveline as a reference for ground distance and corrected for
 138 the angle between the ground and the horizontal, resulting in an RMSE of
 139 0.029 m. The ultrasound sensor showed poorer accuracy compared to LiDAR
 140 for the crop height estimation with an RMSE of 0.32 m.

141 Montes et al. (2007) proposed to use a spectrometer on a combine har-
 142 vester or a tractor to estimate the phenotype of the crop in plant breeding
 143 experiments. They claimed that crop properties such as the dry matter
 144 content, the water content, the biomass yield and the protein content could
 145 theoretically be estimated. The use of stereo-cameras has also been described
 146 in the literature. Liu et al. (2019) implemented a controller capable of iden-
 147 tifying the presence of wheat or beans crop. The header was then lowered
 148 to a harvesting position when crop was detected. Münch (2019) published
 149 a proactive throughput controller based on a 3D stereo-camera. The stereo-
 150 camera provides a point cloud from which the crop volume can be estimated.
 151 A machine learning algorithm then classifies the state of the crop: stubble,
 152 standing, lodged together with lodging direction. In extremely dusty condi-

153 tions, the system falls back on a vegetation data model and feedback control
154 based on classical machine sensors. Münch (2019) concluded that the predic-
155 tive sensing improved the driving comfort and overall control behaviour, but
156 failed to consistently deliver reliable data. As most of the future autonomous
157 vehicles are likely to be equipped with cameras, this could reduce the number
158 of sensors needed on the machine.

159 The ultrasonic sensors have already been studied for a long time (Hom-
160 burg, 1980; Maertens et al., 2003; Yuan et al., 2018) and judged unfit for the
161 task, thus not commercialized despite being available and affordable. One
162 can see a shift from ultrasonic and laser sensors towards 2 and 3-dimensional
163 LiDAR sensors, which has been suggested to provide far higher accuracy
164 (Saeys et al., 2009) and has recently become within reach for commercial-
165 ization on agricultural machinery. So far, those sensors have mostly been
166 used for phenotyping and the proposed processing techniques are not really
167 suitable for real time estimation as they rely on 3 dimensional field recon-
168 struction which is not precise at high driving speed as shown by Saeys et al.
169 (2009). Therefore, the aim of this study was to investigate the potential
170 of using a single LiDAR revolution for real-time estimation of crop height
171 and density. It builds upon the work by Saeys et al. (2009) and proposes
172 two methods, one exploiting the distribution of the detection in the vertical
173 direction to estimate the crop height and density and one exploiting the spa-
174 tial distribution of the detection in the transversal direction for crop density
175 estimation. Different to the work reported by Saeys et al. (2009), tests were
176 done on real crop from different varieties and with different crop heights to
177 evaluate the robustness of the proposed methods. The influence of the dis-

178 tance and angle with regards to the crop is investigated. The robustness
179 of the favoured method towards changes in the LiDAR position (height and
180 angle) is quantified. It is also investigated whether the LiDAR detections are
181 more informative on the number of tillers or on the ears dry mass. Multiple
182 varieties and crop height were considered for the dataset to ensure robustness
183 in multiple field conditions.

184 **2. Materials and Methods**

185 *2.1. Data Acquisition*

186 An LMS 111 LiDAR sensor (Sick, Waldkirch, Germany) LiDAR sensor
187 was used. It was configured with an angular step of 0.5° and a frequency of
188 50 Hz. The spot size at 2 m is 9.7 mm and at 4 m it is about 19 mm. The
189 LiDAR emits a pulsed laser beam over a range of wavelengths between 895
190 nm and 915 nm. The sensor was mounted on a pole looking down to the
191 crop. The height of the sensor could be adjusted to 3 different heights (1.75
192 m, 2.83 m, 3.89 m) with regards to the ground. The angle of the LiDAR was
193 changed in steps of 10° from 90° (looking down on the crop) until 30° with
194 regards to the horizontal (Figure 1). Data were recorded for 10 seconds for
195 each angle and height.

