Distributed Online Optimization for Voltage Control in Low Voltage Distribution Networks

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Abstract—In order to reduce the communication burden in online optimization in low voltage distribution networks (LVDNs), a distributed online optimization approach is proposed. Hamilton–Jacobi–Bellman equation and local measurements are integrated to seek the approximation to optimum. Voltage control is implemented as the embodiment of the proposed approach, which is validated in a realistic 103 nodes LVDN.

Index Terms—online optimization, model-free, voltage control, active distribution system, distributed control

I. INTRODUCTION

The development and application of smart meters have led to the modernization of low voltage distribution networks (LVDN) in the context of smart grids [1]. By enabling bidirectional communication and fundamental computation capability, the implementation of active distribution system (ADS) has extended the control of energy consumption to the household level, offering new opportunities to improve the efficiency and reliability of the grid [2]. Shapeable/deferrable loads and distributed generators are expected to play a key role in such a revolution. With the rising trend of renewable penetration and the popularity of smart household appliances, distributed generators and shapeable loads spring up in LVDNs, which enlarges the serviceableness of ADS. For instance, plug-in electrical vehicles, flexible residential loads such as HVAC system or dishwashers can be actively controlled to adapt their consumption, in order to help LVDNs implement voltage regulation and demand response without relying on the backbone distribution network much. The main challenge is managing such a large group of users with ensuring (i) system constraints, (ii) user satisfaction, (iii) computational tractability [3].

Communication infrastructure and efficient algorithms, are prerequisite technologies to enable the aforementioned applications in LVDNs. With numerous users, communication is becoming one of the limiting parameters that define the computational tractability, especially for online optimization applications in LVDNs [4]. In order to tackle this, coordinated methods are proposed by some works [5], [6]. These approaches employ a predefined active power or voltage-dependent characteristics, to seek optimization based on LVDN sensitivity analysis offered by a central controller.

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Nevertheless, as the main algorithm is still centralized, it still can be so computational intensive that the central controller can barely handle it. Reference [7] confirmed such an issue and proposed a GPUs-based approach that can solve optimization problems in a parallel manner. Nevertheless, GPUs are expensive, and this approach does not effect a permanent cure to this issue.

Distributed optimization is supposed to be a promising solution. Although some recent works proposed instructive approaches about distributed online optimization [8], [9], there are very few papers about distributed online optimization in LVDNs. Reference [10] proposed an online distributed algorithm to identify and track a time-varying social-welfare maximization problem for distribution networks. Nevertheless, it still relies on a centralized network operator to collect and send incentive-based signals. In general, there are three challenges in the design of distributed online optimization algorithms in a LVDN. Firstly, there should be a mechanism to guarantee the consensus among users. This is not an issue in centralized control as the controller acts as a coordinator. Moreover, frequency is not an effective reference for demand response anymore in LVDNs due to the high R/X ratio [11]. Secondly, compared to transmission systems and backbone distribution networks, the users and the topology of LVDNs change more often, which brings difficulties to system modeling. Therefore, model-free is appealing as it offers the potential to make LVDNs plug-and-play. Thirdly, the distributed optimization algorithm has to be computationally solvable for individual users, as it is not practical to assume all the users in LVDNs have a powerful computation capability.

This paper proposes a distributed online optimization approach that addresses the three aforementioned challenges. Dynamic programming is employed in this paper to formulate the optimization problem. From the perspective of dynamic programming, the consumption changes of user i can be regarded as movements with costs. As the case illustrated in Fig. 1, when user i wants to change its consumption P_t^i (generation is regarded as negative consumption), there is always a cost J_t^i depending on its direction of change and nodal voltage. If the change is supposed to make the corresponding nodal voltage V_t^i move towards the bounds, J_t^i will be more expensive than making V_t^i move towards to the

Fig. 1. The different cost due to the different adjustment direction

rated voltage. Therefore, the optimization problem between (i) system constraints and (ii) user satisfaction is transferred to a problem about seeking an optimal trajectory for user i to implement its scheduled changes according to its profile. Consequently, Hamilton–Jacobi–Bellman (HJB) equation [12] is employed to solve the optimal trajectory problems.

