Statistics of velocity and temperature fluctuations in two-dimensional Rayleigh-Bénard convection

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We investigate fluctuations of the velocity and temperature fields in two-dimensional (2D) Rayleigh-Bénard (RB) convection by means of direct numerical simulations (DNS) over the Rayleigh number range $10^6 \leq Ra \leq 10^{10}$ and for a fixed Prandtl number $Pr = 5.3$ and aspect ratio $\Gamma = 1$. Our results show that there exists a counter-gradient turbulent transport of energy from fluctuations to the mean flow both locally and globally, implying that the Reynolds stress is one of the driving mechanisms of the large-scale circulation in 2D turbulent RB convection besides the buoyancy of thermal plumes. We also find that the viscous boundary layer (BL) thicknesses near the horizontal conducting plates and near the vertical sidewalls, δ_u and δ_v , are almost the same for a given Ra , and they scale with the Rayleigh and Reynolds numbers as $\sim Ra^{-0.26 \pm 0.03}$ and $\sim Re^{-0.43 \pm 0.04}$. Furthermore, the thermal BL thickness δ_θ defined based on the root-mean-square (rms) temperature profiles is found to agree with Prandtl-Blasius predictions from the scaling point of view. In addition, the probability density functions of turbulent energy $\varepsilon_{u'}$ and thermal $\varepsilon_{\theta'}$ dissipation rates, calculated, respectively, within the viscous and thermal BLs, are found to be always non-log-normal and obey approximately a Bramwell-Holdsworth-Pinton distribution first introduced to characterize rare fluctuations in a confined turbulent flow and critical phenomena.

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I. INTRODUCTION

Buoyancy-driven fluid flows are ubiquitous in geophysics, astrophysics, and process technology. A paradigmatic example that has been widely used to study this type of flow is the so-called Rayleigh-Bénard (RB) convection, i.e., a fluid layer heated uniformly from below and cooled from above [1–4]. In a closed sample, the RB system is determined by three control parameters: the Rayleigh number $Ra = \alpha g \Delta H^3 / (\nu \kappa)$, the Prandtl number $Pr = \nu / \kappa$, and the aspect ratio $\Gamma = D / H$. Here α is the thermal expansion coefficient, g is the acceleration of gravity, Δ is the sustained temperature difference between the bottom and top plates of the sample, ν is the kinematic viscosity, κ is the thermal diffusivity, and H and D are the height and width of the sample, respectively. The response of the system is characterized by the global heat and momentum transports, which are expressed as the nondimensional Nusselt number Nu and Reynolds number Re , respectively. The scalings of the Nusselt and Reynolds numbers have been extensively studied over the past few years, and we refer interested readers to the recent papers by Ahlers *et al.* [1] and by Chillà and Schumacher [3] for more details.

A prevalent structure in turbulent RB convection is large-scale circulation (LSC, also denoted as the “wind” of turbulence). Krishnamurti and Howard [5] first observed the generation of this large-scale flow, and they argued that the mean flow is apparently maintained against viscous dissipation by the Reynolds stress divergence of the tilted plume motions [5]. This statement motivated researchers to experimentally investigate the turbulent energy production

fields in an RB system by using the particle image velocimetry (PIV) technique, and their results showed that the mean wind is driven by the buoyancy of plumes [6–8]. However, due to the restriction of the PIV technique, they could not get the full information on the velocity field.

The boundary layers (BLs) in the vicinity of the conducting plates contribute to the main resistance for the heat transfer, thus they deserve special attention. The viscous and thermal BL thicknesses in RB convection can be defined in several ways: the most popular style is to define them through the time-averaged velocity and temperature profiles near the plates, i.e., the so-called slope method; another way to define the BL thicknesses is the one employed by the classical laminar boundary layer theories known as 99% criterion [1]. Recently, Petschel *et al.* [9,10] introduced the dissipation layers in RB convection. In addition to the methods illustrated above, BL thicknesses can also be determined from the profiles of root-mean-square (rms) values of velocity and temperature fluctuations close to the plates. The properties of this type of BL are used to predict turbulent heat transport and distinguish scaling theories that are based on different assumptions in RB convection [11]. In comparison with those theories, experimental studies have shown some ambiguous observations [12–15]. Additionally, for the Ra dependence of the kinetic BL thickness, Qiu and Xia found that it scales differently near the sidewalls and the bottom plates [16,17].