196 Tests were done in different fields near Leuven, Belgium. The first field
197 was seeded with Barley, Monique (W. von Borries-Eckendorf GmbH & Co.
198 KG, Leopoldshöhe, Germany) at three different densities (100, 150, 220
199 kg/ha). A total of 155kg/ha of nitrogen was applied in 2 passes. A second
200 field was seeded with 2 different varieties of wheat: Sacramento (R.A.G.T
201 Seeds Ltd, Saffron Walden, United Kingdom) was seeded at 100 and 220

Figure 1: Configuration for the LiDAR mounting positions. The different angle and height adjustments are shown.

202 kg/ha and Bergamo (R.A.G.T Seeds Ltd, Saffron Walden, United Kingdom)
 203 was seeded at 150 kg/ha. Both varieties received a total of 170 kg of ni-
 204 trogen per hectare applied in 3 passes. Finally, the third field was seeded
 205 with wheat (Bergamo) at 220 kg/ha. Measurements started in the beginning
 206 of July 2019 when the crops were maturing. During each measurement, a
 207 sample was collected around the measurement area. A cardboard box, from
 208 which the bottom was cut open, was placed in the crop. All the crop stalks
 209 in the box area were harvested as close to the ground as possible. The di-
 210 mensions of the box were 0.45 m x 0.68 m, resulting in a sampled surface of
 211 0.306 m² (Figure 2).

212 The ears were separated from the straws and both fractions were weighed
 213 separately in the field right after harvest. The scale, a Ranger 3000 from
 214 Ohaus (Nänikon, Switzerland), had a readability of 0.5 g and a measurement

Figure 2: Crop sample collection method. The left image represents the placement of the box in the crop and the right one, the result after harvesting the sample.

range from 10 g to 1500 g. The crop height was also measured with a tape
measure after each acquisition at the location where the samples were col-
lected. The samples were dried in an oven for 24 hours at 103 degrees Celsius
(ASABE Standard S358.2, 2006). Both ears and straw samples were weighed
separately after drying and counted.

2.2. Data Processing

The focus of this article is to demonstrate the performance of crop height
and density estimation from a single LiDAR scan (single revolution of the
LiDAR laser) and for different angles (θ) and heights with regards to the
ground. The coordinate system is converted from polar to Cartesian. The
dimension transversal to the measurement plane is called x and the one in
the longitudinal direction is called y (Figure 3).

The image is first rotated to compensate for a small lateral slope of the
ground or rotation of the sensor. The rotation angle (α) is calculated based
on the slope of the regression for the top of the crop surface.

$$\alpha = \arctan(\text{slope}) \quad (1)$$

Figure 3: Schematic of the experiment. The dots and stars represent a scan at respectively 90° and 60°. The bottom and top planes represent respectively the ground and top of the crop plane measured in the field. α is the angle between the measured top of the crop and the horizontal. The angle φ_{lim} between the dashed lines corresponds to the restricted field of view the centre of the LiDAR scan. θ is the angle between the ground and the scanning plane.

230 The coordinates corrected for the angle (α) between the top of the crop
 231 and the horizontal are then calculated as:

$$\begin{bmatrix} x_{horz} \\ y_{horz} \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} \quad (2)$$

232 Crop height estimation

233 The method used to estimate the crop height involves calculation of the
 234 inter-percentile distance. The field of view of the LiDAR was restricted to
 235 ± 60 degrees to only consider the detections under the sensor. The crop
 236 height is then expressed as follows:

$$CH = (Perc_{p_{gnd}}(y_{horz}) - Perc_{p_{toc}}(y_{horz})) \times \sin(\theta) \quad (3)$$

237 Where $Perc_{p_{gnd}}(y_{horz})$ corresponds to the p_{gnd}^{th} percentile calculation over

the data y_{horz} . The parameters p_{gnd} being the percentile for the ground and p_{toc} the percentile for the top of the crop. The values 0% and 1% were tested for p_{gnd} and the values 100%, 99.5%, 99%, 98%, 95%, 90%, 85% were tested for p_{toc} . The percentile function (*Perc*) takes for a q -th percentile the $q/100$ value between the minimum and the maximum of the sorted input array. If the normalized ranking does not match the location of q exactly, the value is interpolated from the two nearest neighbours. Percentile output for the 100% or 0% percentile correspond respectively to the minimal and the maximal value of y_{horz} . This method has already been proposed for reconstructed fields (Hämmerle and Höfle, 2014; Friedli et al., 2016), but its performance for single measurements and for different sensor heights and inclinations has not yet been reported.

