Word problem-solving approaches in mathematics textbooks: A comparison between Singapore and Spain

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Abstract

Singaporean children are the best performers on international achievement tests in mathematics (i.e., the TIMSS). Their excellent results could be due at least partly to certain characteristics of the textbooks used there (Oates, 2014). Therefore, these materials could be taken as a good benchmark to describe and evaluate aspects of the textbooks of other regions that could be improved. For this reason, the word problem-solving approaches proposed by primary education math textbooks from Spain were compared with those from Singapore based on the presence of the problem-solving steps that are most characteristic of a genuine word problem-solving approach. The results show that the Singaporean textbook include much more reasoning than the Spanish textbooks, while they include fewer problem-solving steps related to strategies and checking. We conclude that Singaporean textbook provide better scaffolding for high-quality learning of word problem solving than Spanish textbooks.

Key words: Word problem solving; Mathematics textbooks; Primary education; Assessment and evaluation; Educational systems.

Introduction

Since the 1990s different international survey assessment programmes, such as the Trends In Mathematics and Science Study (TIMSS) sponsored by the International Education Agency (IEA), have provided reports of the level of mathematical competence in primary education students in a large number of countries. The results obtained in these assessments allow the comparison of participants' achievement between countries. According to the latest TIMSS cycle in 2015, Singapore belongs to the countries whose children achieve a considerably higher level of competence in mathematics than those of other countries, including Spain. More specifically, children from Singapore were the best performers in the world, attaining a score of 618 on the assessment, whereas children from Spain attained a below-average score of 505 (in position 32 of 48 participating countries, Mullis, Martin, Foi & Hooper, 2016).

Mathematics textbooks could be one of the elements of these educational systems that may help to account for these results, given that textbooks have historically been considered and currently are considered a major element of mathematics education around the world (Haggarty & Pepin, 2002; Mullis, Martin, Foi & Arora, 2012; Oates, 2014). Various studies have already been set up with a view to analyse math textbooks from two or more countries or regions and link the results of this comparative analysis to student learning outcomes (Depaepe, De Corte & Verschaffel, 2009; Mayer, Sims & Tajica, 1995; Stigler, Fuson, Ham & Kim, 1986; Xin, 2007). The present study provides such an analysis for the math curricular subdomain of word problem solving, which is generally considered the cornerstone of mathematical competence (Grønmo, Lindquist, Arora & Mullis, 2013). More concretely, this comparative study analyses how Spanish primary school textbooks approach the teaching of arithmetic word problems compared to the Singaporean textbook.

Word problem solving in mathematics textbooks

Textbooks are an important element in educational systems for several reasons. First, they are an instructional resource that is generally and frequently used by teachers in most countries around the world. For example, the data on the 57 countries that participated in the 2011 TIMMS[1] indicate that, on average, 96% of teachers use a textbook, and for the great majority of the teachers, textbooks are the main basis for their educational practice (Mullis et al., 2012). More specifically, 70% and 74% of Singaporean and Spanish teachers, respectively, use textbooks as the basis for instruction, and 23% and 22%, respectively, use them as a supplement. Thus, in both countries, the textbook can be considered a core element for the teaching of mathematics. Nevertheless, the

[1] The latest TIMSS cycle in 2015 did not provided any information about this issue.

creation and supervision of these textbooks do not receive the same amount of attention from the educational administration in Spain as they do in Singapore. Singaporean textbooks are "state-approved, and although a number of publishers co-exist in the system, all must meet State criteria – criteria which were more restrictive when initially formulated but have been successively relaxed as the truly fundamental elements have become clear" (Oates, 2014, p. 5). In Spain, "quality and compliance with the curriculum is driven by the market", trusting that "when a country has a free market for textbooks and these are produced commercially, publishers have to strive for consistency and quality otherwise schools would not choose to buy their products" (Eurydice, 2011, pp. 48-49). Second, textbooks represent such a strong specification of the curriculum that they in fact determine to a great extent what is actually taught in class (Oates, 2014). In this sense, Apple (1992) notes that 'the curriculum in most schools is not defined by courses of study or suggested programmes, but by […] the standardized, grade-level-specific text […]', because it 'dominates what students learn; they set the curriculum, and often the facts learnt, in most subjects' (pp. 565-569). If this is the case, it is understandable that certain researchers are interested in analysing the differences between textbooks from different countries as a way to investigate the curriculum as (potentially) implemented in those countries (e.g., Erbas, Alacaci & Bulut, 2012; Lessani, Yunus, Tarmiz & Mahmud, 2014; Mayer et al., 1995).

For the present study, we decided to focus on a particular element or aspect of the mathematics curriculum, namely, problem solving, and, more particularly, word problem solving. Problem solving is the cornerstone of the mathematics curriculum, as noted in the educational curricula of many countries and in the theoretical frameworks supporting the main international achievement assessments, such as the TIMSS (e.g., Grønmo et al., 2013). Arithmetic word problems are a specific case of mathematic problems, which can be defined as 'verbal descriptions of problematic situations that give rise to one or more questions whose answers can be obtained by applying mathematical operations to the numerical data present in the problem' (Verschaffel, Depaepe & Van Dooren, 2014, p. 641). As explained by these authors, while not all word problems are necessarily genuine 'problems' (as defined above) for the children, they are universally considered a privileged vehicle for promoting children's mathematical problem-solving competencies (Verschaffel et al., 2014).

Different models have been proposed to understand the mental processes that need to be realized to solve a (word) problem. For instance, the model by Verschaffel, Greer and De Corte (2000) proposes that two contrasting approaches can be followed to solve a word problem: a genuine or a superficial approach. Problems that are difficult from the mathematical and/or situational point of view can typically only be solved thoughtfully or genuinely. Difficult math problems are those in which it is necessary to reason out the relations among the numbers involved. For example, in the following problem, ‘David is 52 years old, and 26 years older than Ann. How old is Ann?’, the problem solver must realize that if David is older than Ann, then Ann will be younger than David, and therefore subtraction must be used to find Ann’s age, although the use of the word ‘older’ could induce one to think of addition. On the other hand, situationally (rather than mathematically) difficult problems would be those requiring applications of knowledge from real life to be solved correctly (see Verschaffel et al., 2000). For example, with the problem ‘James wishes to tie together two posts that are separated by 12 metres using pieces of rope that are 1.5 metres long. How many pieces of rope does he need?’, the problem solver should realize that by tying the pieces of rope together, part of their length will be used, and therefore the answer 12/1.5= 8 is not correct, as at least 9 pieces will be needed.

