RESEARCH ARTICLE

Stochastic Distributed Optimization of Shapeable Energy Resources in Low Voltage Distribution Networks under Limited Communications

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Summary

Due to the rising renewable penetration rate, modern low voltage distribution network (LVDN) calls for active control with tractable computation and limited communication. To tackle this, the paper proposes a novel stochastic distributed optimization approach. The computation of optimum is completely decentralized, with a global broadcast signal is used as public reference in order to guarantee the consistency among individual shapeable energy resources. The proposed approach employs Bernoulli trials to imitate the searching process in classical gradient descent approach, and player compatible relationship is employed to play the role of gradients to indicate the direction of the search. Working in a model-free manner without relying on iterations, the proposed approach offers an approximate optimization to minimize the accumulated compensation of reshaping/deferring the shapeable energy resources in a given LVDN while respecting the system constraints. A 103 nodes test network based on a realistic Belgian semi-urban distribution network is used for validation. With two different profiles and a special case of communication failure, the proposed approach is validated and benchmarked with a classical AC optimal power flow algorithm. The results prove that the proposed approach is able to deliver a good approximation to the theoretical optimum with reasonable gap.

KEYWORDS:

Distributed optimization, Low voltage distribution network, Shapeable/deferrable energy resources, Player compatible relationship

1 | INTRODUCTION

With the development of distributed energy resources (DER) and smart devices, an increasing amount of new users are connected to the modern distribution system. To clear elaboration, in this paper, the term users denotes all the participants connected to the distribution system, which includes but not limited to photovoltaics (PV), residential wind turbines, smart households and so on. Most of the new users in modern distribution network have shapeable/deferrable profiles, they are called shapeable/deferrable energy resources in a distribution network. In the context of smart grid, these shapeable/deferrable energy resources call for active control¹ to have a more efficient operation. Moreover, different from conventional distribution networks, modern communication and computation technologies have extended active control to the end-user level of smart grid: Low voltage distribution network (LVDN). These new trends bring up the demand of optimizing the operations of the shapeable/deferrable users,

in order to increase the network reliability by implementing residential demand response² in LVDNs. Nevertheless, compared to conventional distribution network, active control and the corresponding optimization in LVDN come up with new challenges due to its numerous and multifarious users. Firstly, active control employs real-time control and communication system to effect a better integration of DERs in a LVDN. This normally relies on powerful communication infrastructures that are expensive in both construction and maintenance, especially when users are numerous^{3,4}. Secondly, limited by infrastructure, LVDN is not affluent in computational capability, while from the perspective of computation, more users implies more nodes in optimization problem, which normally leads to high computational complexity. Computational tractability therefore becomes a significant concern that needs to be addressed. Thirdly, as the end-user level of modern power system, LVDNs are organically modified or expanded with the expansion of cities more often than other levels. This brings difficulties to system modeling, which is critical in model-based optimization algorithms^{5,6,7}. Updating models does not effect a permanent cure, therefore model-free algorithms are more compatible with LVDNs. Last but not the least, on one hand, users behaviour in a LVDN is dynamic, which is hard to predict accurately, meanwhile, collecting the data of user behaviour risks infringing users' privacy, which is one of the main concerns about the implementation of smart grid⁸. On the other hand, managing a large group of users with ensuring system constraints and user satisfaction is the key objective of active control in LVDN⁹, which demands access to users behaviour data to some extent.

Distributed control approaches¹⁰ have potential to address the aforementioned challenges. As a classic approach, droop control is a reliable way to implement distributed control without communication¹¹. Nevertheless, most of the DERs have no droop character inherently and the control is affected by cable impedance. Virtual synchronous generators (VSG) can integrate droop control to DERs^{12,13}, while the control itself is passive such that it is not compatible with demand response. Most of, if not all, the key functionalities of demand response require active control with high communication demand^{14,15,16}. Therefore the suitable approach that can address the aforementioned challenges should be able to implement active control in a distributed manner under limited communications. In addition to that, model-free control is preferable as well. There are few studies in the existing literature addressing model-free control methods^{17,18}. And most of these model-free control methods are iterative methods. Generally speaking, iterative-based methods require several iterations to reach a decision, which increases the computational burden and communication overhead. Moreover, they might not reach convergence in case of communication failure. To make the computational burden of iterative-based methods light and to have higher reliability, some studies propose the use of "excitation and response" feedback schemes. Iterative feedback strategy (IFS) is used to improve the performance and the reliability of a distributed voltage control system. In IFS-based voltage control, at each iteration, each agent dynamically adjusts its reactive power (and/or active power), measures the voltage, and sends the new measurement as a feedback to the control system.