Velocity and temperature fluctuations play an essential role in the thermal convection system. Once the Rayleigh number is sufficiently high, convection becomes turbulent, the velocity and temperature fields display complex fluctuations both temporally and spatially, and due to buoyancy of the plumes they interplay with each other. The fluctuations of velocity and temperature in turbulent RB convection have received significant attention from researchers during the past

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couple of years, primarily concentrated on determining the local heat flux [18–20] and the dynamics that drive the cascades of turbulent fluctuations from large to small scales [2,21–23]. In this paper, we want to present the statistical properties of velocity and temperature fluctuations in 2D RB convection by means of direct numerical simulation (DNS), which allow us to get full information on the flow field. Strictly speaking, all convection flows occurring in both natural situations and the laboratory are three-dimensional (3D). Nevertheless, previous investigations have shown that 2D and 3D RB flow share some similarities [24–26], such as the 2D characters of the LSC, the Nu-Ra scaling, the boundary layer profiles, and so forth. Therefore, 2D simulations are conducted to better understand the convection phenomena occurring in the real world. Recently, there has been increasing interest in 2D RB convection [27,28]. For example, it has been used to study the structures of viscous and thermal BLs [29,30], to investigate the flow organization influenced by aspect ratio and velocity boundary conditions [31–33], to compare the differences between 2D and 3D thermal convection [25,26], and to reveal the dynamics of the flow reversals [24,34–36]. Although velocity and temperature fluctuations have been investigated intensively in 3D turbulent RB convection during the past several years [1], to the best of our knowledge a study of velocity and temperature fluctuations in 2D RB convection has not been conducted yet.

The remainder of this paper is organized as follows. The governing equations and numerical scheme are briefly introduced in Sec. II. Section III contains the main results of our numerical simulations: Sec. III A contains general features of the flow field; Sec. III B presents the spatial distribution of the turbulent fluctuations; Sec. III C contains the mean profiles of rms velocity and temperature fluctuations and the scalings of BL thicknesses; and in Sec. III D, as a supplement, turbulent dissipation rates in the BLs are investigated by the means of probability density functions (PDFs). We summarize our findings and present the conclusions in Sec. IV.

II. NUMERICAL METHOD

We deal with the dimensionless Oberbeck-Boussinesq equations for the incompressible RB problem numerically. The governing equations are as follows:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \sqrt{\frac{\text{Pr}}{\text{Ra}}} \nabla^2 \mathbf{u} + \theta \mathbf{z}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta = \sqrt{\frac{1}{\text{PrRa}}} \nabla^2 \theta, \quad (3)$$

where $\mathbf{u} = u\mathbf{x} + v\mathbf{z}$ ($\mathbf{x}$ and $\mathbf{z}$ are the horizontal and vertical unit vectors, respectively) is the velocity, t is the time, p is the pressure, and θ is the temperature. The physical variables length, temperature, velocity, and time are normalized by the height of the cell H , the temperature difference Δ , the free-fall velocity $\sqrt{\alpha g \Delta H}$, and the free-fall time $\sqrt{H/(\alpha g \Delta)}$. No-penetration and no-slip velocity boundary conditions are applied to all four solid walls. For temperature θ the two vertical sidewalls are imposed to be insulated, i.e., no heat

TABLE I. Parameters of the different simulations (columns from left to right): Ra, grid resolutions in horizontal and vertical directions $N_x \times N_z$, Nu, Re, and the averaged time of the simulations τ_0 in free-fall time units $\sqrt{H/(\alpha g \Delta)}$.

Ra	$N_x \times N_z$	Nu	$\text{Nu}_{\varepsilon_u}$	$\text{Nu}_{\varepsilon_\theta}$	Re	τ_0
1×10^6	129×129	6.87	6.89	6.89	38	7500
3×10^6	193×193	9.33	9.35	9.34	70	7500
1×10^7	257×257	13.28	13.30	13.29	156	6000
3×10^7	385×385	18.86	18.88	18.87	296	4500
1×10^8	513×513	26.21	26.22	26.22	596	3000
3×10^8	769×769	35.96	36.20	35.96	1145	3000
1×10^9	1025×1025	51.28	51.43	51.31	2269	2500
3×10^9	1537×1537	71.57	71.54	71.71	4330	1200
1×10^{10}	2049×2049	105.8	106.0	105.7	9916	1000

flux, while the top and bottom plates are held at constant temperatures of -0.5 and $+0.5$, respectively.