To solve such mathematically and/or situationally difficult problems in a genuine way, problem solvers must follow a series of steps. First, they must read the problem to extract the relevant information. Then, they must apply the necessary reasoning to understand the situation being described, first situationally (by building a situational model involving the characters, their intentions and actions) and then mathematically (by relating the situational information to their mathematical knowledge to generate a mathematically correct model of the problem). Once these two types of understanding have been reached, the problem solvers must infer one or several arithmetic operations from the mathematical model that will solve the problem and then perform these operations. Finally, they must verify whether the answer ‘fits’ both the mathematical and situational reasoning previously carried out. If the answer ‘fits’, then it can be accepted as valid and reported as the solution to the problem; if not, revision of one or more steps of the word problem-solving cycle is needed.

In contrast, other word problems can be solved in a mindless or superficial way. In this superficial solution process, only the given numerical information is selected, while the rest of the information in the statement of the problem is largely ignored. Then, the problem solvers choose the operation(s) to be performed, taking as a clue some superficial characteristic of the problem statement (e.g., a key word such as ‘more than’ or ‘earn’ as an indication that addition must be used) or a contextual indication (applying the arithmetic that they are currently learning in class). Once the operation has been performed, the answer is considered to be the solution to the problem, without any verification.

Although Verschaffel et al. (2000) argues that children should learn to solve all problems in a genuine way, they warn that because the superficial approach also leads to success when solving easy problems, children may be inclined to avoid investing more cognitive effort than absolutely necessary to solve these problems and thus to choose the less demanding - and educationally less valuable - road to solve them.

Textbooks could have an important impact on learning to solve arithmetic word problems in two different ways: (1) the variety of problems they propose with respect to their mathematical and situational difficulty and (2) the word problem-solving approaches they propose for solving them. Most studies that have analysed word problems in textbooks have focused on describing the variety of the word problems they propose, analysing whether the textbooks include word problems at all levels of mathematical and situational difficulty. Their results have shown, first, that countries such as Japan or Russia offer a greater diversity of problems with addition or multiplication structures than countries such as the US or Spain (Orrantia, González & Vicente, 2005; Stigler et al., 1986; Vicente, Manchado and Verschaffel, 2018; Xin 2007) and, second, that situationally challenging problems are very scarce in textbooks (Orrantia et al., 2005; Pongsakdi, Brezovszky, Veermans, Hannula-Sormunen & Lehtinen, 2016). Furthermore, only a few studies have analysed the problem-solving approaches that textbooks propose for solving word problems. In the case of Singapore, several studies (Beckmann, 2004, Hoven & Garelick, 2007) have analysed their textbooks in a general way, focusing on the use of 'bar representations' as a support for the mathematical reasoning necessary to move from concrete to abstract mathematics. Fan and Zu (2007) analysed problem-solving approaches proposed in textbooks (hereafter abbreviated PSAPTs) for lower-secondary education in China, Singapore and the United States, using Polya's four-step model for problem solving (with heuristics in each phase or step, Polya, 1973), operationalized into concrete heuristics. The results showed that the Chinese and American textbooks were more explicit in labelling the different steps to be followed, that the heuristics most employed in the textbooks of each country were similar (e.g., 'draw a diagram,' 'use an equation,' and 'restate the problem') and that the Singaporean textbook hardly proposed any steps for revising the answer. Finally, Sánchez and Vicente (2015) descriptively analysed PSAPTs for primary education in textbooks from three different publishers in Spain. To do so, they operationalized the model for word problem solving by Verschaffel et al. (2000) into six steps that the problem solver had to follow to solve the word problems: extract the information, perform situational and mathematical reasoning, choose a problem-solving strategy, select the operation, present the answer and check the answer presented. These six steps correspond to the different phases into which the genuine solving process can be divided, some of which are skipped in superficial solving, according to Verschaffel et al.'s (2000) model (which is described in more detail on page 3): 'Information' refers to the extraction of relevant information; 'Reasoning' refers to the situational and mathematical comprehension of the problem; 'Strategies' and 'Operations' refer to the deductions that the problem solver has to make based on the mathematical model of the problem; 'Answer' refers to finding the answer; and, finally, 'Checking' refers to the verification step.

Their results showed that most of the proposed models consisted only of the three steps associated with Verschaffel et al.'s (2000) superficial model, namely, (1) extract the information, (2) select the operation and (3) present the answer, rather than the six steps from Verschaffel et al.'s (2000) genuine mathematical modelling cycle.

The present study

In this study, we analyse the problem-solving approaches proposed in the most frequently employed textbooks in the educational systems of Spain and Singapore, because while Spanish textbooks—as many others all over the world—probably have much room for improvement in different ways, Singaporean textbook can be considered a point of reference in the design of curricular material due to the way the Singaporean educational system has evolved, the role textbooks have played in this evolution, and the attention textbook design has received from the educational authorities there (Oates, 2014).