When distributed control works under limited communication condition, especially in model-free manner, the controls become stochastic due to the limited information acquisition¹⁹. There are some existing works on stochastic controls in energy system. Explicit Congestion Notification (ECN)²⁰ was originally used to choose routers in a communication network. Its applications in energy system has been proposed and implemented²¹ to control the distributed charging among electric vehicles in parking spots. Then this mechanism was extended again and renamed as "GECN (Grid Explicit Congestion Notification)"²², that used GECN to balance the usage of on-load tap changers (OLTC) and elastic loads in primary voltage control. Later, a hardware validation was provided by²³. Consensus-based approaches are proposed to solve distributed optimization problems by making different DERs converge to a common value. It can achieve global optimality via limited, time varying peer to peer communication. Some works on model predictive control (MPC) employed discrete time control approaches, in which the control is not only determined by minimizing a cost function based on current state, but also finite steps of future time employing a predictive model. It can handle multi-variable control problems with tunable parameters, and explicitly consider the constraints. However the computation load might be still too high for the users in LVDNs. Mean Field Theory (MFT) uses stochastic approaches to estimate the global status, which finds its applications in the distributed control of energy system now and then. For instance, MFT-based electrical vehicle charging control²⁴, demand response of thermostatically controlled loads in MFT manner²⁵, and demand management employing MFT in smart grid is studied as well²⁶, with a bang-bang switching control as its application. Nevertheless, as most of the MFT-based approaches only implement dualistic controls (on/off) among users, they need extra mechanisms when controlling shapeable users in LVDNs. A conceptual comparison of the approaches mentioned above is given by Table 1 with the corresponding references.

To further improve the computational tractability and reduce the communication demand of distributed optimization in LVDNs, this paper proposes a stochastic optimization method. Inspired by Stochastic Gradient Descent (SGD)⁴¹, the proposed approach is based on classical gradient descent approach but without collecting gradient information from the whole network. Instead of employing a randomly chosen gradient in SGD, the proposed approach makes users use a local pseudo-gradient.

TABLE 1 Comparison of the approaches in literature review

Approaches	Control structure	Computational overhead with number of users	Communication overhead with number of users	Model dependency	Active control
Droop control ^{11,27}	Decentralized	No growth	No / low growth	High, sensitive to R/X ratio of the network	No
Iterative feedback strategy ^{28,29,30}	Centralized	Iterations take more time or more rounds	Linear growth	Model-free	Yes
Explicit Congestion Notification ^{21,22,23}	Decentralized	No growth	Linear growth	Model based for demand response	Yes
Consensus-based approaches ^{31,32,33,34}	Hybrid	No / minor growth due to massive data	Dramatic growth or slow system response	Model based for fast response	Yes
Classical model predictive control ^{35,36,37,38}	Hybrid	High growth	Depends on control structure	Simplified model	Yes
Mean Field Theory approaches ^{39,40,25,26}	Decentralized	No growth but high anyway	Linear growth	Model-free	Yes
This paper	Decentralized	No growth	Almost no growth	Model-free	Yes

This local pseudo-gradient is given by player compatible relationship (PCR)⁴² and personalized to each individual user. PCR considers the preference and operation status of individual users as local stochastic parameters, and affects users' behaviour accordingly, which will be elaborated later. Consequently, the decision-making process of the control is completely decentralized and does not call for the support of massive communication in a centralized manner. The proposed approach considers the optimization of residential demand response in a dynamic programming way, which implements voltage control in the given LVDN while minimizing the accumulated cost for compensation. Bernoulli trials and Strong Law of Large Numbers (SLLN) are employed to guarantee the coherence among users. Compared to the mentioned existing works, the proposed approach is not based on iterations, and can achieve good approximation to the theoretical optimization results among multifarious users under harshly limited communication.

Comparing to the existing works, the main contribution of this paper is three-fold. Firstly, a novel model-free distributed method to deliver good approximation to the optimization result in LVDNs is proposed. All the personal information is strictly used locally, making it possible to establish a plug-and-play system that takes user preference into account while respecting consumer privacy. Secondly, the communication demand of the method is reduced, only simple measurements are needed, which makes the proposed method compatible with the control in most energy systems. Thirdly, a stochastic way is proposed to implement boundary conditions in optimization problems in a model-free manner without relying on iterations. The paper is structured as follows: the fundamental mechanism is elaborated in Section 2; Section 3 describes the detailed implementation in LVDNs; Section 4 provides the validation of voltage control in a 103 nodes test system with two different profiles; and Section 5 concludes the paper.

2 | FUNDAMENTAL MECHANISM

The actions of the users in this paper are determined by Bernoulli trials. A Bernoulli trial has two possible outcomes: 0 or 1. The user will interpret these values respectively as doing nothing or adjusting its output, and take an action accordingly.

Each user i has its own consumption (generation is regarded as negative consumption) ζ_t^i at time t , which is given by:

$$\zeta_{t+1}^i = \zeta_t^i + u_t^i, \quad (1)$$

where u_t^i is the adjustment of user i . Assume that every user i has a scheduled consumption profile $\{\zeta_t^{i'}\}$. If there is no external control implemented, then ζ_{t+1}^i is given by

$$\zeta_{t+1}^i = \zeta_t^i + u_t^i, \text{ where } u_t^i = \zeta_{t+1}^{i'} - \zeta_t^i. \quad (2)$$