The contribution of this paper is threefold. Firstly, mathematical details are given to indicate the mechanism of employing HJB equations to implement distributed online optimization for voltage control in LVDNs. Secondly, reactive power changes in LVDNs are regarded as uncertainties in the proposed approach, in order to reduce the computation efforts, which is elaborated in section III. Thirdly, local measurements are used to replace the iterations in conventional applications of HJB equations, which makes the algorithm personalized to every user and affordable in the computational aspect. The paper is organized as follows: the preliminaries and mathematical mechanisms are given in section II, specific control scheme is elaborated in section III, case study is illustrated in section IV and section V concludes the paper.

II. PRELIMINARIES AND MATHEMATICAL MECHANISM

Consider an interacting system with n users. The set of users is denoted by $N = \{1, 2, \dots, n\}$, and the time is denoted by $t \in [t_0, t_0 + T]$, while $T > 0$ is the length of a typical control period. The operation state of user $i \in N$ at t is x_t^i , assuming the admissible control of i at t is u_t^i , then the evolution function of user i can be described as a stochastic differential equation (SDE)

$$dx_t^i = [\alpha_t x_t^i + \beta_t u_t^i]dt + \beta_t W x_t^i dw_t^i, \quad (1)$$

where $\alpha_t, \beta_t \in \mathbb{R}$ are decay/growth parameters, and w_t^i is a Wiener process with a multiplier W that depends on state x_t^i .

As aforementioned, for user i , there is a personal cost function J_t^i related to its own state x_t^i and the corresponding control u_t^i . The cost of u_t^i depends on V_t^i , and the corresponding mapping is $r_i(x)$. Then J_t^i is given by

$$J_t^i(x_t^i, u_t^i) = l(x_t^i) + r_i(V_t^i)u_t^{i2}, \quad (2)$$

where $l(x_t^i)$ is a cost based on the distance between x_t^i and its desired value $x_t^{i'}$. In order to make (2) convex, $l(x_t^i)$ is given by

$$l(x_t^i) = d(x_t^{i'} - x_t^i)^2, \quad (3)$$

where d is a weight parameter. Therefore, the optimal trajectory of $x_t^i, t \in [t_0, t_0 + T]$ must satisfy

$$v^i(x_t^i) = \inf_u \int_0^T l(x_t^i) + r_i(V_t^i)u_t^{i2} dt. \quad (4)$$

Consequently, in order to figure out the optimal trajectory, the optimal admissible control u_t^{i*} that always respects the state constraints and minimizes $v^i(x_t^i)$ needs to be found. Assume $\beta_t \neq 0$ in (1), then the Hamiltonian of x_t^i is given by

$$H_t(x_t^i) = \inf_u \{J_t^i(x_t^i, u_t^i) + \partial_x v^i(x_t^i)(\alpha_t x_t^i + \beta_t u_t^i)\}, \quad (5)$$

Then according to the Bellman's Principle of Optimality, (4) satisfies the corresponding HJB-Flemming equation as

$$\partial_t v^i(x_t^i) + H_t(x_t^i) + \frac{\beta_t^2 W^2 x^2}{2} \partial_{xx}^2 v^i(x_t^i) = 0 \quad (6)$$

As J_t^i is strictly convex, it can be derived from (5) that

$$\partial_u J_t^i(x_t^i, u_t^{i*}) + \partial_x v^i(x_t^i)\beta_t = 0. \quad (7)$$