Materials and hypotheses

Sample

Our sample consisted of textbook pages from the publishers of the most used mathematics textbooks in Singaporean and Spanish elementary schools (according to Clark, 2013 and Vicente et al., 2018, respectively); these textbooks feature word problem-solving models structured into different steps with so-called 'worked out' examples in which the steps of the problem-solving process are explicitly identified and applied. According to the textbook authors' instructions, children must read these textbook pages under the supervision of the teacher. Thus, the children's task on these pages is not solving the problem or completing part of the solution process themselves but following a 'worked out' sequence of steps to solve a word problem. In the case of Singapore, we analysed "My pals are here" (Kheong, H., Ramakrishnan, Wah, Choo & Soon, 2015), a textbook published by Marshall Cavendish, for grades 1 to 6 (two pupil textbooks accompanied by two workbooks per grade). With a percentage of use of 86% (Clark, 2013), this mathematics textbook was by far the most used in Singaporean elementary schools. For Spain, according to Vicente et al. (2018), three textbooks were used in more than 90% of elementary schools: those published by Santillana (43.16%), SM (25.76%) and Anaya (21.30%). Thus, the textbooks "La casa del Saber" (Alzu, López-Sáez, Henao & Juan 2010) by Santillana, "Trampolín, Tirolina y Timonel" (Peña et al., 2010) by SM, and "Abre la puerta" (Ferrero, Gaztelu., Martín & Martínez, 2010) by Anaya, were the ones chosen for our analysis for grades 1 to 6 (one pupil textbook and three workbooks per grade for each textbook). In the following section, we will refer to each of these textbooks by using the publisher's name.

In sum, 174 PSAPTs from the Singaporean textbook were analysed as well as 74 PSAPTs in Santillana, 84 in SM and 51 in Anaya. These PSAPTs included 747 steps in the single Singaporean (publisher's) textbook, 278 steps in Santillana, 373 steps in SM and 161 steps in Anaya. To illustrate the PSAPTs analysed, an example from each set is provided. The first PSAPT (see Figure 1) is taken from the fourth-grade Singaporean textbook (p. 76) and is based on the following word problem: 'Mr Gan and Mr Fong had $4536 altogether. Mr Gan's share was twice as much as Mr Fong's. How much was Mr Gan's share?'

Figure 1. Example of a PSAPT taken from the Singaporean mathematics textbook by Marshall Cavendish, grade 4 (book A), p. 76.

The second example is taken from a Spanish textbook (Santillana, 6th grade p. 132):

Figure 2. Example of a PSAPT taken from the Spanish mathematics textbook by Santillana, grade 6, p. 132.

Problem Solving

Represent the information with drawings

Solve the following problems by representing the unknown information with a drawing. Then check to see if your answer is correct

In the two classes of 6th year they were collecting food for a charity campaign. In 6B they collected 9 kg more than in 6A and the two classes together collected 71 kg of food. How many kilos were collected in each class?

▶ We do not know how many kilos were collected by class 6A. We can represent that missing information with an image

1. We write down the information in the problem.

   Kilos collected by 6A: ●

   Kilos collected by 6B: ● + 9

2. We express the condition of the problem: the sum of the two amounts is 71 kg., and we calculate.

   ● + ● + 9 = 71

   2 × ● + 9 = 71

   2 × ● = 71 − 9 = 62

   ● = 62 : 2 = 31

3. We find the solution.

   6.° A ▶ ● = 31 kg

   6.° B ▶ ● + 9 = 31 + 9 = 40 kg

4. Check

   40 = 31 + 9

   31 + 40 = 71

Solution: In class 6A they collected 31 kg of food and in class 6B they collected 40 kg.)

Analysis

The categories used for the analysis were adapted from those used by Sánchez and Vicente (2015). These categories were set up in such a way as to correspond to each of the steps of the word problem-solving process described by Verschaffel et al. (2000), described in more detail on page 3:

1. Information: This category refers to the way in which the textbook author instructs children to handle the information provided in the problem when they extract it or complete it. It includes the following subcategories: a) extract the necessary information in the problem, read it carefully, and explain or paraphrase the problem; b) omit unnecessary information; c) re-order the statements; and c) extract data from graphs or tables.

2. Reasoning: The reasoning step can involve situational reasoning (when specific aspects of the situation that must be taken into account by the learner are considered by the textbook author) or mathematical reasoning (when the mathematical relations are established to deduce the arithmetic operation(s) needed to solve the problem). The reasoning step includes the following subcategories: a) perform situational reasoning (by directing children's attention to the understanding of the qualitative situation of the statement; for example, in the sample problem of the posts and ropes, this would mean highlighting the need to consider that by tying the ropes, part of their original length is lost); b) perform mathematical reasoning about the mathematical relations between the known and unknown quantities stated in the problem; and c) inspect mathematical representations accompanying the word problem (outlines, diagrams and other illustrations that represent the mathematical relations between the sets[2]). It should be noted that other more general types of reasoning are not included in this reasoning category (e.g., metacognitive processes.)

3. Strategies: For steps in this category, a specific strategy for solving a specific problem is shown in the textbook, without indicating how to generalize the application of this strategy to other situations. This category has the following subcategories: a) make a table, b) look for a rule or pattern, c) use simpler problems, d) estimate the answer, e) start from the end, f) trial and error, g) physical modelling, h) rule out possible solutions, i) use the number line, j) use drawings to represent unknowns, k) establish the number of operations necessary to solve the problem and l) look for intermediation questions or operations.

4. Operations: The operations category includes the steps that show the arithmetic operation(s) needed to solve a problem.

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[2] When the drawings were used to represent data or some aspects of solving problems other than the mathematical relationships between the sets, they were considered 'use of drawings', which is a problem-solving strategy.

5. Answer: This category refers to steps that demonstrate the specific way in which the result of the operations must be expressed for it to be considered the solution to the problem

6. Checking: Steps in this category show how to check the answer to see if it is correct. This category has three subcategories: a) generic checking when no criterion is specified for performing the verification; b) specific mathematical checking through operations that are the reverse of those in the Operations step or reasoning whether the answer is correct from a mathematical point of view; and c) specific situational verification, for example, in the problem of the posts and ropes, checking that the number of pieces of rope obtained will include enough length to tie them together.

Hereafter, we will illustrate the distinct categories of our scoring system by applying them to the abovementioned illustrative 'worked out' word problem from the Singaporean and Spanish textbooks.

