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Fire-hose instability of inhomogeneous plasma flows with heat fluxes

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E. S. Uchava,^{1,2,3} A. G. Tevzadze,^{2,4,a)} B. M. Shergelashvili,^{2,5,6} N. S. Dzhililov,⁷ and S. Poedts^{8,9}

AFFILIATIONS

¹Faculty of Exact and Natural Sciences, Tbilisi State University, Tbilisi 0179, Georgia

²Evgeni Kharadze Georgian National Astrophysical Observatory, Abastumani 0301, Georgia

³Nodia Institute of Geophysics, Tbilisi State University, Tbilisi 0171, Georgia

⁴Kutaisi International University, Kutaisi University Campus, Kutaisi 4600, Georgia

⁵Space Research Institute, Austrian Academy of Sciences, 8042 Graz, Austria

⁶Centre for Computational Helio Studies, Faculty of Natural Sciences and Medicine, Ilia State University, Tbilisi 0162, Georgia

⁷Shamakhy Astrophysical Observatory of the ANAS, Baku AZ-1000, Azerbaijan

⁸Center for Mathematical Plasma Astrophysics, KU Leuven, B-3001 Leuven, Belgium

⁹Institute of Physics, University of Maria Curie-Skłodowska, PL-20-031 Lublin, Poland

^{a)}Author to whom correspondence should be addressed: aleko@tevza.org

ABSTRACT

We study the effects of heat flows and velocity shear on the parallel firehose instability in weakly collisional plasma flow. For this purpose, we apply an anisotropic 16-moment MHD fluid closure model that takes into account the pressure and temperature anisotropy, as well as the effect of anisotropic heat flux. The linear stability analysis of the firehose modes is carried out in the incompressible limit, where the MHD flow is parallel to the background magnetic field, while the velocity is sheared in the direction transverse to the flow direction. It seems that an increase in the velocity shear parameter leads to higher growth rates of the firehose instability. The increase in the instability growth rate is most profound for perturbations with oblique wave-numbers $k_{\perp}/k_{\parallel} < 1$. Combined action of the velocity shear and heat fluxes introduces an asymmetry of the instability growth in the shear plane: perturbations with wave-vectors with a component in the direction of the velocity shear grow significantly stronger as compared to those with components in the opposite direction. We discuss the implications of the presented study on the observable features of the solar wind and possible measurements of local parameters of the solar wind based on the stability constraints set by the firehose instability.

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I. INTRODUCTION

It is well known that in low density, ionized, rarefied flows, magnetic field effects can dominate over the particle collision effects. Such systems can exhibit different values of temperature and pressure when measured along and normal to the direction of the background magnetic field. In such flows, the anisotropic effect of the gyration of the charged particles around magnetic field lines dominates over the isotropic particle collision process leading to a plasma with anisotropic thermodynamic properties.

Anisotropic ionized flows are prone to a number of kinetic instabilities that tap energy from the magnetic anisotropy and grow due to various destabilization mechanisms. Flows where the thermal energy dominates over the magnetic energy are prone to the firehose

instability.^{1–5} When the pressure parallel to the magnetic field is sufficiently higher than the perpendicular pressure, transverse kink perturbations of the magnetic field can become unstable and grow exponentially in time. Also, it has been shown that in viscous dissipation, anisotropy plays a significant role in the heating of the solar coronal plasma shear flows,⁶ even when other effects are abandoned.

The firehose instability has gained much attention recently,^{7–15} since it is believed to be one of the primary kinetic instabilities affecting the solar and stellar wind dynamics. The importance of the firehose instability in various astrophysical situations is also recognized.¹³ Still, much of the observational evidence about the development of the instability comes from the measurements of pressure anisotropies in solar wind fluctuations.^{16–20}

It has been shown that the nonlinear development of the microscopic firehose instability can affect the large-scale dynamics of astrophysical plasmas.²¹ A more detailed analysis of the instability development at nonlinear amplitudes can be conducted by means of numerical simulations.^{13,22} Moreover, direct numerical simulations of the classical firehose instability using a hybrid-kinetic numerical approximation confirm the predictions of the standard linear theory on the exponentially growing instability.²³ In the isotropic limit, the plasma nonequilibrium thermalization manifested by periodic or rapid aperiodic variations of the system entropy in time leads to strong coupling of different MHD wave modes (described in a similar mathematical framework as for waves in shear flows) and also to the development of parametric instabilities.²⁴

The theoretical framework for many of the advances in the analysis of magnetically anisotropic flows using a fluid description is the adoption of the Chew–Goldberger–Low (CGL) approximation.²⁵ This limit exploits the simplicity of a fluid description by using a closure model that leads to two different equations of state, viz., parallel and perpendicular to the magnetic field. Thus, the CGL limit is often referred to as a double adiabatic anisotropic MHD description.²⁶