Crop density estimation : inter-percentile method

In theory, as the yield increases, either the number of ears per square metre increases or the size of the ears increases. This results in less space between the ears, so laser hits will be closer to each other in the transversal and longitudinal direction. As crop density decreases the spacing or size of the ears decrease, resulting in larger spacing between the reflections in the transversal direction and detections deeper in the crop (more laser beams reaching the straw and the ground). The second theory has been investigated by other researchers, mostly by Saeys et al. (2009) with the median, mean, standard deviation of the range of the detection as well as the top of the crop to ground peak distribution ratio. These methods work under the assumption that the distance of the crop and ground to the LiDAR is constant or compensated accurately by knowing the distance to the top of the

crop. If there are enough detected reflections from the ground, the ground to top of the crop detection ratio was working well. Unfortunately, in the first case, top of the crop estimation relies on the field reconstruction and a computationally intensive tree dimensional spline fitting. In the second case, Saeys et al. (2009) showed that only 1% of the detected reflections comes from the ground when there are 400 tillers per square metre. This makes it questionable that there will be enough detected reflections from the ground at higher density. The methods proposed by Jimenez-Berni et al. (2018) and detailed in the section 1, rely either on field reconstruction for the first method or accurate top of the crop estimation to calculate the extinction coefficient in the second method. The purpose here is therefore to focus on a single frame, real time capable methods which do not require accurate top of the crop estimation and which perform equally in multiple varieties, densities, crop height and moisture.

From a farming point of view, it is important to know the disparities in the grain yield from place to place to better compensate for theses disparities in the harvest of the next year. From a machine point of view, there are two main parameters; the volume of crop entering the machine and the difficulty to process it. As the volume is concerned, it depends on the harvested width, the crop height, the number of tillers per square metre and the ears mass per square metre. As the difficulty to process the crop is concerned, it depends mostly on the maturity of the crop and the moisture content. In this paper, we are only investigating the volume of crop entering the machine. Assuming harvesting a constant full header width, the varying parameters that remains are the crop height and crop density. The crop density is defined

288 as a combination of the number of tillers per square metre and the ears
 289 mass per square metre. When predicting the crop density, optimization is
 290 performed separately for both element of this combination.

291 The first method involves linearly correlating the inter-percentile distance
 292 of the detection in the range direction to the crop density. As the crop
 293 density decreases, the laser beam is expected to penetrate more into the crop
 294 leading to more detections which were reflected deeper in the crop, increasing
 295 the spread of the data in the range direction. In order to focus on the
 296 detections from the field of view in the centre of the LiDAR scan, detections
 297 outside the viewing limits ($-\varphi_{\text{lim}} \leq \varphi \leq \varphi_{\text{lim}}$) were removed. To emphasise
 298 the penetration into the crop, the distance travelled by the electromagnetic
 299 wave into the crop was calculated. To do so, the cloud of detections was
 300 first offset in the y direction by a value representative for the top of the crop
 301 position and using the following equation:

$$y_{\text{offset}} = y_{\text{horz}} - \text{Perc}_p(y_{\text{horz}}) \quad (4)$$

302 The percentile value p should be close to 100% in order to represent the
 303 top of the crop. The distance travelled through the crop was calculated to de-
 304 termine the actual Y position of the detections for the percentile calculation.
 305 The adjusted Y position is then calculated as:

$$y_{\text{adj}} = \frac{y_{\text{offset}}}{\sin(\arctan(y_{\text{horz}}/x_{\text{horz}}))} \quad (5)$$

306 The crop density is then predicted based on the inter-percentile distance
 307 using the following equation:

$$CD = a_1 \times (Perc_{p_{\max}}(y_{\text{adj}}) - Perc_{p_{\min}}(y_{\text{adj}})) + b_1 \quad (6)$$

308 The percentile p for the offset, $p_{\max}$ and $p_{\min}$ for the inter-percentile cal-
 309 culation and φ_{lim} were optimized with a pattern search maximizing the coef-
 310 ficient of determination per position (angle and height of the LiDAR). Both
 311 the number of tillers per square metre and the ears dry mass were predicted
 312 and the optimization was conducted independently for each estimator. This
 313 method has the disadvantage of being potentially sensitive to the crop height.
 314 Therefore, an alternative method was also tested which is expected to be
 315 more robust against this.