Nevertheless, the system constraints exist. It might not be possible to let every user to implement $\zeta_t^{i'}$ according to its $\{\zeta_t^{i'}\}$ with respecting the voltage constraints. Therefore u_t^i has to be reshaped/deferred. For users who get reshaped/deferred, a compensation should be given, whose price is denoted by $\varepsilon_t^i > 0$ and varies with time and among users. From the perspective of the distribution network operators (DSO), consequently there is a cost can be written as

$$J_t = \sum_{i=1}^n \varepsilon_t^i \cdot (\zeta_t^{i'} - \zeta_t^i)^2, \quad (3)$$

where n is the total amount of the users under control. Theoretically, the DSO will want to minimize

$$J = \int_0^T J_t dt, \quad (4)$$

where $[0, T]$ is the whole time scope of the control. The practical meaning of (4) is that the DSO wants to make every user in the LVDN execute its desired consumption as much as possible with ensuring the voltage constraints. According to the Bellman Principle of optimality, to minimize (4), all the (3) need to be minimized successively⁴³. In conventional centralized approaches, this can be done by collecting all the ζ_t^i and ζ_{t+1}^i from users, and all the nodal voltage v_t^i , solving an optimization problem with the model of the grid. Nevertheless, this needs complete information and powerful computational capability. In this paper, the changes from ζ_t^i to ζ_{t+1}^i are regarded as a stochastic process. When user i wants to change its consumption from ζ_t^i to ζ_{t+1}^i , it will do a Bernoulli trial according to

$$R_t^i = T(1 - B_t^i), \quad (5)$$

where T denotes Bernoulli trial and R_t^i is its result, with $R_t^i \in \{0, 1\}$ as mentioned before. B_t^i is the probability parameter for user i , which is given by

$$B_t^i = [(1 - |\theta_t^i|) \cdot (-\varepsilon_t^i) + |\theta_t^i| \cdot |\theta_t^i|]_+, \quad (6)$$

where θ_t^i is the local nodal voltage parameter, and given by

$$\theta_t^i = (1 - \sin(\frac{e^{-\frac{\mu}{\alpha}}}{2} \pi)) \cdot \text{sgn}(\mu), \text{ where } \mu = v_t^i - v^r, \quad (7)$$

where sgn stands for the sign of μ , v^r is the rated voltage of the given LVDN, the tunable parameters α and β will be discussed later on in the Remark.1. Then (2) becomes

$$\zeta_{t+1}^i = \zeta_t^i + u_t^i \cdot R_t^i, \text{ where } u_t^i = \zeta_{t+1}^{i'} - \zeta_t^i. \quad (8)$$

Equations (5), (6), (7) and (8) are personalized for every user i , and calculated in parallel independently by users. Assume that $[\zeta_i^{\min}, \zeta_i^{\max}]$ are the operating ranges of user i respectively. For a given LVDN, at a given time slice t , ε_t^i is given as well. Together with the corresponding v_t^i , it leads to a certain B_t^i according to (6) and (7). Then the number of the users who will implement its u_t^i is S_t^n , which is given by

$$S_t^n = \sum_{i=1}^n R_t^i. \quad (9)$$

Let $X_t^i = 1 - B_t^i$. Since R_t^i is independent for all user i in n , and follows the Bernoulli distribution $B(1, X_t^i)$ according to (5), then S_n follows Poisson binomial distribution⁴⁴. Then according to the Strong Law of Large Numbers (SLLN) and Kolmogorov Convergence Criterion⁴⁵, the actual S_n always meets its expectation $E(S_n)$ when n is large enough. Hence, it can be concluded that statistically, user i will always try to implement its $\zeta_t^{i'}$ with considering its corresponding voltage situation and cost. Users with milder v_t^i or high compensation price θ_t^i execute their u_t^i at a statistically higher priority, while users with severe v_t^i or cheaper θ_t^i are more likely to be reshaped/deferred. And further discussion is given in Remark.2.

Furthermore, for easier elaboration, let $\Delta\zeta_t^i$ denote the desired changes of user i (which is the u_t^i in (8)). $\Delta\zeta_t^i$ varies with users. Then the realized adjustment of every user $A_t^i = \Delta\zeta_t^i \cdot R_t^i$ are different from each other. The total adjustment in one control cycle

A_t^s is the sum of adjustments by the individual users:

$$A_t^s = \sum_{i=1}^n A_t^i = \sum_{i=1}^n \Delta \zeta_t^i \cdot R_t^i, \text{ where } \Delta \zeta_t^i = u_t^i. \quad (10)$$

According to Binomial Sum Variance Inequality for Multiple Variables⁴⁶, the coefficient of variance (CV) of A_t^s is then given by

$$CV(A_t^s) = \frac{\sigma(A_t^s)}{E[A_t^s]} = \frac{\sqrt{\text{Var}[A_t^s]}}{E[A_t^s]} = \frac{\sqrt{\sum_{i=1}^n \Delta \zeta_t^{i2} \cdot \text{Var}[|R_t^i|]}}{\sum_{i=1}^n \Delta \zeta_t^i \cdot \overline{X_t^i}} \leq \frac{\Delta \zeta^{max}}{\Delta \zeta^{min} \cdot \sqrt{n} \cdot \overline{X_t^i}} \quad (11)$$

$$\text{where } \Delta \zeta^{max} = \max_i \Delta \zeta_i, \Delta \zeta^{min} = \min_i \Delta \zeta_i$$

For specific n and $\overline{X_t^i}$, $CV(A_t^s)$ reaches its minimum when every user has the same $\Delta \zeta_t^i$, which implies almost completely accurate control result. Namely, from the perspective of mathematics, the control prefers if the users have $\Delta \zeta_t^i$ in similar volume or larger n .