The purpose of the present paper is to study the properties of the firehose instability in weakly magnetized anisotropic shear flows with heat fluxes. Waves and instabilities of anisotropic MHD shear flows have been studied in the framework of the nonmodal approach in the CGL limit.^{27,28} The CGL closure model neglects heat fluxes as low frequency phenomena and uses the double adiabatic approximation for the analysis of higher frequency modes. On the other hand, the velocity shear of the flow is acting on the shearing time scales: these time scales are usually much longer than the short time scales described within the CGL limit and are comparable to the time scales of thermal processes. Therefore, we here employ a so-called 16-moment anisotropic MHD model that can be effectively used to analyze low frequency phenomena in such flows.^{29,30} This approximation enables us to study anisotropic plasmas with heat fluxes using the fluid description. The method is an extension of the 13-moment Grad method³¹ for the case, taking into account higher order dispersive effects. These include two anisotropic heat flux vectors that describe transport of parallel and perpendicular thermal energies and viscosity tensor that describes the collision-free viscosity effect. Viscous contribution can be often neglected in the low frequency limit thus leading to the fluid model that represents the apparent generalization of the CGL-MHD model with the addition of anisotropic heat fluxes. Indeed, the 16-moment formalism has already proven to be successful in analyzing waves and instabilities in weakly collisional media.^{32–36}

In this paper, we present the stability analysis of the low frequency incompressible perturbations to the anisotropic MHD shear flows with heat fluxes. Effects of the velocity shear and heat fluxes on the dynamics of linear perturbations are characterized by lower frequency compared to the compressibility effects. Thus, we adopt an incompressibility approximation and study the properties of fire-hose instability when compressibility effects can not directly affect this instability in the linear limit.

The physical model of the problem is described in Sec. II, where the steady-state flow is described. Section III presents the linear perturbations and the stability analysis of both the static case and the sheared flows are described. The effects of the velocity shear and anisotropic heat fluxes are summarized in Sec. III.

II. PHYSICAL MODEL

The incompressible anisotropic MHD system can be described in the 16-moment approximation by using the following simplified fluid model:^{29,30,35}

$$\frac{d\mathbf{V}}{dt} + \frac{\nabla P_{\perp}}{\rho} + \frac{\mathbf{B} \times (\nabla \times \mathbf{B})}{4\pi\rho} = \frac{\nabla_{\parallel}}{\rho} \left((P_{\perp} - P_{\parallel}) \frac{\mathbf{B}}{B^2} \right), \quad (1)$$

$$\frac{d\mathbf{B}}{dt} - \nabla_{\parallel} \mathbf{V} = 0, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

and the incompressibility condition

$$\nabla \cdot \mathbf{V} = 0, \quad (4)$$

where the following notations are used for the shortness

$$d/dt \equiv \partial/\partial t + \mathbf{V} \cdot \nabla,$$

$$\nabla_{\parallel} \equiv \frac{\mathbf{B} \cdot \nabla}{B}.$$

The equations of state in the parallel and perpendicular to the magnetic field directions now include the heat fluxes

$$\frac{d}{dt} \left(\frac{P_{\parallel} B^2}{\rho^3} \right) = -\frac{B^2}{\rho^3} \left(\nabla_{\parallel} \left(\frac{S_{\parallel}}{B} \right) + \frac{2S_{\perp}}{B^2} \nabla_{\parallel} B \right), \quad (5)$$

and

$$\frac{d}{dt} \left(\frac{P_{\perp}}{\rho B} \right) = -\frac{B^2}{\rho} \nabla_{\parallel} \left(\frac{S_{\perp}}{B^2} \right). \quad (6)$$

The CGL MHD equations can be derived by setting the heat flux parameters to zero ($S_{\perp} = S_{\parallel} = 0$) in Eqs. (5) and (6). The full closure of the system of Eqs. (1)–(6) can be accomplished through the 16-moment closure model that provides two more equations for the heat fluxes, namely,

$$\frac{d}{dt} \left(\frac{S_{\parallel} B^3}{\rho^4} \right) = -\frac{3P_{\parallel} B^2}{\rho^4} \nabla_{\parallel} \left(\frac{P_{\parallel}}{\rho} \right), \quad (7)$$

and

$$\frac{d}{dt} \left(\frac{S_{\perp}}{\rho^2} \right) = -\frac{P_{\parallel}}{B\rho^2} \left(\nabla_{\parallel} \left(\frac{P_{\perp}}{\rho} \right) + \frac{P_{\perp} P_{\perp} - P_{\parallel}}{\rho P_{\parallel} B} \nabla_{\parallel} B \right). \quad (8)$$

For the sake of completeness, we show a connection between the fluid velocity of the flow $\mathbf{V}$ and the velocity of individual particles $\mathbf{u}$ in the kinetic regime using the particle distribution function $f(\mathbf{u}, \mathbf{r}, t)$

$$\mathbf{V} = \left(\int \mathbf{u} f d^3 \mathbf{u} \right) / \left(\int f d^3 \mathbf{u} \right).$$

Hence, parallel and perpendicular heat fluxes can be defined using the third moments as follows:

$$S_{\parallel} = \frac{m}{2B^2} \int d^3 \mathbf{u} (\mathbf{u} \cdot \mathbf{B})^2 u_{\parallel} f,$$

$$S_{\perp} = \frac{m}{2} \int d^3 \mathbf{u} \left(u^2 - \frac{(\mathbf{u} \cdot \mathbf{B})^2}{B^2} \right) u_{\perp} f,$$

where m and $\mathbf{u}$ are the mass and velocity of individual particles, while $u_{\parallel}$ and $u_{\perp}$ stand for the particle velocity components parallel and perpendicular to the magnetic field, respectively.