316 Crop density estimation : transversal variance method

317 This method builds on the hypothesis that when the crop density de-
 318 creases, the distance between the tillers increases and that the variance of
 319 the detections in the transversal direction would capture this effect. Similar
 320 to the previous method, a minimal and maximal viewing angle φ_{lim} is de-
 321 fined and detections outside the field of view are removed. Next, a window
 322 is moved laterally over the transversal direction and the transversal variance
 323 is calculated as follows for each of these windows:

$$Var(x) = \frac{1}{n} \times \sum_{i=1}^n (x_i - \bar{x})^2 \quad (7)$$

324 With n the number of samples, x_i the i^{th} sample and $\bar{x}$ the average value
 325 of the samples. The total averaged variance for a scan is then expressed as:

$$Var_x = \frac{1}{n_{\text{wind}}} \times \sum_{j=1}^{n_{\text{wind}}} Var(x_j) \quad (8)$$

326 With x_j the x position of the detection located in the j^{th} window and n_{wind}
 327 the number of windows. The relation between the average variance Var_x and
 328 the crop density is then estimated through linear regression:

$$CD = a_2 \times Var_x + b_2 \quad (9)$$

329 Again, the parameters φ_{lim} , the step size and window size were optimized
 330 with a pattern search maximizing the coefficient of determination (R^2) of the
 331 linear regression for each angle and height independently. Both the number
 332 of tillers per square metre and the ears dry mass were predicted and the
 333 optimization was conducted independently for each estimator.

334 3. Results and Discussion

335 3.1. Crop properties

336 Data was acquired in three varieties of cereal. Each variety has specific
 337 characteristics in terms of crop height, yield and number of tillers per square
 338 metre. In Table 1, an overview of the crop parameters is presented. We can
 339 see that the barley variety has a high crop height and low grain yield and has
 340 the lowest single ear mass. Sacramento has a small to medium crop height
 341 with a grain yield that varies depending on the test location. The number
 342 of tillers does not fluctuate much across the test location. Therefore, the
 343 density variation is mostly coming from the large variation in the single ear
 344 weight. The Bergamo variety had a high crop height coupled with a high
 345 yield, while Sacramento had a lower crop height and a lower yield. However,
 346 the correlation between crop height and ears dry mass was low with a Pearson
 347 coefficient of 0.27. This can be explained by the fact the barley had a high

crop height and low grain yield, which balances the dataset. In general, taller varieties tend to have lower yields. However, within a field and single variety, the crop height is partly driven by the growing conditions (fertilizer, sun light, water...) which also influences the yield. The low correlation between the crop height and density reduces the probability to predict the crop density from the crop height and vice et versa. The ears dry mass per square metre for Bergamo varies significantly depending on the sample batch. Therefore, additional samples were taken at the test location resulting in the collection of 9 samples. In addition, the number of tillers per square metre correlates to the ears dry mass per square metre with a Pearson coefficient of 0.77. The wet base moisture content of the crop varied from 45 to 8 percent for the grain and from 62 to 6 percent for the straw. As barley was already mature at the time of the experiment, the moisture does not fluctuate much. For the wheat varieties, measurements were conducted during the ripening period.

The test locations were selected for the flatness of the terrain. Therefore, the rotation angle (α) is rather small and ranges from -2.1 to 5.7 degrees for LiDAR inclinations (θ) larger than 70 degrees and from -2.1 and 11.1 for LiDAR inclinations (θ) smaller or equal than 70 degrees. Also, the crop height was relatively constant for each test location and the variation mostly comes from the maturing of the crop where the ears tend to bend downwards as the crop matures.