Plug (2) into (7), then

$$2r_i(V_t^i)u_t^{i*} + \partial_x v^i(x_t^i)\beta_t = 0. \quad (8)$$

Therefore, u_t^{i*} can be written as

$$u_t^{i*} = -\frac{\partial_x v^i(x_t^i)\beta_t}{2r_i(V_t^i)} \quad (9)$$

So far, the equation set that user i needs to solve is

$$\left. \begin{aligned} \partial_t v^i(x_t^i) + l(x_t^i) + r_i(V_t^i)u_t^{i*2} + \partial_x v^i(x_t^i)(\alpha_t x_t^i + \beta_t u_t^{i*}) \\ + \frac{\beta_t^2 W^2 x^2}{2} \partial_{xx}^2 v^i(x_t^i) = 0 \\ u_t^{i*} = -\frac{\partial_x v^i(x_t^i)\beta_t}{2r_i(V_t^i)} \\ x_0 = x_0^i \\ x_t^i \in [x^{min}, x^{max}] \end{aligned} \right\}, \quad (10)$$

where $[x^{min}, x^{max}]$ is the constraint feasible domain of x_t^i . u_t^{i*} is obtained by solving (10). One can see from (9) and (4) that u_t^{i*} is a function of x_t^i , which means user i can employ u_t^{i*} as the policy for its feedback control, in order to implement the optimal trajectory of x_t^i . Nevertheless, V_t^i gets changed right after u_t^{i*} is implemented. Therefore, u_t^{i*} needs to be updated timely in order to find the actual optimal trajectory of x_t^i . Conventionally, HJB equation is usually combined with additional Fokker-Planck-Kolmogorov PDE [13], or McKean-Vlasov SDE [14], in order to estimate the influence of u_t^{i*} . These approaches need to be computed iteratively to obtain the optimal trajectory, which is too intensive for users in LVDNs. To tackle this issue, (10) is combined with the local measurements of V_t^i . Between each measurement, V_t^i is assumed to remain unchanged. Although this leads to approximation, the algorithm is streamlined, which will be elaborated in the control scheme.

III. CONTROL SCHEME

A. Problem statement

Assume there is a LVDN with $n > 100$ multifarious users including normal households, households with PV, small residential wind turbines. Each user has a certain degree of flexibility that is shapeable/deferrable, whereas the flexibility varies along time due to different user habits. Due to privacy concern, users are not willing to share their electricity consumption data, some users are even alleging that knowing the topology of distribution network is a kind of infringement of privacy as well. From the perspective of regulation, all the nodal voltages have to be kept within the system constraints. Since voltage regulation consumes flexibility, and flexibility may affect users' comfort, the control needs to be optimized in order to implement voltage regulation with respecting every user's scheduled consumption as much as possible. Additionally, all the users are equipped with smart meters but in different models and conditions, which means they have different computational capabilities.

B. Consideration on reactive power

Most of the papers about voltage control consider that the active power and reactive power can be controlled separately. This is true in transmission systems and backbone distribution systems but not necessarily in LVDNs. There are three reasons to this. Firstly, most of the users in LVDNs do not have such an ability. For instance, the appliances in a household including washing machine, dishwasher, are not able to actively change their reactive power, although their consumption can be shaped or deferred due to demand response. This leaves no space for a household user to response the reactive power regulation unless additional dedicated devices are installed. Secondly, as [15] stated, although distributed generators are able to adjust their reactive power injection, it is still not a feasible option, unless there is market in place for VAR from DERs or active regulation of voltage is made a standard (IEEE 1547) for DERs to participate in energy market. Thirdly, most of the DSOs have set a power factor range of $[0.9, 1]$ to every qualified user in LVDNs. Nevertheless, this does not mean reactive power can be ignored in the voltage control of LVDNs. As reactive power change is more like an associative effect of the active power change in LVDNs, in this paper, reactive power changes are regarded as uncertainties. It enters the control via the multiplier W in (1), yielding an equivalent change on x_t^i . Note that it does not mean the proposed approach is not able to consider active power and reactive power separately. Actually, another PDE set similar to (10) can be established for reactive power, and the corresponding cost can be used as weight to indicate the adjustment ratio between active power and reactive power. This will double the computation effort whereas it does not necessarily yield a significant improvement on performance, due to the aforementioned reasons and high R/X ratio in LVDNs.