In the example problem from the Singaporean textbook, we identified the following steps: Information (gathering the information provided by the problem about the amount shared by Mr. Gan and Mr. Fong and about who owned the biggest quantity); Reasoning (representing the mathematical structure of the problem and splitting the total amount into three 'units', two of them belonging to Mr. Gan and the other to Mr. Fong); Operations (solving the problem by means of a division and a multiplication, starting from the mathematical model generated in the Reasoning step); Answer (providing a statement that answers the question raised in the problem) and Checking (checking that the quantity obtained by Mr. Gan is double that of Mr. Fong and that both quantities sum to the total amount by means of a mathematical estimation). It is important to note that none of the steps included in this PSAPT could be considered to belong to the Strategies category; for example, although 'making a representation' could in some cases be considered a Strategy, given that the representation included in this PSAPT was related to the mathematical structure of the problem, it was included in the Reasoning step.

The steps included in the example of the PSAPT taken from a Spanish textbook were categorized as follows: Information (writing down the information in the problem); Strategies (using a drawing - a red dot- to represent the unknown quantities); Operations (choosing the necessary operations and using them in an equation); Answer (expressing the solution of the problem) and Checking (fitting the answer into the mathematical situation). In this case, Reasoning was not coded because none of the solution steps promoted any mathematical or situational reasoning (as defined above). It is noteworthy that the strategy proposed ('represent the data using drawings') could not be considered Reasoning because red dots were merely intended to symbolically represent the unknown quantities in a mathematical equation instead of representing the mathematical relations between the sets of the problem.

To simplify the data analysis in the present study, and given that, according to Verschaffel et al.'s (2000) model, the superficial way of solving problems lies exclusively in the solution steps of 'Information', 'Operations' and 'Answer', we decided to focus our analysis of the PSAPTs on the presence and precise nature of the other three steps, i.e., 'Reasoning', 'Strategies' and 'Checking', which have been characterized as specific to the genuine word problem-solving process.

Procedure

First, we extracted the textbook pages that explicitly proposed (the 'worked out' parts of) a model for solving word problems, which were usually – but not always – identified with a title at the top of the page, such as 'Problem Solving'. We then extracted each step of each of these PSAPTs found on these textbook pages from each publisher and each school year and assigned them a category and/or subcategory. To check whether there was an adequate degree of reliability, two researchers analysed 20% of the sample separately and calculated the reliability index as measured by Cohen's kappa coefficient. As a result, the reliability index was 1 for all main categories, except for Information (κ = .92), Reasoning (κ = .91) and Strategies (κ = .84), while for the subcategories, it was 1 for all of them except extract information (κ = .9), performing mathematical reasoning (κ = .82), make a table (κ = .84) and look for intermediate operations (κ = .97).

Measures

To compare the results of the Singaporean textbook with the Spanish textbooks, two measures were created: a) the percentage of PSAPTs in which each of the six categories of steps appeared (for example, the percentage of PSAPTs in which the step 'Reasoning' was included) and b) the percentage of PSAPTs in which a specific sequence of steps occurred (an example of a less complete and a more complete sequence would be 'Information-Operations-Answer' and 'Information-Reasoning-Strategies-Answer-Checking', respectively).

These two measures allowed us to observe the relative number of steps devoted to 'Reasoning', 'Strategies' and 'Checking' in the PSAPTs of each of the textbooks as well as the specific sequences of steps that most frequently formed the PSAPTs in each textbook, thus enabling us to determine which textbooks proposed approaches that combined two or even all three steps characteristic of the genuine approach.

Two independent variables were related to these two measures: the educational system (Singapore vs Spain) and the grade for which the textbook was designed. To facilitate interpretation of the data, grades were categorized into groups of two, thus establishing three levels of school grades (grades 1/2, 3/4 and 5/6) for this independent variable. Finally, in line with the characteristics of the sample, to compare the textbook data between the two countries, we used personalized tables with Pearson's chi-squared test and chi-squared tests for the analysis of differences between specific categories of the different textbooks and school grades.