Nevertheless, users do not always have $\Delta \zeta_t^i$ in similar volume. Moreover, from the perspective of user i , (8) is only able to tell whether the desired change u_t^i should be deferred, but it is possible for the u_t^i to be executed partially in order to minimize (3) with respecting the system constraints. Namely, u_t^i can be reshaped as well. To do this, during the period $(t, t+1)$, the corresponding u_t^i can be divided into k segments, and go through (6), (7) and (8) k times successively with the parameters updated after each time. This implies downsized step, and (8) becomes

$$\zeta_{t+1}^i = \zeta_t^i + \sum_{j=1}^k \frac{u_t^i}{k} \cdot R_{t+\frac{j}{k}}^i, \text{ where } u_t^i = \zeta_{t+1}^{i'} - \zeta_t^i. \quad (12)$$

Then (10) becomes

$$A_t^s = \sum_{j=1}^k A_{t+\frac{j}{k}}^s = \sum_{j=1}^k \sum_{i=1}^n \Delta \zeta_t^i \cdot R_{t+\frac{j}{k}}^i, \text{ where } \Delta \zeta_t^i = \frac{u_t^i}{k}. \quad (13)$$

Meanwhile, the corresponding CV of each $A_{t+\frac{j}{k}}^s$ is given by

$$CV(A_{t+\frac{j}{k}}^s) \leq \frac{\Delta \zeta^{max}}{k \cdot \Delta \zeta^{min} \cdot \sqrt{n} \cdot \overline{X_{t+\frac{j}{k}}^i}}, \quad (14)$$

which means the CV is reduced when the step is downsized, implying a more accurate result. Nevertheless, downsized step also means more computational efforts. Moreover, another notable difference between (8) and (12) is its expectations along B_t^i . More specifically, for user i , if its u_t^i is not segmented, the expectation of ζ_{t+1}^i can be written as

$$E(\zeta_{t+1}^i) = \zeta_t^i + u_t^i \cdot (1 - B_t^i), \text{ where } u_t^i = \zeta_{t+1}^{i'} - \zeta_t^i. \quad (15)$$

If its u_t^i is segmented, according to (12), the expectation of ζ_{t+1}^i however becomes

$$E'(\zeta_{t+1}^i) = \zeta_t^i + \sum_{j=1}^k \frac{u_t^i}{k} \cdot (1 - B_{t+\frac{j}{k}}^i), \text{ where } u_t^i = \zeta_{t+1}^{i'} - \zeta_t^i. \quad (16)$$

Therefore, $|E'(\zeta_{t+1}^i)| \leq |E(\zeta_{t+1}^i)|$ when $B_t^i \rightarrow 1$, which means that when v_t^i approaches its bounds, user i is expected to execute u_t^i partially when the decision making process is segmented according to (12). This is preferable when v_t^i is approaching its bounds, as there is no explicit model to indicate the corresponding change on v_t^i after $\Delta \zeta_t^i$ is executed, the decision making process needs to be more conservative. Whereas (8) is more preferable when v_t^i is close to v^* , as the computational effort is less and it is more likely that user i can execute its u_t^i completely in order to minimize (4). Therefore, a mechanism that can make smooth transition between (8) and (12) needs to be proposed.

FIGURE 1 The variations of θ with the changes of α and β respectively

Remark 1. Equation (6) works in a see-saw manner between v_t^i and ε_t^i . Its practical meaning is that the control will value the cost of user i being reshaped/deferred more than its voltage situation when v_t^i is close to v^r , while the further v_t^i is from v^r , the less consideration will be given to the cost of being reshaped/deferred, as ensuring v_t^i remains within its bounds has higher priority than reducing the cost.

Equation (7) is the configuration function of the control, and parameters α and β are tunable. By tuning α and β , the control can have different boundary conditions on voltage, and different emphasis in the control. The corresponding effect of changing α and β imposes on θ_t^i is given by Fig. 1. The theoretical feasible region of α is $\mathbf{R}_+$, while $\beta \in \{2n, n = 1, 2, 3, 4, \dots\}$. One can conclude from Fig. 1 that α defines the anchor points of $\theta = \pm 0.5$, while β determines the gradients around $\theta = \pm 0.5$. According to (8) or (12), the expectation of ζ_{t+1}^i at different v_t^i can be configured.

Remark 2. By employing (6), (7), (8) and (12), the proposed mechanism actually establishes Player Compatible Relationships (PCR)^{34,47} among users in the given LVDN. A PCR is defined as

Definition 1. For every user i , there is a finite strategy set $\mathbb{S}_i$, where strategy $s_i \in \mathbb{S}_i$. The set of mixed strategy σ_i is M_i and the set of strictly mixed strategies, where every pure strategy in $\mathbb{S}_i$ has nonzero probability, is denoted by M_i° . For s_i , the correlated strategies of the other $N - 1$ users is s_{-i} . s_i^* is the optimal strategy of user i . The cost of user i is indicated by J^i , the utility of user i is $U^i = -J^i$. For user $i \neq j$ and the corresponding strategies $s_i^* \in \mathbb{S}_i, s_j^* \in \mathbb{S}_j$, if it holds for every correlated strategy $\sigma_{-j} \in M_{-j}^\circ$ and for every $\sigma_{-i} \in M^\circ(\mathbb{S}_{-i})$ satisfying $s_{-i} \upharpoonright_{\mathbb{S}_{-ij}} = s_{-j} \upharpoonright_{\mathbb{S}_{-ij}}$, that

$$U_j(s_j^*, \sigma_{-j}) \geq \max_{s'_j \in \mathbb{S}_j \setminus \{s_j^*\}} U_j(s'_j, \sigma_{-j}), \text{ while } U_i(s_i^*, \sigma_{-i}) > \max_{s'_i \in \mathbb{S}_i \setminus \{s_i^*\}} U_i(s'_i, \sigma_{-i}), \quad (17)$$

then we say i is more *player compatible* with s_i^* than j is with s_j^* , which is denoted as $(s_i^*|i) \gtrsim (s_j^*|j)$. And this compatibility relation is transitive and asymmetric, readers are referred to the appendix section in the work of D.Fudenberg⁴⁷ for the corresponding mathematical proofs.