3.2. Crop height estimation

The crop height estimation results for a mounting angle of 90 degrees with regards to the ground are shown in Figure 4. We can see that the

Variety	Crop Height (m)		Ears Moisture (%,wet base)		Ears Dry Mass (g/m ²)		Straws Dry Mass (g/m ²)		Number of Tillers (#/m ²)		Single Ear Mass (g/ear)		Number of measurements (#)
	Min	Max	Min	Max	Mean	Std	Mean	Std	Mean	Std	Mean	Std	
Monique	0.80	0.80	12	14	880	41.18	365	21.25	401	20	2.20	0.21	2
Monique	0.75	0.75	12	15	634	12.26	260	13.08	266	9	2.38	0.04	2
Monique	0.70	0.70	12	16	676	56.41	300	7.36	398	45	1.73	0.34	2
Sacramento	0.64	0.67	12	45	720	127.40	298	53.24	360	53	2.00	0.10	5
Sacramento	0.68	0.69	10	36	994	32.67	424	39.99	361	17	2.75	0.08	5
Sacramento	0.68	0.69	8	26	895	30.07	392	10.16	397	20	2.26	0.08	5
Bergamo	0.68	0.70	8	41	923	34.66	457	76.85	433	87	2.22	0.43	5
Bergamo	0.70	0.78	11	45	1146	139.35	708	73.97	606	47	1.89	0.21	9

Table 1: Overview of the crop properties

373 1% percentile (dashed lines) for the further detection systematically leads
 374 to a higher root mean squared error in the estimation of the crop height.
 375 Taking the furthest detection ($p_{gnd} = 0\%$, solid line) appears to be the best
 376 configuration. Detections below the ground level are highly unlikely and
 377 could only be explained as double bounces from the laser beam in the crop
 378 or failure from the LiDAR. For the top of the crop, the range also leads
 379 to the best results. Data were acquired in the absence of dust. In the
 380 presence of dust there is a chance that targets are detected between the
 381 LiDAR and the crop. In this scenario, a higher percentile value may have
 382 to be chosen or a noise removal algorithm should be considered. A trade-
 383 off between robustness and precision may be necessary for combine harvester
 384 applications. The best prediction is for a height of 1.75 m, an angle of 90° and
 385 the 0%-100% percentile distance with an RMSE of 0.082 m. When removing
 386 the systematic error, the RMSE decrease to 0.058 m. When the 0-99.5%
 387 percentile range proposed by Jimenez-Berni et al. (2018) was used, the error
 388 increased to an RMSE of 0.09 m. This error is considerably higher than the
 389 0.017 m error they reported and the 0.05 m error that Yuan et al. (2018)
 390 reported. This can be explained by the lower number of detections available

Figure 4: Evolution of the crop height estimation error for different percentiles values. The dashed line corresponds to the value $p_{gnd} = 1\%$ and the solid line to the value $p_{gnd} = 0\%$. The marker type corresponds to the different mounting heights (dots correspond to 1.75 m, crosses correspond to 2.83 m and diamonds correspond to 3.89 m)

391 for a single scan compared to multiple scans in a reconstructed field.

392 To reduce the impact of this trade-off one might consider applying a cor-
393 recting factor to the estimated crop height. Figure 5 confirms that the range
394 is the best estimate for the crop height. In addition, we can see that a correc-
395 tion factor should be dependent on external parameters such as crop density
396 or variety. The distance from the 98% percentile to the 100% percentile and
397 from the 0% percentile and the 2% percentile is different for the high density
398 (stars) and the low density (dots) despite the fact that the distributions of
399 the detections over the height are very similar (solid line on the right of the
400 figure). To explore the influence of the measurement height and angle based
401 on the best performing method, the 0%-100% inter-percentile distance, the

Figure 5: Crop height estimation in two different densities for a mounting height of 1.75 m and a viewing angle of 90 degrees. The dots correspond to the lowest density (620 g m^{-2}) and a measured crop height of 0.70 m. The stars correspond to the highest density (1248 g m^{-2}) and a measured crop height of 0.78 m. The solid lines on the right of the figure correspond to the distribution of the detection in the range direction for the two densities.

RMSE for the different combinations of height and angle is plotted. Figure 6 shows a decrease in the crop height estimation error as the angle with the ground decreases. There is almost no influence of the height of the LiDAR on the estimation error.

Figure 7 shows the crop height estimation error together with the Pearson correlation coefficient between the estimated crop height and the measured crop density, and the percentage of detections reaching the last 0.1 m above the ground. At about 60° inclination, only half of the laser beams are reaching the ground. While the RMSE increases with decreasing viewing angles, the error for the different heights is rather constant (tip of error-bars). As the

Figure 6: Crop height estimation error (RMSE) for different mounting positions.