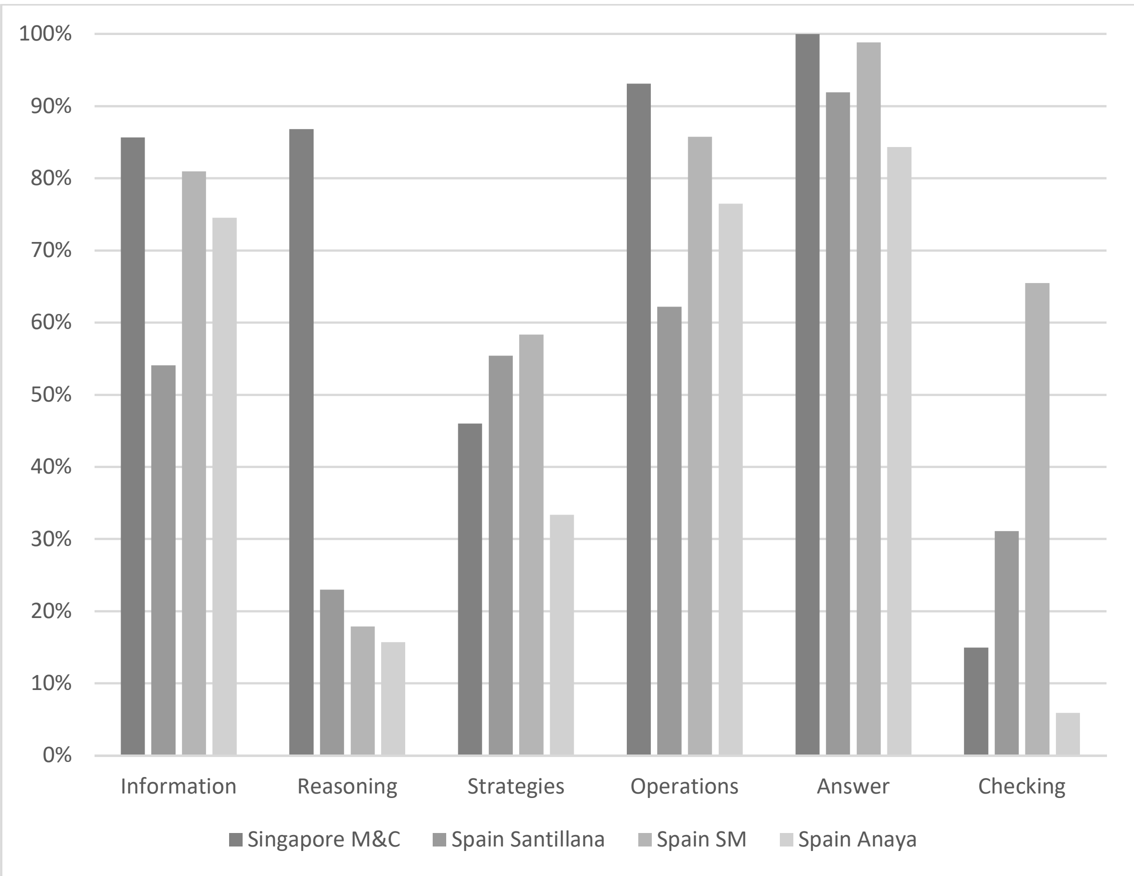
Hypotheses

In line with certain previous results (Sánchez and Vicente, 2015, Beckmann, 2004, Fan & Zu, 2007, Hoven & Garelick, 2007), our general hypothesis was that the Singaporean textbook would follow a more genuine instructional approach towards word problem solving than the Spanish textbooks. This hypothesis leads to the following more specific predictions. First, we predicted that the Singaporean textbook and the three Spanish textbooks would include the Information, Operations and Answer steps (the steps that are included in both the genuine and superficial approach) in a similar proportion of PSAPTs (prediction 1a); however, we predicted a higher proportion of the PSAPTs from the Singaporean textbook than from the Spanish textbooks would include an explicit reference to the problem-solving steps of Reasoning, Strategies and Checking (steps that are unique to the genuine approach) (prediction 1b). Consequently, PSAPTs from the Singaporean textbook would also include more sequences involving the steps of 'Reasoning', 'Strategies' and/or 'Checking' than those from the Spanish textbooks (prediction 1c). With regard to the influence of grade, we expected that the above differences between the Singaporean and Spanish textbooks would gradually decrease and even disappear in the higher grades (prediction 2). This prediction is based on the observation that in a good textbook, the amount of scaffolding for word problem solving gradually decreases as children move to the upper grades of elementary school since the learners should by then have 'internalized' the competent problem-solving model, so there should be less need for the textbook author to include all of the scaffolding elements each time a word problem is presented.

Results

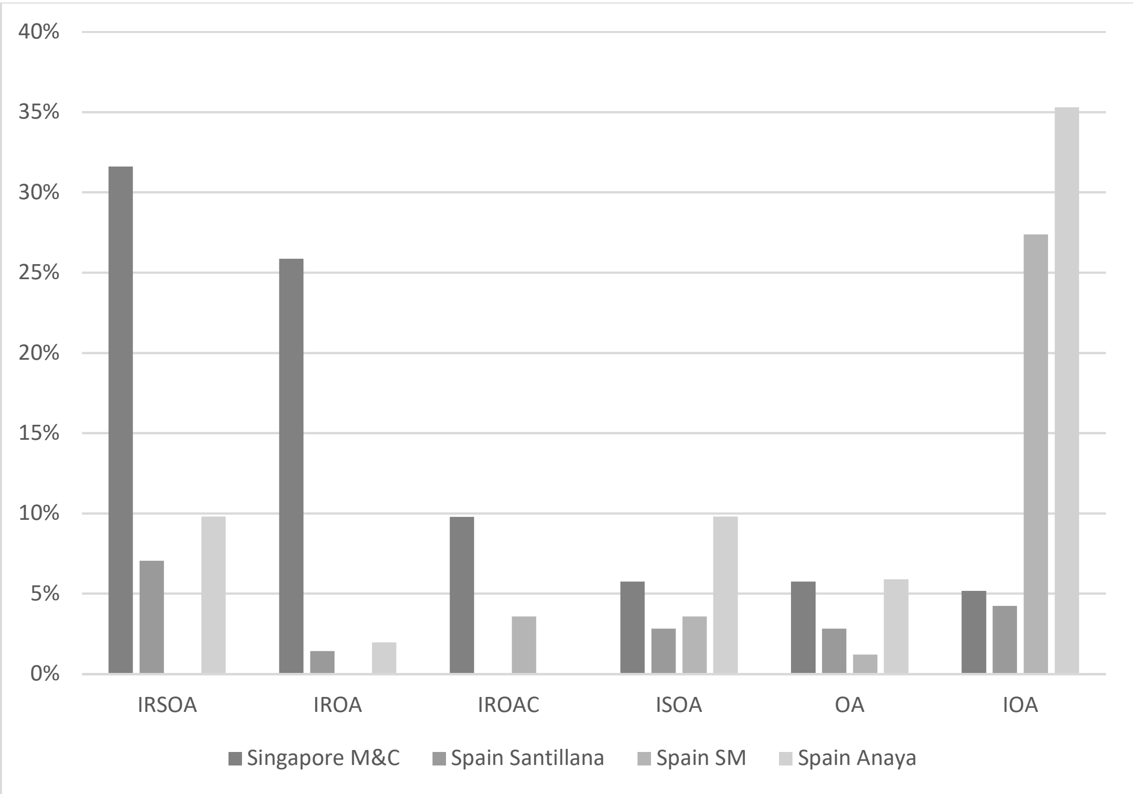
An analysis using custom tables showed overall differences between the Singaporean and Spanish textbooks in the proportion of PSAPTs that included the different steps being analysed, $\chi2(15, n=1441) = 143.1$, $p < .001$. In this sense, while there were no significant differences between the Singaporean and Spanish textbooks for the three steps shared by the genuine and superficial problem-solving processes (confirming our prediction 1a), differences related to the three genuine approach steps were found. For the first measure of our study, regarding the presence of the three genuine problem-solving process steps (prediction 1.b), in all of the PSAPTs proposed in the four textbooks analysed, the results showed significant differences in the proportion of PSAPTs that included the steps of Reasoning, Strategies and Checking: $\chi2(3, n=144) = 97.05$, $p < .001$; $\chi2(3, n=192) = 7.88$, $p < .05$; and $\chi2(3, n=118) = 71.06$, $p < .001$, respectively. First, Reasoning was present in more of the word problem-solving models used in the Singaporean textbook than in Santillana, $\chi2(1, n=1104) = 37.24$, $p < .001$; Anaya, $\chi2(1, n=105) = 43.34$, $p < .00$; and SM, $\chi2(1, n=103) = 48.94$, $p < .001$. Second, although Strategies was present in more of the PSAPTs used in Santillana and SM than in those used in the Singaporean textbook, only the differences between the Spanish Anaya textbooks and the other two Spanish textbooks were significant: $\chi2(1, n=88) = 5.50$, $p < .02$ and $\chi2(1, n=91) = 6.87$, $p < .01$ for Santillana and SM, respectively. Finally, Checking was significantly more present in two of the Spanish textbooks (Santillana and SM) than in the Singaporean textbook, $\chi2(1, n=46) = 5.65$, $p < .02$ and $\chi2(1, n=81) = 32.11$, $p < .001$, respectively. However, Checking was significantly less present in the third Spanish textbook, Anaya, than in the Singaporean textbook, $\chi2(1, n=21) = 3.86$, $p = .05$. Taken together, these results (shown in Figure 3) do not confirm prediction 1b.