In this paper, the PCR is determined by two factors: the nodal voltage v_t^i and the compensation (the cost if from the perspective of DSO) ε_t^i of being reshaped/deferred. Having v_t^i closer to v^r or more expensive ε_t^i will give user i upper hand in a certain PCR chain, which means they are less likely to be reshaped/deferred than other subservient users, as they are relatively more helpful with minimizing J_i .

Remark 3. The philosophy of the proposed mechanism is regarding the whole LVDN as a surface where the optimization algorithm searches on. Similar to the classical gradient descent approach, in which the users play the role of variables while the PCR acts as the gradient. Then when users are adjusting ζ_t^i , they have the tendency to statistically converge to the optimal result. According to (11) and (14), and the fact that $\zeta_t^{i'}$ and ε_t^i vary with time t , ζ_t^i tracks the points that can minimize J_t with highest probability. Therefore, the results are the stochastic approximation to the theoretical optimum instead of the optimum itself.

Since the calculations and Bernoulli trial are implemented by users in a parallel manner, the increase of user amount does not exert influence on computational load. The fundamental mechanism can be embodied into different controls in energy system, as long as it can form a resource allocation game⁴⁸. A congestion game is the game in which the payoff of each player depends on the resources it chooses and the number of players choosing the same resource⁴⁹.

3 | DISTRIBUTED OPTIMIZATION OF SHAPEABLE/DEFERRABLE ENERGY RESOURCES

3.1 | Setup and implementation

In LVDN, due to the high R/X ratio, voltage is highly related to active power distribution⁵⁰. Moreover, users, especially households in LV distribution network do not have dedicated devices of which the reactive power output can be controlled. In this paper, the change of reactive power is regarded as an affiliated change to active power. This does not mean that the reactive power is ignored in the control, as v_t^i is the systemic result of both the active power and reactive power changes. To simplify the case, in this paper, according to the EU standard EN61000-3-2, all the users in the given LVDN have a power factor (PF) ranging [0.9, 1], and they are assumed to be constant.

In addition to the fundamental mechanism elaborated in Section 2, there are several auxiliary mechanisms needed in order to the implement stochastic distributed optimization of shapeable/deferrable energy resource in a given LVDN.

Firstly, as aforementioned in Section 2, an automatic mechanism needs to be designed to shift between (8) and (12). Actually, the shift is a risk evaluation. As there is no explicit model to indicate the corresponding change on v_t^i when ζ_t^i changes, the control need to use variable-step when v_t^i is moving close to the bounds or there is a considerable risk that the changes on ζ_t^i might lead to the bounds violation of v_t^i . In order to do this, a global broadcast is introduced. Several key voltage monitoring points are allocated in the whole LVDN evenly, the measured voltage information is aggregated by a broadcaster, and the broadcaster processes the mean voltage $\hat{v}_t$ in the manner of (7), which can be written as

$$\hat{\theta}_t = (1 - \sin(\frac{e^{-(\frac{\mu}{\alpha})^\beta}}{2}\pi)) \cdot \text{sgn}(\mu), \text{ where } \mu = \hat{v}_t - v^r, \quad (18)$$

Then the corresponding $\hat{\theta}_t$ is broadcast to all the user as a public information, such that users have an update about the system-wide situation. When user i receives $\hat{\theta}_t$, it will combine $\hat{\theta}_t$ with its own θ_t^i , as

$$\hat{B}_t^i = |(1 - |\theta_t^i|) \cdot \hat{\theta}_t + |\theta_t^i| \cdot \theta_t^i|. \quad (19)$$

Consequently, for user i , the k in (12) is given by

$$k = \lceil \hat{B}_t^i \cdot k^{max} \rceil, \quad (20)$$

where k^{max} is the maximum amount of the segments. Equation (19) works in a see-saw manner, and implies that user i evaluates its local nodal voltage with priority, in addition to the global average one. When $\hat{B}_t^i = 0$, according to (20), (12) degrades to (8). From Fig. 1 one can observe that θ_t^i remains zero when the corresponding v_t^i is around v^r . And the segmentation starts when v_t^i moves away from v^r .

Secondly, although it is very unlikely, it is possible for v_t^i to be out of the bounds for some reason. After all, the control is stochastic and v_t^i could have already been out of the bounds before the control is implemented. Therefore, a mechanism that can bring v_t^i back within the bounds is needed, which is given by

$$\zeta_{t+1}^i = \zeta_t^i + \eta \cdot (\zeta_t^{max} - \zeta_t^{min}) \cdot \text{sgn}(\theta_t^i), \quad (21)$$

where η is a preset coefficient and $\eta \in (0, 0.1]$. (21) regulates v_t^i by reshaping ζ_t^i according to the sign of θ_t^i only when v_t^i is out of the bounds.