412 median of the detected reflections corresponding to the ground decreases, the
 413 correlation between the estimated crop height and the crop density jumps
 414 from a value close to the correlation in the input data, as mentioned in
 415 subsection 3.1, to a negative value. As fewer laser beams are reaching the
 416 ground the correlation gets stronger. The negative sign indicates that a
 417 smaller estimated crop height corresponds to a higher crop density. This
 418 makes sense when none of the detected signals have been reflected by the
 419 ground. It reflects the fact that when the density increases, the laser beams
 420 have more difficulty penetrating through the crop, thus reducing the inter-
 421 percentile distance. For angles equal to or below 40° this correlation shows
 422 limitations. This theory is further confirmed by the high Pearson coefficient
 423 (0.60 ± 0.03) between the percentage of detection reaching the last 0.1 m
 424 above the ground and the crop density (both number of tillers and ears dry
 425 mass per square metre) for an angle of 90° and both a height of 1.75 m

Figure 7: Influence of the incidence angle on the crop height estimation (dotted line), percentage of detected reflections corresponding to the ground level (dashed line) and Pearson correlation between estimated crop height and measured crop density (solid line). The arrows correspond to the range of the data. The lines connect the median values. The whiskers from the box plot (percentage of detection at ground level) correspond to the 5%-95% intervals

and 2.83 m. However, it does not significantly influence the accuracy of the crop height estimation at 90° as the Pearson correlation between the number of point reaching the ground and the error is 0.21 (± 0.02). The Pearson correlation between the crop density and the estimation error is 0.11 (± 0.16).

3.3. Crop density estimation: inter-percentile method

As indicated in the previous section and reported by other researchers (Ehlert et al., 2009; Saeys et al., 2009; Jimenez-Berni et al., 2018), there is a correlation between the range of the detection and the crop density, especially when the ground cannot be reached. Figure 8 shows that the dry mass of the

Figure 8: Inter-percentile based ears dry mass per square metre estimation error (RMSE) for different mounting positions

ears can be estimated based on the inter-percentile method, particularly for higher mounting positions. Indeed, the best prediction reached an RMSE of 97 g m⁻² and an R² of 0.68 for an angle of 40 degrees and a height of 3.89 m. In addition, when predicting the number of tillers per square metre, the R² was in average 35% lower than when predicting the ears dry mass per square metre. For this reason, results when predicting the number of tillers are not discussed.

The working principles of this method are illustrated in Figure 9, where the positions of the percentiles for the high and low density at the best performing position are plotted. The position of the percentile, p_{max} corresponds to the position of the main peak in the distribution. The position of the percentile, p_{min} captures the end of the tail of the distribution. We can conclude that the inter-percentile distance reflects the length of the tail of

Figure 9: Inter-percentile based crop density estimation for two different densities at 3.89 m and 40 degrees. The dots correspond to the lowest density (880 g m^{-2}) and the stars correspond to the highest density (988 g m^{-2}). The solid lines on the right of the figure correspond to the distribution of the detection in the range direction. The crop height for both densities is 0.69 m. The predicted density is 825 g m^{-2} for the lowest density and 1199 g m^{-2} for the highest density.

448 the distribution.

449 As the best results for crop height estimation were obtained at an angle of
 450 90° , this would mean that two different LiDARs would have to be mounted
 451 on the combine harvester under different viewing angles, which may not
 452 be desirable. If one mounting angle has to be selected, the best trade-off
 453 between performance for crop height and crop density estimation would be
 454 to mount it at 3.89 m height under an angle of 80° . This would result in an
 455 RMSE for crop height estimation of 0.16 m and an RMSE for crop density
 456 estimation with the inter-percentile method of 109 g m^{-2} , which are quite
 457 poor performances.

458 3.4. Crop density estimation: variance based method

459 In Figure 10 the working principle of the variance based method is il-
460 lustrated for a height of 2.83 m and an angle of 80 degrees and for two
461 different densities. The crop density has little influence on the transversal
462 distribution as the solid lines on the top of Figure 10 look very similar. The
463 variance (dashed line on the top part of the figure) is constantly higher for
464 the lower density than it is for the higher density regardless of the position
465 in the transversal direction. Averaging the variance over multiple locations
466 in the transversal direction improves the precision and the robustness of the
467 estimation.