Figure 3. Percentage of models in which each problem-solving step appears, by textbook.



Regarding the second measure of the study, a frequency analysis of the different specific approaches proposed in the Singaporean and Spanish textbooks (as a result of the different combinations of problem-solving steps) showed important differences. To simplify the analysis, we compared the approaches of the three Spanish textbooks to only the six most frequent approaches found in the Singaporean textbook.

Figure 4. Frequency of the different combinations of steps in the Spanish and Singaporean textbooks, in order of higher to lower frequency in the Singaporean textbooks. Note: I= Information; R= Reasoning; S= Strategies; O= Operations; A = Answer; C = Checking



Whereas in the Singaporean textbook, only 13 different sequences were proposed, in the Spanish textbooks, 23 combinations of problem-solving steps were proposed in Santillana, 16 in SM and 18 in Anaya. Regarding prediction 1c, two fundamental differences were found between the textbooks of the two educational systems under study. First, the most frequent specific approaches were completely different (see Figure 4): whereas in the textbooks from the Singaporean publisher, the most frequent approach was Information-Reasoning-Strategy-Operation-Answer, followed by the same approach without Strategy, in the Spanish textbooks, the most frequent approach for two of the three publishers (SM and Anaya) was the Information-Operations-Answer approach (27.38% and 35.29%, respectively), followed by the Information-Strategy-Operations-Answer-Checking approach (21.83%) and the Information-Reasoning-Strategy-Operations-Answer approach (9.80%) for SM and Anaya, respectively. For Santillana, the most frequent approaches were the single-step 'Answer' approach

(9.86%), followed by the Information-Reasoning-Strategies-Operations-Answer approach (7.04%). These results partially confirm our prediction 1b.

A closer look at this result by analysing the subcategories included in each of the problem-solving steps reveals further differences between the textbooks. On the one hand, no important differences were found in the categories of Information, Reasoning and Checking. First, the Information category in both the Spanish and Singaporean textbooks was mainly reduced to the mere extraction of data. Second, setting aside the considerable differences between the two countries in the percentage of steps devoted to Reasoning, approximately 85% of the Reasoning steps in the Singaporean textbook and the Santillana and SM textbooks and 100% of the steps in the Anaya textbook were done based on a mathematical representation of the problem, with the rest being completed through mathematical reasoning, with no situational reasoning appearing in either case. Finally, in the Singaporean textbook and in the Spanish SM and Anaya textbooks, most of the Checking steps involved a specific mathematical verification (87.50% for the Singaporean textbook and 85.45% and 100% for SM and Anaya, respectively), while the rest were generic with no situational corroboration. Again, Santillana was different from the other Spanish textbooks, given that only 30.43% included specific mathematical checking, with the rest using generic checking. On the other hand, some important differences were found in Strategies: although in the Singaporean textbook, Santillana and SM, 'look for intermediate operations' was the most frequent sub-category (and the second most frequent sub-category in Anaya), its weight within the step was much greater in the Singaporean textbook (87,50%) than in Santillana (44.23%), SM (31.75%) and Anaya (41.18%). As a result, there was much less variety of strategies in the Singaporean textbook than in the Spanish Santillana and SM textbooks; in fact, the Singaporean textbook included only three more strategies: 'make a table' (7.50%), 'physical modelling' (3.75%) and 'trial and error' (1.25%). In contrast, 13 different strategies were found in the textbooks by Santillana, and 10 were found in the textbook by SM, of which 'physical modelling' (13.46%) and 'estimate the answer' (7.69%) were the next most frequent categories in Santillana, and 'make a table' (23.81%) and 'estimate the answer' (9.52%) were the next most frequent in SM. Anaya was the most similar Spanish textbook to the Singaporean textbook because it included only two sub-categories beyond 'look for intermediate operations': 'physical modelling' (47.06%) and 'estimate the answer' (11.77%).