A. Steady-state equilibrium flow

We consider a stationary, weakly collisional MHD flow parallel to the uniform background magnetic field in the x -direction, with a velocity profile that is sheared linearly in the transverse (y -) direction

$$\mathbf{V}_0 = (Ay, 0, 0), \quad \mathbf{B}_0 = (B_0, 0, 0), \quad (9)$$

where A is the transverse shear parameter of the parallel velocity. Assuming the similar physical origins of the pressure and heat flux anisotropy in the rarefied flow, we introduce the pressure anisotropy parameter as follows:

$$\alpha = P_{\perp 0}/P_{\parallel 0} = S_{\perp 0}/S_{\parallel 0}. \quad (10)$$

The pressure and heat flux anisotropy parameters may be different due to the contribution from different plasma species (e.g., protons and electrons). Still, we neglect these differences within the adopted one's fluid description and use pressure anisotropy parameter as a guiding anisotropy factor for further analysis. The described flow matches the average configuration of the solar and stellar winds locally, where the flow convexity and turbulent component can be neglected. Indeed it is known that, while being stable global configurations, such flows can exhibit a number of micro-instabilities depending on the magnetic field, anisotropy, and heat flux parameters.

III. LINEAR STABILITY ANALYSIS

For the purpose of stability analysis, we introduce linear perturbations of the background incompressible parallel shear flow embedded in a uniform magnetic field as follows:

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_0 + \mathbf{V}', & \mathbf{B} &= \mathbf{B}_0 + \mathbf{B}', \\ P_{\parallel} &= P_{\parallel 0} + P'_{\parallel}, & P_{\perp} &= P_{\perp 0} + P'_{\perp}, \\ S_{\parallel} &= S_{\parallel 0} + S'_{\parallel}, & S_{\perp} &= S_{\perp 0} + S'_{\perp}, \end{aligned} \quad (11)$$

where $\mathbf{V}' \ll \mathbf{V}_0$, $\mathbf{B}' \ll \mathbf{B}_0$, etc., ρ is a uniform constant density, and

$$C_{\parallel}^2 \equiv P_{\parallel 0}/\rho, \quad C_{\perp}^2 \equiv P_{\perp 0}/\rho. \quad (12)$$

In the present paper, we study the effect of background stationary heat fluxes on the stability of local fire-hose instability. Thus, we introduce linear perturbations over the nonzero heat fluxes:

$$S_{0\parallel} \neq 0, \quad S_{0\perp} \neq 0.$$

Defining the Alfvén velocity as

$$V_A = B_0/\sqrt{4\pi\rho}, \quad (13)$$

we may introduce parallel and perpendicular plasma beta parameters as follows:

$$\beta_{\parallel} = 4\pi P_{\parallel 0}/B_0^2, \quad \beta_{\perp} = 4\pi P_{\perp 0}/B_0^2. \quad (14)$$

Effects of the heat fluxes can be described by the nondimensional heat flux parameters³⁵

$$\gamma_{\parallel} = \rho S_{\parallel 0}/P_{\parallel 0}^2, \quad \gamma_{\perp} = \rho S_{\perp 0}/P_{\perp 0}^2. \quad (15)$$

In the present paper, we introduce parallel and perpendicular heat flux parameters that also account for the magnetic field of the plasma

$$q_{\parallel} = 2\gamma_{\parallel}\beta_{\parallel}^{1/2}, \quad q_{\perp} = 2\gamma_{\perp}\beta_{\perp}^{1/2}. \quad (16)$$

Hence, upon substituting Eq. (12) into the Eqs. (1)–(6) and neglecting all the nonlinear terms, we can obtain the linear system of partial differential equations describing the dynamics of perturbations in the anisotropic MHD flow. This system can be analyzed in the shearing sheet limit, where spatial Fourier expansion with time dependent wave-numbers can be employed as follows:

$$\begin{pmatrix} P'_{\parallel}(\mathbf{r}, t)/P_{\parallel 0} \\ P'_{\perp}(\mathbf{r}, t)/P_{\perp 0} \\ S'_{\parallel}(\mathbf{r}, t)/P_{\parallel 0} \\ S'_{\perp}(\mathbf{r}, t)/P_{\perp 0} \\ \mathbf{V}'(\mathbf{r}, t)/V_A \\ \mathbf{B}'(\mathbf{r}, t)/B_0 \end{pmatrix} \propto \begin{pmatrix} ip_{\parallel}(\mathbf{k}, \tau) \\ ip_{\perp}(\mathbf{k}, \tau) \\ s_{\parallel}(\mathbf{k}, \tau) \\ s_{\perp}(\mathbf{k}, \tau) \\ \mathbf{v}(\mathbf{k}, \tau) \\ i\mathbf{b}(\mathbf{k}, \tau) \end{pmatrix} \exp\left(i\mathbf{k}(\tau)\frac{\mathbf{r}}{L}\right). \quad (17)$$