468 As can be seen from Figure 11, this method performs better than the
469 inter-percentile method, as it performs well regardless of the position and
470 when optimizing for each angle and height independently. The best predic-
471 tion reached an RMSE of 82 g m⁻² and an R² of 0.77 for an angle of 80° and
472 a height of 3.89 m.

473 In addition, the performance remains good at an angle of 90°, where the
474 best crop height estimation was obtained. If we consider a height of 2.83 m
475 and an angle of 90° the RMSE is 94.37 g m⁻² and the R² is 0.71. This method
476 offers a good compromise between height and density estimation as the R²
477 for density estimation at 90° is above 0.7, regardless of the LiDAR mounting
478 height. Again, when predicting the number of tillers per square metre, the
479 R² was in average 22% lower and at best equal than when predicting the
480 ears dry mass per square metre. For this reason, results when predicting the
481 number of tillers are not discussed.

482 In Table 2, the effect on the crop density estimation performance of op-

Figure 10: Variance based density estimation for two different densities at 2.83 m and 80 degrees. The dots correspond to the lowest density (724 g m^{-2}) and the stars correspond to the highest density (1041 g m^{-2}). The crop height is 0.67 m for the lowest density and 0.68 m for the highest density. The solid and dashed lines on the top of the figure correspond, respectively, to the distribution of the detection in the transversal direction and the estimated variance. The predicted density is 785 g m^{-2} for the lowest density and 1052 g m^{-2} for the highest density.

Figure 11: Variance based crop density estimation error (RMSE) for different mounting positions

timizing the model parameters for multiple positions together is illustrated for the variance based method. The reference position was selected to be at the in-between position at 2.83 m and 80° angle in the area where the best performances were found (Figure 11). It should be noted that the angle has less influence on the performance than the height. Considering all the varieties, the R^2 does not vary significantly when considering the neighbouring angles of 70° and 80° at 2.83 m height with R^2 values of, respectively 0.63 and 0.67 compared to the original value of 0.65 when only considering the 80° angle. When considering the neighbouring height position, the R^2 significantly drops to 0.37 and 0.22 respectively for the heights of 1.75 m and 3.89 m. When considering more than 2 heights or angle positions at once, the R^2 is unacceptable with values of 0.25 or 0.07.

When considering a single variety, the Sacramento variety, the estima-

496 tion performance increases significantly. The R^2 increased from 0.65 to 0.8
 497 and the RMSE decreased from 101 g m⁻² to 44 g m⁻². To reach this perfor-
 498 mance, the pattern search optimized the model parameters at the following
 499 values: window step size = 2.6 m, window size = 4.16m and $\varphi_{lim} = \pm 65^\circ$.
 500 However, when considering multiple positions heights, the R^2 dropped more
 501 significantly, but the RMSE remained lower than when considering multiple
 502 varieties. This could be explained by the lower range of predicted densities.
 503 Indeed, the ears dry mass ranges from 620 g m⁻² to 1247 g m⁻² when con-
 504 sidering all the varieties and from 724 g m⁻² to 1041 g m⁻² when considering
 505 only the Sacramento variety. However, the conclusion remains the same: The
 506 algorithm is more sensitive to height variation than to angle variation and
 507 does not perform any more when considering more than 2 positions at once
 508 especially when including the height of 3.89 m. This may be attributed to
 509 the large steps in mounting height. In practice, one would select one mount-
 510 ing height, but the distance and angle to the crop may vary within a field.
 511 Therefore, it would be interesting to investigate the robustness of the method
 512 against smaller height variations.

513 In Figure 12, the reference ear dry mass values for Sacramento are plot-
 514 ted against the calculated variances for the best performing configuration
 515 at 2.83 m mounting height and 80° angle. While considerable variation can
 516 be observed between neighbouring runs, the variation is larger within a run.
 517 Averaging the predicted density over multiple scans or over time may reduce
 518 this variation and improve the estimation accuracy.