The second hypothesis of our study was related to the results by grade level. If we compare the results of the textbooks used in our sample by educational grade, significant differences can be observed in all of the grades: $\chi2(15, N=1344) = 298.66$, $p < .001$; $\chi2(15, 1399) = 195.72$, $p < .001$; $\chi2(10, 1275) = 150.78$, $p < .001$ for grades 1/2, 3/4 and 5/6, respectively. These differences were mainly due to the following three reasons. First, in all grades, the Singaporean textbook shows significantly more approaches in which Reasoning was included as a step in the problem-solving process than all of the Spanish textbooks, except Santillana in grades 5/6. It is noteworthy that this difference decreases at higher grades. Second, while in grades 1/2, the Santillana and Anaya textbooks contained more strategies than the Singaporean textbook, and in grades 3/4, the Santillana and SM textbooks included more strategies than the Singaporean textbooks, no significant differences were found in grade 5/6. Finally, while in grades 1/2, both the Santillana and Singaporean textbooks included significantly more Checking steps in their PSAPTs than Anaya and SM, in grades 3/4 and 5/6, SM included significantly more Checking steps than the rest of textbooks. Furthermore, in the Santillana and Singaporean PSAPTs, the presence of Checking steps decreased with the educational grade, especially in the Singaporean textbook in grades 3/4 and 5/6 and in Santillana in grades 5/6.

Taken together, these results (see Table 1) only partially support our prediction 2; considering the three genuine approach steps, the percentage of approaches with Reasoning, Strategies and Checking steps increased with grade in the Spanish textbooks, while in the Singaporean textbook, the percentages of Checking steps decreased with grade, Strategies steps increased with grade and Reasoning steps increased from grade 1/2 to 3/4 but decreased from 3/4 to 5/6.

		Information	**Reasoning**	**Strategies**	**Operations**	**Answer**	**Checking**
L 1/2	M&C	73.17	73.17 $^{\text{Sa, Sm, A}}$	12.20	100.00 $^{\text{Sa}}$	100.00	39.02 $^{\text{A, Sm}}$
	Santillana	77.78	0.00	44.44 $^{\text{Sm, M}}$	55.56	100.00	44.44 $^{\text{A, Sm}}$
	SM	89.66	0.00	17.24	93.10 $^{\text{Sa}}$	96.55	0.00
	Anaya	100.00 $^{\text{M}}$	0.00	36.36 $^{\text{Sm, M}}$	90.91 $^{\text{Sa}}$	95.45	4.55
L 3/4	M&C	95.24 $^{\text{Sa, Sm, A}}$	92.86 $^{\text{Sa, Sm, A}}$	23.81	90.48 $^{\text{Sa}}$	100.00	16.67 $^{\text{A}}$
	Santillana	43.33	13.33	46.67 $^{\text{M}}$	56.67	86.67	36.67 $^{\text{Sm, M}}$
	SM	55.17	31.03 $^{\text{Sa}}$	68.97 $^{\text{Sa, A, M}}$	79.31	100.00	100 $^{\text{Sa, A, M}}$
	Anaya	55.17	27.59 $^{\text{Sa}}$	31.03	65.52	75.86	6.90
L 5/6	M&C	86.81 $^{\text{Sa}}$	69.23 $^{\text{Sm}}$	70.33	91.21	100.00	3.30
	Santillana	50.00	50.00 $^{\text{Sm}}$	73.08	73.08	92.31	15.38 $^{\text{M}}$
	SM	100.00 $^{\text{Sa}}$	23.08	92.31	84.62	100.00	100.00 $^{\text{Sa, M}}$

Table 1. Percentage of models in which each problem-solving step appears, by country and grade. The superscripts indicate the textbooks with respect to which the difference was statistically significant.

With regard to the most frequent approaches, relatively little variability between grade levels was found in the Singaporean textbook. In contrast, in the Spanish textbooks, the between grade level variability was higher; in fact, the most frequent PSAPT in the Spanish textbooks differed to a greater extent than in the Singaporean textbook throughout all the educational grades and textbooks (see Table 2). The Spanish textbooks tended to include more steps related to Reasoning, Strategies and Checking in the most frequent approaches in the higher grades. These results do not confirm our prediction 2.

		L1/2	**L3/4**	**L5/6**
M&C	1st	IROA 41.46	IROA 50	IRSOA 50.55
	2nd	IROAC 26.83	IRSOA 16.67	ISOA 10.99
Santillana	1st	IOAC 27.78	ISOAC 25.93	IRSOA 15.38
	2nd	ISA 16.67	A 25.93	RSOA 15.38
SM	1st	IOA 70.31	SOAC 20.69	ISOAC 57.69
	2nd	ISOA 10.34	ISOAC 17.24	IRSOAC 19.23
Anaya	1st	IOA 59.09	IOA 17.24	
	2nd	ISOA 22.73	IRSOA 17.24	

Table 2. Most frequent approaches by educational grade in order of the percentage in which they appear. Note: I= Information; R= Reasoning; S= Strategies; O= Operations; A = Answer; C = Checking.

Discussion

International assessments of competence in mathematics, such as the TIMSS, are usually interpreted as a measure of the efficacy of educational systems in many countries. Countries such as Singapore, whose learners performed at an excellent level in the latest TIMSS, could be taken as a reference to analyse some important elements of other educational systems, among them Spain, whose children performed at a lower level.

The main question of our study was whether the educational system in Spain used textbooks with the quality necessary for sustaining adequate instruction in relation to a central aspect (learning how to solve word problems) of one of the competencies assessed by the TIMSS (mathematics). By adequate instruction, we mean instruction that allows children to solve problems through the genuine modelling proposed in the competent word problem-solving model proposed by Verschaffel et al. (2000). To do so, and with that cognitive model in mind, we compared the PSAPTs of the three most used textbooks in Spain with those of one of the most employed textbooks in Singapore, a country that can be considered a reference for the design of high-quality textbooks (Oates, 2014). Two different aspects were analysed in this comparison. First, the number genuine modelling steps in the PSAPTs was determined (by counting the number of cases wherein the approach shows how to understand the situational and mathematical structure of the problem, how to propose the adequate problem-solving strategies and how to check the answer according to the situational and mathematical understanding of the problem); second, the presence of PSAPTs in which a specific sequence of steps occurred was identified (examples of less and more complete sequences would be 'Information-Operations-Answer' and 'Information-Reasoning-Strategies-Answer-Checking', respectively). To do so, we used the system of analysis previously employed by Sánchez and Vicente (2015), which includes six categories based on the different phases of the theoretical model for problem solving proposed by Verschaffel et al. (2000): Information, Reasoning, Strategies, Operations, Answer and Checking.