C. Online optimization of user i

Every typical control period of user i starts by measuring V_t^i . Then, a voltage deviation coefficient ζ_t^i is given by

$$\zeta_t^i = (1 - \sin(\frac{e^{-(\frac{\mu}{V_0})^2}}{2}\pi)) \cdot \text{sgn}(\mu), \text{ where } \mu = V_t^i - V^r, \quad (11)$$

where V^r is the rated voltage and ζ_t^i ranges $(-1, 1)$. Recall (2) and (3), let $d = 1$, the specific cost function for user i is

$$J_t^i(x_t^i, u_t^i) = (x_t^{i'} - x_t^i)^2 + r_i(\zeta_t^i)u_t^{i2}, \quad (12)$$

where $r_i(\zeta_t^i)$ is given by

$$r_i(\zeta_t^i) = \begin{cases} 2 - |\zeta_t^i| & , (x_t^{i'} - x_t^i)\zeta_t^i \geq 0 \\ 1 + 5 \cdot |\zeta_t^i| & , (x_t^{i'} - x_t^i)\zeta_t^i < 0 \end{cases}. \quad (13)$$

The values 2 and 5 in (13) are empirical values to make sure neither $(x_t^{i'} - x_t^i)^2$ nor $r_i(\zeta_t^i)u_t^{i2}$ is too dominating. The minimization problem by plugging (12) into (4) yields the optimal trajectory of adjusting the consumption of user i from x_t^i to $x_t^{i'}$. And user i solves (10) together with (12) to seek the corresponding solution.

As associated reactive power changes are regarded as uncertainty, parameter W is employed to deal with it. From the perspective of computation, high W urges the optimal trajectory to stay closer to the median of $[x^{min}, x^{max}]$, which leaves margin in the control. Due to model-free, user i has no knowledge about how its reactive power changes along its active power changes, whereas margins created by W in the control helps user i with this uncertainty. As users have different characteristics, their power factor ranges different as well. Therefore, W is personalized to every user and denoted by W^i as

$$W^i = 1 + 5 \cdot \frac{1 - \phi_t^i}{0.25}, \quad (14)$$

where ϕ_t^i is the statistical power factor coefficient of user i . Similar to (13), the value 5 is empirical, and the principle of its value is making W^i has same feasible domain with $r_i(\zeta_t^i)$. ϕ_t^i is given by

$$\phi_t^i = \min(1.01 \cdot \phi_{t-1}^i, |\varphi_{t-1}^i|), \quad (15)$$

where φ_{t-1}^i is the measured power factor (PF) at the time $t - 1$. Equation (15) works in a moving average-like manner to evaluate the range of the power factor of user i . The current configuration of (14) is for $\phi^i \in [0.75, 1]$, which is fierce enough in a typical LVDN. Even lower PF will not result in failure of the control, but just no more margin can be given.

D. Control implementation

To implement the proposed distributed online optimization, user i measures V_t^i and solves the PDE to update u_t^{i*} . Therefore, the faster the PDE is solved, the better approximation to the optimum is yielded. A typical PDE is solved within 1 second by using the approach from [16] on a Intel I7-6600U processor. Although users cannot be expected to have such capability, three PDEs per minute are enough to give an

Fig. 2. Topology of the test network. Cable lengths are drawn to scale.

admissible control, which will be illustrated in the case study. When user i is computing u_{t+1}^{i*} , the current u_t^{i*} operates a feedback control to track the optimum. According to (12), with same $(x_t^{i'} - x_t^i)^2$, the cheaper $r_i(\zeta_t^i)$ is, the larger u_t^{i*} is given. Larger u_t^{i*} means user i has a faster trajectory to reach its $x_t^{i'}$, allowing it make a larger adjustment within one time window, to implement the scheduled consumption changes. A time window is the interval time between the updates of u_t^{i*} . In this way, the distributed online optimization is implemented.