3.2 | Algorithm Description

The control law of user i is illustrated in Algorithm 1. User i firstly evaluates v_t^i with the broadcast signal $\hat{\theta}_t$ to decide the segments number of u_t^i , calculating the corresponding k . Then u_t^i is segmented into k parts, and go through the loop from the line 4 to line 18 successively until all the k segments of u_t^i are executed. The condition of line 6 checks whether u_t^i happens to be deemed to make v_t^i move towards v^r , if so, u_t^i will be executed without any reshape or deferral.

Since v_t^i varies with user i , users may have different k . Algorithm 1 is executed by users in parallel independently, different k therefore results in different execution speed. For timing, a time interval Δt between t and $t + 1$ is divided into k^{max} time windows evenly, and each time window has the length of $\Delta t / k^{max}$. When Algorithm 1 is executed by user i , each round of loop from line 4 to 18 takes one time window, and keep operating ζ_{t+1}^i during the remaining time windows if $k < k^{max}$ until $t + 1$.

Algorithm 1 Control law of user i **Input:**

Nodal voltage, v_t^i ;
 Current consumption, ζ_t^i ;
 The price of the compensation, ε_t^i ;
 Scheduled consumption profile, $\zeta_t^{i'}$;
 Broadcast signal, $\hat{\theta}_t$;

Output:

Optimized consumption at $t + 1$, ζ_{t+1}^i ;

- 1: θ_t^i = equation (7) with v_t^i as input;
- 2: $\hat{B}_t^i$ = equation (19) with θ_t^i and $\hat{\theta}_t$ as input;
- 3: k = equation (20) with θ_t^i and $\hat{B}_t^i$ as input;
- 4: **for** $j=1:k$ **do**
- 5: $\theta_{t+\frac{j-1}{k}}^i$ = equation (7) with $v_{t+\frac{j-1}{k}}^i$ as input;
- 6: **if** $\frac{u_t^i}{k} \cdot \theta_{t+\frac{j-1}{k}}^i > 0$ **then**
- 7: $R_{t+\frac{j-1}{k}}^i = 1$;
- 8: **else**
- 9: $B_{t+\frac{j-1}{k}}^i$ = equation (6) with ε_t^i and $\theta_{t+\frac{j-1}{k}}^i$ as input;
- 10: $R_{t+\frac{j-1}{k}}^i$ = equation (5) with $B_{t+\frac{j-1}{k}}^i$ as input;
- 11: **end if**
- 12: **if** $v_{t+\frac{j-1}{k}}^i$ is out of bounds **then**
- 13: $\zeta_{t+\frac{j-1}{k}}^i$ = equation (21) with $\theta_{t+\frac{j-1}{k}}^i$ as input;
- 14: **else**
- 15: $\zeta_{t+\frac{j-1}{k}}^i = \zeta_{t+\frac{j-1}{k}}^i + \frac{u_t^i}{k} \cdot R_{t+\frac{j-1}{k}}^i$;
- 16: **end if**
- 17: Execute $\zeta_{t+\frac{j-1}{k}}^i$ and get $v_{t+\frac{j-1}{k}}^i$;
- 18: **end for**
- 19: Return ζ_{t+1}^i

4 | CASE STUDY

4.1 | Grid Topology

The case study is implemented in a three phase 230/400-V reference grid, based on the topology of a real semiurban feeder in the region of Flanders, Belgium⁵¹. The schematic diagram is illustrated by Fig. 2. To make the network more multifarious, new users such as residential wind turbines and small PV farms are added, the number of nodes are increased from 62 to 103. The impedance values are calculated according to Belgian standard for underground distribution cables with an assumed operating temperature of 45°C, which is given in Table 2. All of the main feeder cables are of type EAXVB 1 kV 4 · 150mm². A 250 kVA

FIGURE 2 Topology of the test network. Cable lengths are drawn to scale.

10/0.4 kV transformer is assumed with an impedance of 0.013+0.038j pu. From feeders to each individual user, a cable EXVB 1kV 4 · 16mm² is used with a length of 15m. To simplify, the three phase system is assumed symmetrical, then the analysis can

focus on single phase. The nominal line-to-neutral voltage v^r of all the users is 230V and the voltage variance constrain is $\pm 10\%$ of the v^r .

4.2 | Profiles and Scenarios

Two kinds of consumption profiles are used in validation: the practical profile that is based on realistic measured data and the generated one that is based on randomly generated data. The former one is used to test the performance of the proposed approach with practical scenarios, while the later one validate the effectiveness of the proposed approach under extreme scenario. The price of compensation $\varepsilon_t^i \in (0, 1]$ is randomly generated, with a general rule that ε_t^i tends to be expensive if the corresponding v_t^i is close to its bounds.

4.2.1 | Practical profile

A model of domestic electricity use is used to generate high resolution household consumption profile, which is based upon a combination of patterns of active occupancy^{52,53}. Lighting and appliances, occupant's behaviour, month of the year and weekday or weekend have been taken in account for the profile generation. Household profiles in this paper are assumed on a weekend day of June, whose source database is based on a realistic measurement of 22 domestic dwellings around the town of Loughborough in the East Midlands, UK. It is assumed that normal households randomly have one to four family members, and four to six family members are assumed in high-consuming households.

Profiles of PV panels are taken from a 368kWp PV system on the rooftop of research institute EnergyVille in Genk, Belgium⁵⁴. The system has 24 strings of PV panels, and each string can be recorded individually. The profiles are scaled, randomly selected and combined from the raw data in the week of 6th-12th June, 2016. Since all the PV strings are located close to each other, the data has good correlation which is suitable to be used as a profile in LVDNs.