Our hypothesis was that the PSAPTs in the Singaporean textbook would be more genuine than those of the Spanish textbooks, which implied that no differences would be found between the Spanish and Singaporean textbooks in terms of Information, Operations and Answer steps but that more Reasoning, Strategies and Checking steps would appear in the Singaporean textbook, indicating a higher percentage of PSAPTs that included these steps. These differences were also expected to be found in the three educational grades into which the six years of study were grouped, although we predicted that these differences would gradually decrease and even disappear at the higher grades.

Can the Singaporean PSAPTs be considered more genuine than the Spanish PSAPTs?

The results only partially confirmed our predictions. Regarding the first prediction, the PSAPTs in the Singaporean textbook presented more reasoning than those in the Spanish textbooks if we rely on the higher percentage of PSAPTs in which each of the six categories of steps appeared but not a greater number of strategies or checking steps. This finding implies that the Singaporean approaches were more genuine than the Spanish approaches, mainly because of the inclusion of mathematical reasoning, but not as much because of the type of strategies proposed or the verification of the solution. This greater call for reasoning was most likely due to the problem-solving teaching method on which the Singaporean textbook is based, in which mathematical graphic representations and, more specifically, the bar representation, play a fundamental role. In fact, these bar models made up 85% of the Reasoning steps found in the textbooks. To better understand the role of these graphic representations in the way in which children learn how to solve word problems, we can take the two sample models shown in the Methods section. The first model, which was taken from the Singaporean primary education textbook, includes a graphic representation to prompt the children to reason that the total has three equal parts, one of which belongs to Mr. Fong, which is the key to the problem. However, the second example, taken from a Spanish textbook, posits the problem-solving process in a more symbolic way than the Singaporean problem, even though, as seen in Figure 2, it instructs children to 'Represent the information with drawings.' In fact, whereas in the Singaporean PSAPT, a 'drawing' is understood to be a schematic representation of the mathematical structure of the problem, in the Spanish textbook, it is understood as an alternative way of presenting the 'X' that is typical in mathematical equations.

Furthermore, even though some of the steps related to the genuine approach (i.e., strategies and checking) were more present in Spanish approaches, the lack of reasoning stimulated by the Spanish textbooks undermines the contribution of these processes to a truly genuine resolution of the problem. First, although Spanish textbooks propose more approaches that include strategies than the Singaporean textbook, the high variability of the strategies proposed by the Spanish textbooks leads us to think that these strategies are not proposed with the real intention that the children learn to apply them, but as specific suggestions that appear in isolation as an example of how to solve specific problems, which are practised immediately after the strategy has been taught, but do not reappear in the textbook. This method contrasts with the (highly) systematic way in which the Singaporean use of 'bar representations' is shown and trained repeatedly throughout elementary school (and probably beyond). Second, the proportion of PSAPTs in which the checking step appeared was greater in the Spanish textbooks than in the Singaporean textbook, but the fact that, in most cases, these checking steps did not form part of a PSAPT in which reasoning was also included limited this verification to the execution of the arithmetic operation, which does not contribute to the genuine process of resolution.

Furthermore, the Singaporean textbook presented better scaffolding than the Spanish textbooks throughout the grades. As predicted, the scaffolding for word problem solving, especially regarding Reasoning and Checking, decreased (at least, to some degree) as the children moved to the upper grades in the Singaporean textbook, supporting the internalization of the competent problem-solving model. However, the trend in Spanish textbooks was the reverse: the higher the grade was, the more steps related to the genuine processing that were included as steps of the different approaches, which might express the (wrong) claim that younger children are not yet developmentally able to grasp and acquire these problem-solving hints.

In sum, the results obtained in this research, analysed with the chosen cognitive model (Verschaffel et al., 2000), suggest that the textbooks used in Singapore can be considered adequate teaching material for upholding the pedagogical basis for learning problem solving, whereas the textbooks used in Spain do not seem to be as suitable because they foster the learning of more superficial than genuine approaches, and they provide less adequate scaffolding for word problem solving throughout grade levels. Given that textbooks play a central role in the development of the mathematics curriculum, i.e., they represent such a strong specification of the curriculum that they in fact determine to a great extent what is actually taught in class (Oates, 2014), some revision of the Spanish textbooks seems to be necessary to ensure that there is real compliance with the curriculum, especially in relation to the development of the ability to solve word problems in a genuine way.

That said, it should be underscored that there is still room for improvement of the Singaporean textbook in two different ways. First, although it is true that the Singaporean textbook proposes models from which the children can learn to reason, that reasoning is exclusively mathematical. Thus, to increase even further the genuine nature of the PSAPTs in the Singaporean textbook, situational reasoning should be included as part of the problem-solving process, including situationally difficult problems (for example, the problematic items described by Verschaffel et al., 2000) in the instructional regimen of children. Second, in line with the results by Fan and Zu (2007), the completeness of the PSAPTs in Singapore could be improved by more systematically including checking as a final step of the word problem-solving process, especially in grades 3 to 6.

In short, it does seem clear that, in relation to PSAPTs, the differences found between the Singaporean textbook and the Spanish textbooks are very important and suggest that the former may be considered to have better scaffolding for high-quality learning to solve word problems than the latter.

Limitations and future studies

This research has some limitations that should be considered when interpreting the conclusions, and they point to the need for additional studies. It is necessary to emphasize that the scope of the conclusion drawn in this research is limited since, although its object of analysis is an important part of textbooks, it is also true that it is a small part. For this reason, the object of study should be extended to other important aspects of textbooks, such as the diversity - in terms of the semantic-mathematical structures - of the problems they propose for practising the models analysed, the instructional help they provide the children for solving the problems in a genuine way, and the cognitive level (as defined by the TIMSS) of other mathematical tasks besides word problems. It would be very much of interest to know whether, as one would expect, the word problems included in the editions of the Spanish textbooks analysed here can be solved superficially in most cases.

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