Here, L corresponds to the characteristic length-scale of the flow and τ is the nondimensional time variable

$$\tau = tV_A/L, \quad (18)$$

$k_y(\tau)$ is the nondimensional shearing wave-number of the perturbation harmonics

$$k_y(\tau) = k_{y0} - Rk_x\tau,$$

and R is the normalized velocity shear parameter

$$R = AL/V_A.$$

Spatial Fourier Harmonics of the physical quantities ($p_{\parallel}$, $p_{\perp}$, $s_{\parallel}$, $s_{\perp}$, etc.) derived from Eq. (17) are in general complex valued functions. Still, there are specific phase differences between different quantities in the complex plane. In uniform compressible flow without heat fluxes, the phase of pressure and velocity perturbations differs by $\pi/2$. This means that perturbations with real valued velocity harmonics will have purely complex pressure harmonics. To account for these standard differences, we introduce velocity and heat flux harmonics with real and pressure and magnetic field harmonics with purely complex factors [see Eq. (17)].

In this framework, we may derive the system of differential equations governing the linear dynamics of perturbation harmonics of the incompressible anisotropic shear flow system

$$\dot{v}_x(\tau) = Rv_y(\tau) - \beta_{\perp}\frac{k_{\perp}^2}{k_x}p_{\perp}(\tau) - \frac{\Delta\beta k_x^2 + k^2}{k_x}b_x(\tau), \quad (19)$$

$$\dot{v}_y(\tau) = \beta_{\perp}k_y p_{\perp}(\tau) + k_y b_x(\tau) - (1 + \Delta\beta)k_x b_y(\tau), \quad (20)$$

$$\dot{p}_{\perp}(\tau) = k_x v_x(\tau) + Rb_y(\tau) - k_x s_{\perp}(\tau) + iq_{\perp}k_x b_x(\tau), \quad (21)$$

$$\begin{aligned} \dot{s}_{\parallel}(\tau) &= -\frac{3iq_{\parallel}}{2}k_x v_x(\tau) + 6Rv_y(\tau) - 3\beta_{\perp}\frac{k_{\perp}^2}{k_x}p_{\perp}(\tau) \\ &\quad - 3\frac{2\Delta\beta k_x^2 + k^2}{k_x}b_x(\tau) - \frac{3iq_{\parallel}}{2}Rb_y(\tau), \end{aligned} \quad (22)$$

$$\dot{s}_{\perp}(\tau) = \beta_{\parallel}k_x p_{\perp}(\tau) + \Delta\beta k_x b_x(\tau), \quad (23)$$

$$\dot{b}_x(\tau) = Rb_y(\tau) + k_x v_x(\tau), \quad (24)$$

$$\dot{b}_y(\tau) = k_x v_y(\tau), \quad (25)$$

$$0 = k_x b_x(\tau) + k_y b_y(\tau) + k_z b_z(\tau). \quad (26)$$

where

$$\begin{aligned} k_{\perp}^2 &= k_y^2 + k_z^2, \\ k^2 &= k_x^2 + k_{\perp}^2, \\ \Delta\beta &\equiv \beta_{\perp} - \beta_{\parallel}, \end{aligned} \quad (27)$$

and the dot denotes the temporal derivative, e.g., $\dot{\psi}(\tau) \equiv d\psi/d\tau$. Despite the fact that we have normalized perturbation harmonics to remove complex parameters due to intrinsic phase differences, Eqs. (19)–(26) still explicitly include complex parameters. Complex parameters are due to heat fluxes $q_{\perp}$ and $q_{\parallel}$. Equation (21) indicates that for nontrivial solutions ($p_{\perp} = s_{\perp} = v_x = b_x = b_y = 0$), pressure, velocity, magnetic field, and heat flux perturbations cannot be simultaneously real valued variables. Thus, heat fluxes to the anisotropic flow break intrinsic phase differences between pressure and velocity harmonics and introduce nonremovable phase differences between the harmonics of different physical quantities. Note that the incompressibility condition [Eq. (4)] is already applied in Eqs. (19)–(26). Here, we have chosen the perpendicular pressure perturbation component $p_{\perp}$ for further analysis. Parallel pressure perturbation can be calculated from the condition $d/d\tau(\mathbf{k} \cdot \mathbf{v}) = 0$. Interestingly, none of the Eqs. (19)–(21) and (23)–(26) contain parallel heat flux perturbation ($s_{\parallel}$) or parallel heat flux parameter ($q_{\parallel}$). Thus, under the considered approximation, the ordinary differential equation (ODE) system (19)–(26) can be split into two parts: parallel heat flux perturbations are driven by other perturbations [see Eq. (22)]. This choice defines that only the perpendicular heat flux parameter appears in further analysis. Still we note that in the present case, parallel and perpendicular heat flux parameters are not independent parameters and are linked through Eqs. (10) and (16).