The profiles of residential wind turbines are obtained from Elia, a Belgian transmission system operator. The data was measured from Belgian onshore wind turbines on the 6th and 7th of June, 2016, the same time period as the profiles of PV and households. Then it is scaled and assigned randomly to the residential wind turbines in test network.

Four small residential generators are connected to the test network to represent other distributed generation users, such as CHP generators. Their profiles are generated randomly according to occupants' behaviour.

4.2.2 | Generated profile

Generated scenario is used to validate the robustness of the proposed approach under severe situations, as the practical profile is rather moderate. The profile is generated randomly according to the corresponding operating ranges of different users, which are shown in Table 3. Generation is regarded as negative consumption. Profiles are generated in a Markov manner step by step, namely although users behaviours are random, they still have reasonable continuity.

4.2.3 | Configuration of the tunable parameters

There are four tunable parameters in Algorithm 1, which are α and β in (7), k^{max} in (20) and η in (21). For case study, α is set to 15 and $\beta = 4$. $k^{max} = 4$, as the control resolution is one minute, a typical aforementioned time window in the control law of user i is 15 seconds, which is sufficient for a common controller with basic computational capability. $\eta = 0.05$, which means when (21) is triggered, it reshapes ζ_t^i with 5% of the operating range as the step.

TABLE 2 Cable Parameters

Cable Type	Impedance at 45° C
EAXVB 1kV 4 · 150mm ²	0.227+0.078j Ω/km
EXVB 1kV 4 · 16mm ²	1.265+0.083j Ω/km

TABLE 3 Operating Ranges of Users in the Generated Profile

User Type	Operating Range (kW)
Houses without PV	0 ~ 9
Houses with PV	-4 ~ 9
Small PV farms	-16 ~ 0
Residential wind turbines	-30 ~ 0
High-consuming unit	5 ~ 13
Residential generators	-8 ~ 0

4.2.4 | Powerflow simulation

The validation is implemented in MATLAB, based on the backward-forward sweep method⁵⁵. Although the control is simulated step by step, the effect is processed as continuous. Namely, to make the simulation more realistic, user behaviours are affected by the control imposed as well. In every control cycle, one third of the control from previous time remains as an after effect, instead of just obtaining original profile rigidly.

4.2.5 | Benchmark

A centralized controller can derive the optimal adjustment for each participating user, providing a benchmark for the proposed method^{56,57}. The controller solves an optimization problem using the full non-linear network model (Alternating Current Optimal Power Flow, ACOPF) to do this. It requires two-way communication; each participating user sends a range to the central controller, within which it can adjust its power consumption or generation. The optimization problem is exactly same to the proposed approach: minimizing the cost function (3) with ensuring all the v_t^i within system constraints. In order to obtain the theoretical optimal results to be compared with, several assumptions are given to the benchmark approach: instant and accurate communication between the controller and users, perfect information on grid topology, real-time update on ϵ_t^i and immediate execution of the control.

4.3 | Results

4.3.1 | Practical profile

The evolution of all the 103 nodal voltages with practical profile is illustrated by Fig. 3. The evolution of original profile without control is given by Fig. 3(a), which is suffering from overvoltage issues. The nodal voltage evolution of the proposed approach is given by 3(b), while the one with ACOPF is shown in Fig. 3(c). Comparing Fig. 3(b) and Fig. 3(c), one can observe that the proposed approach yields almost the same nodal voltage evolution with the benchmark ACOPF, which suggests that the proposed approach is able to give a good approximation to the theoretical optimum but with limited communication and computational efforts. In order to show the gap between the approximation and the theoretical optimum more clear, a metric that sums all the compensation costs of the 103 nodes hourly is illustrated in Fig. 3(d). It can be observed that the biggest gap between the approximation and the theoretical optimum appears during 12 ~ 16h, with an average cost of 3.86 versus 2.16 per minute, which is not a very significant gap as it is the accumulate cost of the whole network. The reason to this phenomenon is that the time period 12 ~ 16h was encountering a severe overvoltage on most of the nodes, as a model-based approach, ACOPF has the precise indication of the relationship between consumption and voltage, while as a model-free, non iteration-based approach, the proposed approach has to evaluate the probability of violating the voltage bounds. Therefore, the proposed approach statistically implemented a relatively more conservative control than ACOPF, which causes higher cost. This phenomenon can be clearly observed from Fig. 3(e), the statistical box-plot of hourly nodal voltage. One can observe that the nodal voltages are concentrated closer to the bound during 12 ~ 16h with the control of ACOPF, while the ones controlled by the proposed approach are relatively more dispersive, which implies more reshape and deferral. This is one of the harshest scenarios for the proposed approach, as it is too risky to allow the nodal voltage be even closer to the bound when both v_t^i and $\hat{\theta}_t$ are critical.