Variety	Position [Height (m), Angle (°)]	R ²	RMSE (g/m ²)
All	[2.83, 80]	0.65	101.26
All	[1.75, 80] and [2.83, 80]	0.37	136.04
All	[2.83, 80] and [3.89, 80]	0.22	150.25
All	[2.83, 70] and [2.83, 80]	0.63	103.16
All	[2.83, 80] and [2.83, 90]	0.67	98.66
All	[2.83, 70] and [1.75, 80] and [2.83, 80]	0.25	148.68
All	[1.75, 80] and [2.83, 80] and [3.89, 80] and [2.83, 70] and [2.83, 90]	0.07	165.79
Sacramento	[2.83, 80]	0.8	43.68
Sacramento	[1.75, 80] and [2.83, 80]	0.52	67.92
Sacramento	[2.83, 80] and [3.89, 80]	0.14	91.21
Sacramento	[2.83, 70] and [2.83, 80]	0.55	65.32
Sacramento	[2.83, 80] and [2.83, 90]	0.61	61.04
Sacramento	[2.83, 70] and [1.75, 80] and [2.83, 80]	0.44	73.24
Sacramento	[1.75, 80] and [2.83, 80] and [3.89, 80] and [2.83, 70] and [2.83, 90]	0.11	93

Table 2: Overview of the robustness analysis. Crop density was predicted using data from multiple LiDAR height and angle mounting positions. The regression coefficient and the parameters for the variance based algorithm were unique for each set of positions.

Figure 12: Correlation between calculated variance and reference crop density for Sacramento variety at an angle of 80° and a height of 2.83 m. Each symbol corresponds to a different acquisition time or location. The solid line is the regression line ($y = -4813 \cdot x + 3504$) and the dashed lines indicate the 95% confidence interval around this line. The prediction error (RMSE) is 43 g m^{-2} and the R^2 is 0.8.

519 4. Conclusions and Future perspectives

520 The best results for crop height estimation were obtained for a mounting
521 height of 1.75 m and a viewing angle of 90 degrees with respect to the hor-
522 izontal, resulting in an average error (RMSE) of 0.082 m. The crop height
523 could not be estimated with sufficient accuracy for angles sharper than 70
524 degrees. As the angle sharpens, fewer detections are reaching the ground
525 leading to larger errors. The range gave the most accurate height estima-
526 tion. However, for field operation, smaller percentile values for the top of the
527 crop could be more suitable to cope with false detection (e.g. dust). When
528 smaller percentiles are used, a calibration will be required to estimate the
529 crop height from the calculated inter-percentile distance. The two proposed
530 methods for crop density estimation were successful in predicting the crop
531 density. However, the best results were obtained with the variance based
532 method, resulting in an RMSE of 82 g m⁻² and an R² of 0.77 for an angle of
533 80 degrees and height of 3.89 metres. The positions at which the variance
534 based method performed best were also more compatible with accurate crop
535 height estimation. The ears dry mass per square metre could be predicted
536 accurately for different varieties, although a variety specific calibration gave
537 better results. To better understand the influence of the variety and par-
538 ticularly the weight of a single ear on the performance, acquiring data in
539 different varieties having the same yield range could provide more insight
540 in the absolute density estimation. As most fields are seeded with a single
541 variety and absolute measurement is not necessary for feed-rate control, this
542 method might provide sufficient performance for relative yield measurement
543 as a basis for feed-rate control. If absolute measurement is needed, online

544 calibration based on the yield sensor of the combine harvester could be an
545 option. In this study, the mounting height was found to have a larger influ-
546 ence on the density estimation than the angle. However, large steps in height
547 were considered, so an experiment with smaller steps is required to better
548 quantify the robustness of the method against relevant variations in the dis-
549 tance to the crop. In addition, using a multi-plane LiDAR may be interesting
550 as the angle and the distance to the crop could be more precisely estimated
551 from these measurements to correct for their impact on the relation between
552 the variance and the crop density. Calculation of the variance over multiple
553 LiDAR measurements plane may increase the crop height and density esti-
554 mation as well. Also, as the LiDAR is measuring over a very narrow region
555 in the longitudinal direction, it is expected that averaging over sequential
556 scans in the driving direction will further improve the estimation of crop
557 height and density. As the proposed methods take less than a millisecond to
558 estimate the crop density and height, they could easily be implemented for
559 online estimation on a combine harvester. Future research is recommended
560 to investigate the added value of exploiting the distribution of the detections
561 both in the longitudinal, transversal and vertical directions.

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