A. Firehose instability in the static case

In the zero-shear limit ($R = 0$), k_y and thus also all coefficients in the differential equations become time-independent. Hence, we may use a spectral expansion of the linear perturbations in time $\propto \exp(i\omega\tau)$ and obtain the dispersion equation of the incompressible, anisotropic MHD system, which reads as follows:

$$[\omega^2 - (1 + \Delta\beta)k_x^2]D_0 = 0, \quad (28)$$

where

$$\begin{aligned} D_0 &\equiv \omega^4 - (1 + \beta_{\perp})k^2\omega^2 + \alpha\beta_{\perp}q_{\perp}k_x k_{\perp}^2\omega \\ &\quad + \left[(1 + \Delta\beta)k_x^2 + (1 + \beta_{\perp} - \alpha^2\beta_{\parallel})k_{\perp}^2\right]\beta_{\parallel}k_x^2. \end{aligned} \quad (29)$$

The first obvious solution of this dispersion equation is given by the firehose mode

$$\omega_{F\pm} = \pm c_F k_x, \quad (30)$$

where

$$c_F^2 \equiv 1 + \Delta\beta, \quad (31)$$

stands for the square of the characteristic speed of the firehose mode, when positive. It seems that kink deformations of the parallel magnetic

structures that are described by the firehose mode do not feel the effect of heat fluxes ($q_{\parallel}$, $q_{\perp}$). Hence, the linear stability criterion for firehose instability ($c_F^2 < 0$) can be set by the balance of the parallel and perpendicular plasma beta parameters^{4,5}

$$\beta_{\parallel} > 1 + \beta_{\perp}. \quad (32)$$

The growth of linear perturbations is described by ω_{F+} or ω_{F-} , depending on whether the streamwise wave-number k_x is positive or negative, respectively.

The mechanism of the classical microscopic anisotropic MHD instability can be illustrated using a simple sketch shown in Fig. 1. Transverse perturbations of the magnetic field ($B'_{\perp} \neq 0$) can be shown as kink perturbations of the parallel magnetic structure. The response to this perturbation consists of a combined action of the parallel and perpendicular pressure and the magnetic field. If the perpendicular pressure is lower than the parallel pressure to the extent that even the magnetic field cannot compensate for the kink deformation, the perturbation will grow and instability will be developed. Interestingly, the heat fluxes ($S_{\parallel}$, $S_{\perp}$) do not affect this process, at least not in the static/uniform flow limit.

Another set of modes in Eq. (28) are described by $D_0 = 0$ solutions: these are incompressible slow modes modified by the thermal effects. In the static case, these modes are decoupled with the firehose instability and it do not explicitly affect their dynamics.

B. Firehose instability in nonuniform flows

The dynamic behavior of linear perturbations in shear flows ($R \neq 0$) is described by Eqs. (19)–(26). Employing the low frequency limit, we use the following approximation for time variation of perturbation harmonics

$$\dot{\psi}(\mathbf{k}, \tau) \approx -i\omega(\tau)\psi(\mathbf{k}, \tau). \quad (33)$$

FIG. 1. Sketch illustration of the firehose instability in the flow parallel to the magnetic field. Instability occurs when the transverse effective pressure acting on the perturbed magnetic tube is less than the parallel pressure: $P_{\perp 0} + B_0^2/4\pi < P_{\parallel 0}$ and $c_F^2 < 0$. In this case, transverse Alfvénic deformation of the magnetic field lines grows in time exponentially. The direction of the axis adopted for the description of the flow is shown in the left bottom corner. Arrows on the axis indicate the direction of the equilibrium heat fluxes, subject to linear perturbations.

This limit is somewhat similar to the adiabatic limit, when temporal variations of the background are slow enough ($R \ll 1$) to introduce a parameter similar to the spectral frequency $\omega(\tau)$. Such a method is widely used in the adiabatic approximation, where frequency varies slower than the oscillations it describes

$$|\dot{\omega}(\tau)| \ll \omega^2(\tau). \quad (34)$$

In our case, the source of time variation of frequency (or growth rate in the case of instability) is the velocity shear parameter leading to the wave-number variation in the shearing sheet limit

$$\omega(\tau) = \omega(k_x, k_y(\tau), k_z). \quad (35)$$

Using this assumption

$$\dot{\omega}(\tau) = \frac{\partial \omega}{\partial k_y} \frac{\partial k_y(\tau)}{\partial \tau}. \quad (36)$$

and substituting Eq. (36) into condition (34), we derive the following:

$$|\omega^2| \gg \left| R k_x \frac{\partial \omega}{\partial k_y} \right|. \quad (37)$$

In this limit, we can derive the adiabatic dispersion equation of the shear flow system and obtain

$$[\omega^2 - (1 + \Delta\beta)k_x^2]D_0 + iRk_x k_y D_1 = 0, \quad (38)$$

where

$$D_1 \equiv (1 + \beta_\perp)\omega^2 - \alpha\beta_\perp q_\perp k_x \omega + (\beta_\perp^2 - \beta_\parallel \beta_\perp - \beta_\parallel)k_x^2, \quad (39)$$

shows the modification to the standard dispersion equation. The implications of the velocity shear effects on the firehose instability set by Eq. (38) are obvious. Unlike the static/uniform flow result, the firehose solutions now do depend on the velocity shear R as well as on the heat flux parameter $q_\perp$. In addition, the velocity shear effect may lead to asymmetry of the growth rates in the shearing plane [see the last term of Eq. (38)].

Modes described by Eq. (38) can be coupled due to the velocity shear of the flow. Low frequency resonance can occur between unstable firehose and MHD wave modes. A similar effect was studied in the CGL limit²⁷ and it was shown that a moderate shear parameter is needed for such resonant coupling to occur.