FIGURE 3 Benchmark between the proposed approach and ACOPF with the practical profile

4.3.2 | Generated profile

In order to prove that the proposed approach is compatible to all the cases, the evolution of all the 103 nodal voltage with generated profile is illustrated by Fig. 4. Compared to the practical profile, the users in generated profile have larger operating ranges, and more drastic changes and voltage variations, which can be observed from Fig. 4(a). Similar to the practical case, the comparison of nodal voltage evolution between Fig. 4(b) and Fig. 4(c) proves that the proposed approach can always yields good approximation to the voltage controlled by the centralized approach. The statistic that sums all the compensation cost of the 103 nodes hourly is illustrated in Fig. 4(d), which has an average cost of 2.91 versus 1.39 per minute for the proposed approach and the benchmark ACOPF respectively. The hours with visible gap such as 7 ~ 8h and 21 ~ 23h can be checked from the hourly boxplot of nodal voltages in Fig. 4(e). Similar to the practical case, these hours are the hours where the given LVDN with severe voltage issues, and the proposed approach is therefore more conservative.

Remark 4. The conservativeness of the proposed control is controlled by (7) as discussed in the Section 2. According to Fig. 1, larger α or smaller β will make the control less sensitive to the voltage so that it yields more concentrated nodal voltage

FIGURE 4 Benchmark between the proposed approach and ACOPF with the generated profile

distribution like the benchmark ACOPF, in the hours with severe voltage issues. Nevertheless, as there is no explicit model acting as an indicator, the control would be risk at causing voltage bounds violate on some nodes.

4.3.3 | Special case: communication failure

Since the proposed approach only employs unidirectional communication with low bandwidth, it does not rely on the reliability of communication infrastructures, which increases its robustness. To prove this, a special case is implemented, in which the broadcaster is broken, or the communication fails, so that the users cannot receive the public broadcast signal $\hat{\theta}_t$ anymore. The special case has the exactly same configurations with the regular cases except a functional broadcaster or communication system.

FIGURE 5 The proposed approach with the practical profile under communication failure

The result of the practical profile under communication failure is illustrated in Fig. 5. The nodal voltage evolution is given by Fig. 5(a), comparing to Fig. 3(b) and (c), one can conclude that the control of the voltage doesn't receive much influence from communication failure. Meanwhile, the minor difference can be found in Fig. 5(b), in which the control yields slightly higher cost than the one with communication with an average of 3.89 per minute. Nevertheless, the boxplot of hourly nodal voltage delivers a more interesting result. When the communication is absent, the distribution of the nodal voltage becomes even more similar to the benchmark one, however, the gross compensation cost is not accordingly reduced. On the contrary, it rises. This is because the control becomes relatively more bold when the communication malfunctions, as the general situation is not considered anymore, users lack information to evaluate the situation in a systemic manner. Therefore, although users with upper hand are reshaped or deferred less, subservient users are abused, which deteriorates the Pareto improvement so that degrade the result to a Nash equilibrium, resulting in higher gross cost.

The similar phenomenon takes place with the generated profile and is shown by Fig. 6. Fig. 6(a) illustrates the evolution of the nodal voltage when the communication is absent, while Fig. 6(b) gives the comparison on gross compensation costs and Fig. 6(c) delivers the hourly nodal voltage box-plot. The special case with randomly generated profile confirmed that the proposed approach is able to keep working under the situation of communication malfunction, and it is not limited for some specific profile. Meanwhile, similar to the case in 5, the coordination of users is deteriorated due to lacking of public information, which resulting in a average compensation cost of 3.22 per minute, which is slightly higher than the 2.97 per minute in its corresponding case with functional communication.

5 | CONCLUSION

A stochastic distributed control method for the optimization of shapeable/deferrable energy resources in LVDNs is proposed in this paper, which is able to deliver good approximation to the theoretical optimum in a model-free manner without relying on iterations. Its decision making process is decentralized and personalized to every user, making computational burden independent from the number of users. By employing statistical theories, controller and users couple with each other without relying

FIGURE 6 The proposed approach with the generated profile under communication failure

on advanced communication system, resulting in a flexible, plug-and-play system in which users can join and quit freely. For instance, compared to the benchmark approach, assume the message length is 1 per communication, then the communication burden of one typical control is n for the proposed approach (broadcast), while it is at least $2n$ for the benchmark approach (bidirectional communication). In fact, the communication burden of the benchmark approach is even heavier as more information needs to be delivered by per message than the proposed approach. Additionally, in the proposed approach, the double-blind structure and locally used data firmly secure the privacy protection of users while implementing the control.

The philosophy of the proposed approach is approaching the optimal point while implementing users' profiles. By creating PCR among users in the given LVDN, similar to the gradient descent, in the proposed approach, users move along the PCR, which always points the direction to the potential optimal point. Nevertheless, different from the classical iteration-based search, according to the nodal voltage and the public information, users stochastically leap to the most likely optimal point straightforwardly, then segment the control, in order to statistically approach the practical optimum. In this way, users are always able to keep tracking the optimal point, deliver good approximation to the optimum under limited communication with tractable computational effort. Moreover, as the proposed approach is model-free, its application does not limit to the control in LVDNs, other energy system with shapeable/deferrable users are potential applications as well, especially in large scale networks thanks to the SLLN.

The work in this paper has at least two potential directions for future research. Firstly, although the parameters α and β in (7) and its variations are set to be the same among each user in this paper, they can be different as well. In fact, a trial and error learning mechanism or the extreme search could be used to find better combinations of α and β for different users and given LVDNs, then the performance of the proposed approach can be further improved. This is the first potential future research. Secondly, peer to peer communication can be used to replace the broadcast if applicable, the public information could be delivered in a token passing manner⁵⁸, which can increase the reliability of the communication, although the proposed approach is proven to be able to work in offline mode as well.

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