IV. CASE STUDY

A. Setups

The schematic diagram is shown in Fig. 2. It is a balanced three phase 230/400V reference grid, based on the topology of a real semiurban feeder in Flanders, Belgium. For case study purpose, new users including residential wind turbines and small PV farms are added, increasing the number of nodes from 62 to 103. For profiles, the high resolution consumption profile of households are generated by a model of domestic electricity use, and profiles of PV panels are taken from a 368kWp PV system on the rooftop of a building in Genk, Belgium. All the other engineering details about the testing network can be found in section 4.1 and 4.2 in [17].

For benchmark purposes, a centralized optimization approach is implemented [18]. The controller solves the optimization problem using the full non-linear network model (Alternating Current Optimal Power Flow, ACOPF). Assumptions of perfect topology and voltage knowledge, instant computation, communication and implementation are given to the control, in order to make an ideal online optimization case. Its optimization problem has exactly the same objective with the proposed approach, except considering the cost of control, as doing so will result in dynamic programming problem, which is too computationally intensive to be simulated with the benchmark approach.

B. Results and remarks

For the worst case, random PF changes are added to the profiles. In order to make the comparison fair, it is assumed that the benchmark ACOPF and the proposed approach encounter the same PF changes on users. Different possible PF ranges are given to users upon their serial numbers. Specifically, users

Fig. 3. Evolvement of nodal voltages, sample resolution: 20s

numbered with the multiples of three ranging $\pm[0.85, 1]$, while users numbered with the multiples of five ranging $\pm[0.75, 1]$ and all the rest ranging $\pm[0.9, 1]$. Fig. 3 illustrates the nodal voltages evolvement for all the 103 nodes with the original

Fig. 4. The dissatisfaction rate between ACOPF and the proposed approach

profile, the benchmark ACOPF and the proposed approach respectively. As there is no automatic mechanism to deal with the uncertainty in the benchmark ACOPF algorithm, a tighter constraint is set. Instead of $\pm 10\%$ of the rated voltage, $\pm 7\%$ of the rated voltage is given in order to create margins in the benchmark ACOPF. The proposed approach employs statistic mechanism to address the uncertainty, therefore no additional adjustment needs to be given, and in order to show the aforementioned compatibility of low computational capability, the PDE solving speed is set to 3 PDEs/min for all the users. During the whole simulation horizon, for the benchmark ACOPF, there were 0.43% of the voltage samples violating the voltage bound, and 0.41% for the proposed approach. In order to compare the performance, we propose *dissatisfaction rate* as the key performance indicator, as the original objective function of the benchmark ACOPF was tuned to fit the test case. The *dissatisfaction rate* is given by

$$D_t = \sum_{i=1}^n |x_t^{i'} - x_t^i|. \quad (16)$$

It is an explicit evaluation: with bringing all the nodal voltages within bounds as much as possible, the closer users work at their desired consumption, the smaller number it yields. As shown in Fig. 4, the proposed approach results in a better D_t than the benchmark, with an average of 13.55 versus 14.22. It is not necessarily fair to conclude that the proposed approach is completely better than the benchmark ACOPF, whereas it can be concluded that the proposed approach can give distributed online optimization that is as good as the centralized one with uncertainties with less communication.

V. CONCLUSION

A distributed online optimization approach is proposed. It abstracts voltage control problems in LVDN as dynamic programming problems, then the HJB equation is employed to seek the approximation to the optimum. Instead of introducing extra statistical models to evaluate the influence of the control and solve the corresponding PDEs iteratively, local measurements are employed to reduce the computation demand.

There are at least two directions to extend this research. Firstly, peer to peer communication can be added to allow

users to have different objectives. By contrast, in this paper, global optimization is achieved as all the users have the same objectives, which forms a potential game. Secondly, feedback statistic mechanisms can be employed to estimate the voltage sensitivity, which may be able to make the approximation to optimum more precise.

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