To analyze the effect of shear flow on the classical firehose mode, we introduce the deviation from the static solution (30) as follows:

$$\omega = \omega_{F\pm} + R\omega_{1\pm}. \quad (40)$$

This limit can be especially useful in the low shear limit, when $R < 1$, i.e., the shearing timescale is much longer than the Alfvén wave time scales in the flow. Using Eq. (40) in condition (37) and recalling that fire-hose mode frequency $\omega_{F\pm}$ does not depend on k_y , we get the applicability condition of our adiabatic approximation

$$|\omega^2| \gg R^2 \left| k_x \frac{\partial \omega_{1\pm}}{\partial k_y} \right|. \quad (41)$$

Substituting Eq. (40) into Eq. (38) and neglecting terms of order higher than R^2 in the low shear limit ($R < 1$), we may derive the second-order dispersion equation with respect to $\omega_{1\pm}$ as follows:

$$RA_\pm \omega_{1\pm}^2 + (B_\pm + iRC_\pm)\omega_{1\pm} + iD_\pm = 0, \quad (42)$$

where

$$A_\pm = 4c_F^2(c_F^2 - \beta_\parallel)k_x^2 + (3\Delta_\pm - 2(1 + 2\beta_\perp))k_\perp^2, \quad (43)$$

$$B_\pm = 2\omega_{F\pm}\Delta_\pm k_\perp^2, \quad (44)$$

$$C_\pm = (1 + 2\beta_\perp - 2\Delta_\pm)k_x k_y, \quad (45)$$

$$D_\pm = -\omega_{F\pm}\Delta_\pm k_x k_y, \quad (46)$$

and

$$\Delta_\pm = \pm c_F \alpha \beta_\perp q_\perp + (1 - c_F^2)(1 + 2\beta_\perp) - c_F^2. \quad (47)$$

The solution of Eq. (38) should match the standard firehose solution in the zero shear limit, i.e.,

$$\omega(R=0) = \omega_{F\pm}.$$

This sets the following convergence requirement on the shear flow correction

$$\lim_{R \rightarrow 0} (R\omega_{1\pm}) = 0. \quad (48)$$

Hence, from the two solutions of the reduced dispersion equation (42), we can choose the one obeying the asymptotic condition (48)

$$\omega_{1\pm} = \frac{B_\pm + iRC_\pm}{2RA_\pm} \left[-1 + \left(1 - \frac{4iRA_\pm D_\pm}{(B_\pm + iRC_\pm)^2} \right)^{1/2} \right]. \quad (49)$$

Using the low shear limit ($R < 1$), we may separate the real and complex parts of the solution (49) for stable, unstable, and neutrally stable firehose modes

$$\omega = \pm c_F k_x + \delta_{1\pm} + i\sigma_{1\pm} \quad \text{when } c_F^2 > 0, \quad (50)$$

$$\omega = 2i\sigma_A \quad \text{when } c_F^2 = 0, \quad (51)$$

$$\omega = (\sigma_A + \delta_{2\pm}) + i(\pm|c_F|k_x + \sigma_{2\pm}) \quad \text{when } c_F^2 < 0, \quad (52)$$

where

$$\sigma_A \equiv R \frac{k_x k_y}{2k_\perp^2},$$

describes the shear flow induced transient growth of aperiodic perturbations, while the explicit forms of $\delta_{1\pm}$, $\delta_{2\pm}$, $\sigma_{1\pm}$, and $\sigma_{2\pm}$ are given in the Appendix. Applicability of the results derived in Eqs. (50)–(52) depends on the validity of the adiabatic approximation [see Eq. (41)]. The results are valid if the growth rate of the instability is larger than the velocity shear of the flow: $\omega^2 > R^2$.

IV. DISCUSSION

In order to show the effects of different physical factors and parameters on the firehose instability, we illustrate analytic solutions given by Eq. (52).

Figure 2 shows the comparison of the growth rates of the instability in a static case to flows with different values of the velocity shear parameter. It seems that the velocity shear effect is negligible for strictly parallel in the shear plane ($k_y = 0$) or perpendicular ($k_x = 0$) perturbations [see also Eq. (45)]. However, velocity shear induces a

FIG. 2. The growth rate of the firehose instability as a function of the angle ϕ of the wave vector with respect to the direction of the magnetic field in the shearing plane: $k_x = k_0 \cos(\phi)$ and $k_y = k_0 \sin(\phi)$ with $k_0 = 1$. Here, $\alpha = 0.5$, $\beta_{||} = 2$, $q_{\perp} = 0.2$, $k_z = 1$, $c_F^2 = -0.01$, and $R = 0.05, 0.2, 0.4$ (left to right). The horizontal axis corresponds to the direction parallel and the vertical axis corresponds to the direction perpendicular to the magnetic field, respectively. The red dashed line shows the instability growth rate for the static case ($R = 0$), while the solid blue line shows the velocity shear modification. It seems that an increase in the shear parameter leads to stronger instabilities for perturbations with oblique wave-numbers: $\phi \sim \pi/4$.

significant boost to the instability growth rates for perturbations with wave-vectors oblique to the magnetic field.

Figure 3 shows the growth rates of the instability in the static case and sheared flows for different values of the vertical wave-number. It seems that the effect of the velocity shear is most profound on the vertically uniform perturbations with $k_z = 0$. The increase in the instability growth rates is most profound for nearly parallel perturbations with $k_x/k \lesssim 1$.

Figure 4 shows the growth rates of the instability in flows with different heat flux parameters. Interestingly, the combined action of velocity shear and heat fluxes introduces an asymmetry of the firehose instability growth in wave-number space: perturbations having a transverse component pointing in the direction of the velocity shear ($k_y > 0$) are amplified, while the perturbations with wave vectors pointing in the opposite direction ($k_y < 0$) are somewhat suppressed. Notably weaker asymmetry is introduced in the streamwise direction: perturbations in the $k_x > 0$ area grow stronger in comparison to those in the $k_x < 0$ area.

Finally Fig. 5 shows the growth rates of the instability for different values of the firehose parameter c_F^2 . The effects of the velocity shear

are most profound for marginally unstable perturbations, while the violent firehose processes remain largely insensitive to the velocity shear modifications.

It seems that low frequency processes in the anisotropic flows that are marginally unstable to firehose perturbations can be significantly modified by the velocity shear as well as by heat fluxes of the MHD medium. The parallel firehose instability acquires a transverse component in shear flows and, hence, is affected by the heat fluxes. Thus, the observable features of the MHD fluctuations can be significantly modified in high shear regions of anisotropic flows.

In solar and stellar winds, the combined effect of the velocity shear and heat fluxes can be an additional source of turbulence and anomalous heating in rarefied magnetized outflows. The firehose instability can lead to enhanced turbulence in anisotropic MHD flows. Thus, sustained fluctuations of the solar wind can occur in areas of phase space where no instabilities are present. Therefore, observations of the fluctuations in the solar wind can be a powerful tool for analyzing the physical conditions in the wind using the stability considerations for local perturbations. Indeed, the firehose instability modified by the effects of velocity shear and heat fluxes can draw the energy of

FIG. 3. The growth rate of the firehose instability as a function of the angle ϕ of the wave vector with respect to the direction of the magnetic field. Here, $\alpha = 0.5$, $\beta_{||} = 2$, $q_{\perp} = 0.2$, $R = 0.05$, $c_F^2 = -0.01$, and $k_z = 0, 1, 2$ (left to right). It seems that the velocity shear effect is the strongest for vertically uniform perturbations ($k_z = 0$), while small scale perturbations with $k_z > 1$ remain largely insensitive to the velocity shear effects.

FIG. 4. The growth rate of the firehose instability as a function of the angle ϕ of the wave vector with respect to the direction of the magnetic field. Here, $\alpha = 0.5$, $\beta_{\parallel} = 2$, $k_z = 1$, $c_F^2 = -0.01$, $R = 0.2$, and $q_{\perp} = 0.2, 2, 10$ (left to right). It seems that an increase in the heat flux parameter ($q_{\perp}$) leads to an asymmetry of the instability growth in shear flows.

FIG. 5. The growth rate of the firehose instability as a function of the angle ϕ of the wave vector with respect to the direction of the magnetic field. Here, $\alpha = 0.5$, $\beta_{\parallel} = 2$, $k_z = 1$, $R = 0.4$ and $c_F^2 = -0.01, -0.1, -0.2$ (left to right). The velocity shear effects are most profound for weakly unstable perturbations. Otherwise the growth rate of firehose instability is lower than the velocity shear parameter $\sigma^2 < R^2$ and analytical results derived within the adiabatic approximation are not valid.

the background flow into turbulent fluctuations and, ultimately, into heat via dissipative effects which can be included in the solar/stellar models in terms of statistically proven macroscopic wave heating and pressure gradient quantities.³⁷ It is also interesting to elaborate on the role of the firehose instability in the dynamics of the smaller scale flows like solar coronal jets, for which quasi-oscillatory precursors in the mean intensity variations recently have been observed³⁸ that pretend to be triggers of the instability processes occurring in coronal bright points.

Thus, the analytic solutions derived in the present paper can be used to study the features of the solar wind fluctuations by deducing the physical conditions in the outflow, such as the parameters of the heat fluxes and the azimuthal velocity shear of the radial outflow.

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APPENDIX: FIREHOSE SOLUTION IN NONUNIFORM FLOWS

The modification of firehose solutions in nonuniform flows can be described in a low shear limit ($R < 1$) using Eqs. (50)–(52).

In these equations, we have introduced a number of notations for the shortness of the presentation in the main text of the article. Shear flow modification in the stable ($c_F^2 > 0$) flow configuration to the real

$$\delta_{1\pm} = \frac{k_{\perp}(t)^2}{8c_F k_x A_{\pm} \Delta_{\pm}} \left\{ M_{1\parallel} k_x^2 + \frac{|k_x \Delta_{\pm}|}{k_x \Delta_{\pm}} \sigma_A^2 [2\beta_{\perp} + 1 - 2\Delta_{\pm}]^2 \right\}, \quad (\text{A1})$$

and imaginary parts of the frequency ω

$$\sigma_{1\pm} = \sigma_A \frac{P_{1\parallel} k_x^2 + P_{1\perp} k_{\perp}(t)^2}{2A_{\pm}}, \quad (\text{A2})$$

introducing

$$\eta \equiv \alpha |c_F| \beta_{\perp} q_{\perp}, \quad (\text{A3})$$

and

$$M_{1\parallel} = 8c_F^2 \Delta_{\pm}^2 \left(-1 + \frac{|k_x \Delta_{\pm}|}{k_x \Delta_{\pm}} \right), \quad (\text{A4})$$

$$P_{1\parallel} = 4 \left[(c_F^2 - 1)(c_F^2 - \beta_{\parallel} + 2) - \beta_{\perp} + 1 \right], \quad (\text{A5})$$

$$P_{1\perp} = \Delta_{\pm} + \left(1 + \frac{|k_x \Delta_{\pm}|}{k_x \Delta_{\pm}}\right) [\pm \eta - 2c_F^2(1 + \beta_{\perp})]. \quad (\text{A6})$$

387 Shear flow modification in the unstable flow configuration ($c_F^2 < 0$)
388 to the real

$$\delta_{2\pm} = -\sigma_A \alpha \beta_{\perp} q_{\perp} k_{\perp}(t)^2 \times \frac{(M_{2\parallel} + \eta^2 N_{2\parallel})k_x^2 + (M_{2\perp} + \eta^2 N_{2\perp})k_{\perp}(t)^2}{T_2}, \quad (\text{A7})$$

389 and imaginary parts of the frequency ω

$$\sigma_{2\pm} = \mp \frac{\sigma_A k_{\perp}(t)^2}{2|c_F|} \times \frac{(P_{2\parallel} + \eta^2 Q_{2\parallel})k_x^2 + (P_{2\perp} + \eta^2 Q_{2\perp})k_{\perp}(t)^2}{T_2}, \quad (\text{A8})$$

390 where

$$T_2 = \left[\eta^2 - (2c_F^2(1 + \beta_{\perp}) - (1 + 2\beta_{\perp}))^2\right] \times \left\{4c_F^2(c_F^2 - \beta_{\parallel})k_x^2 - (6c_F^2(1 + \beta_{\perp}) - (1 + 2\beta_{\perp}))k_{\perp}(t)^2\right\}^2 + 9\eta^2 k_{\perp}(t)^4, \quad (\text{A9})$$

$$M_{2\parallel} = 4c_F^2(c_F^2 - \beta_{\parallel})(-2\beta_{\perp}(-2c_F^2 + 1) - 4\beta_{\parallel} + 1) \times (2\beta_{\perp}(2c_F^2 + 4\beta_{\parallel} - 5) - 4\beta_{\parallel} + 3), \quad (\text{A10})$$

$$N_{2\parallel} = 16c_F^2 k_x^2 (c_F^2 - \beta_{\parallel}),$$

$$M_{2\perp} = 3 \left[1 + 2(c_F^2 + \beta_{\parallel})(2c_F^2 - 1)\right]^2 \left[2(c_F^2 - 1)(c_F^2 + \beta_{\parallel}) + 1\right] + \left[6(c_F^2 + \beta_{\parallel})(c_F^2 + 2\beta_{\parallel} - 2) + 1 + 2\beta_{\perp}\right] \times \left[-2\beta_{\perp}(-2c_F^2 + 1) - 4\beta_{\parallel} + 1\right] \times \left[2\beta_{\perp} \times (2c_F^2 + 4\beta_{\parallel} - 5) - 4\beta_{\parallel} + 3\right],$$

$$N_{2\perp} = 48(c_F^2 + \beta_{\parallel})(\beta_{\parallel} - 1) + 4 + 8\beta_{\perp},$$

$$P_{2\parallel} = 4(c_F^2 - \beta_{\parallel})c_F^2 \left[1 + 2(c_F^2 + \beta_{\parallel})(2c_F^2 - 1)\right]^2 \times \left[2(c_F^2 - 1)(c_F^2 + \beta_{\parallel}) + 1\right],$$

$$Q_{2\parallel} = 32c_F^4 (\beta_{\parallel}^2 - c_F^4),$$

$$P_{2\perp} = \left[6(c_F^2 + \beta_{\parallel})(c_F^2 + 2\beta_{\parallel} - 2) + 1 + 2\beta_{\perp}\right] \times \left[1 + 2(c_F^2 + \beta_{\parallel})(2c_F^2 - 1)\right]^2 (2c_F^2 - 1)(c_F^2 + \beta_{\parallel}) + 1),$$

$$Q_{2\perp} = 12\eta^2 + 3 \left[2\beta_{\perp}(-2c_F^2 + 1) - 4\beta_{\parallel} + 1\right] \times \left[2\beta_{\perp}(c_F^2 + 2\beta_{\parallel} - 2) + 1 + 2\beta_{\perp}\right] (c_F^2 + \beta_{\parallel})c_F^2 + 4\beta_{\parallel} - 5) - 4\beta_{\parallel} + 3] - 8 \left[6(c_F^2 + \beta_{\parallel})(c_F^2 + 2\beta_{\parallel} - 2) + 1 + 2\beta_{\perp}\right] \times (c_F^2 + \beta_{\parallel})c_F^2.$$

391 DATA AVAILABILITY

392 The data that support the findings of this study are available
393 within the article and its Appendix.

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