The numerical method is based on a compact fourth-order finite-difference scheme introduced by Liu *et al.* [37], and it has been explained by Zhou [38] and Huang and Zhou [39]. Recently, we have employed the same numerical code to investigate the small-scale cascade properties in the 2D Rayleigh-Taylor system [40–42]. The grid spacing is uniform in both horizontal and vertical directions, and the number of grid points is increased from 129×129 to 2049×2049 as Ra increases from 10^6 to 10^{10} in the present work. The grid resolution has been chosen to achieve a full resolution of the BLs [43] and to resolve the smallest scales of the problem, i.e., the Kolmogorov scale η_K and the Batchelor scale η_B . The details about the grid resolution of the present simulations have been given in Table I of Ref. [44]: for all simulation cases, the thermal BLs (here the thermal BL thickness is calculated as $\delta_{\text{th}} = \frac{H}{2\text{Nu}}$) are resolved with at least 10 grid points, and the grid spacings $\Delta_g < 0.21\eta_K$ and $\Delta_g < 0.49\eta_B$. The fluid is water at 31°C , corresponding to the Prandtl number $\text{Pr} = 5.3$, and the aspect ratio Γ of the cell is equal to 1.

The main parameters of the numerical simulations in this study are summarized in Table I. The integral quantities, i.e., the Nusselt and Reynolds numbers, are calculated as $\text{Nu} = \sqrt{\text{RaPr}\langle\theta\rangle_{V,t}} + 1$ and $\text{Re} = \sqrt{\langle\mathbf{u}^2\rangle_{V,t}\text{RaPr}}$. Here the notation $\langle\cdot\rangle_{V,t}$ denotes the volume and time average. In a RB convection cell, using the governing equations and the rigid boundary conditions, the two exact relations for the kinetic energy dissipation rate $\langle\varepsilon_u\rangle_{V,t} = \nu^3 H^{-4} \text{Pr}^{-2} \text{Ra}(\text{Nu} - 1)$ and the thermal energy dissipation rate $\langle\varepsilon_\theta\rangle_{V,t} = \kappa \Delta^2 H^{-2} \text{Nu}$ can be derived immediately [1–3]. Meanwhile, the nondimensional forms for the two exact expressions are given, respectively, by $\langle\tilde{\varepsilon}_u\rangle_{V,t} = (\text{Nu} - 1)/\sqrt{\text{PrRa}}$ and $\langle\tilde{\varepsilon}_\theta\rangle_{V,t} = \text{Nu}/\sqrt{\text{PrRa}}$. The mean heat transfer calculated in $\text{Nu}_{\varepsilon_u} = \sqrt{\text{RaPr}\langle\tilde{\varepsilon}_u\rangle_{V,t}} + 1$ and $\text{Nu}_{\varepsilon_\theta} = \sqrt{\text{RaPr}\langle\tilde{\varepsilon}_\theta\rangle_{V,t}}$ is presented in Table I as well. One sees that for all of the simulations, the differences among Nu, $\text{Nu}_{\varepsilon_u}$, and $\text{Nu}_{\varepsilon_\theta}$ are less than 1%, thus guaranteeing the accuracy of our simulations. To further check the accuracy, we compare the response parameters Nu and Re with numerical results from the literature. Figure 1 shows a semilogarithmic plot of $\text{Nu}/\text{Ra}^{1/3}$ as a function of Ra compared with 3D simulations. As previously reported, the Nu-Ra scalings for the 2D and

FIG. 1. Ra dependence of the compensated Nusselt number $Nu/Ra^{1/3}$, the simulation results for 2D RB convection are indicated by solid circles, and the 3D numerical results conducted in a cylindrical convection cell by Lakkaraju *et al.* [20] ($Pr = 5.2$) are marked with open circles. Note that the 3D data have been multiplied by a factor of 0.79, which agrees well with the constant 0.78 suggested by van der Poel *et al.* [26].

3D cases are very similar, differing only by a constant factor in the present parameter range [26]. The global momentum transport scales as $Re \sim Ra^{0.60 \pm 0.01}$ (not shown here), which is consistent with the numerical results of 2D Boussinesq RB convection calculated by Sugiyama *et al.* [45].

III. RESULTS AND DISCUSSION

A. Flow fields

Although the present study concerns a 2D situation, the dynamics in the system is rather complex. Indeed, the main convection roll exhibits irregular reversals and could reorient its direction due to the energy and size growth of

corner-flow rolls [24,34]. The results of Huang and Zhou [39] further revealed that the competition between the corner-flow rolls and the LSC generates countergradient heat transport, which leads to an anomalous $Nu-Pr$ relation in the 2D domain. Typical snapshots of the instantaneous velocity and temperature flow fields for three different Ra spanning four orders of magnitude are plotted in Figs. 2(a)–2(c). They illustrate the general features of the thermal plumes and the LSC in 2D RB convection systems. At the lowest Ra, thermal plumes extend approximately over the entire cell, and they extract thermal energy from the boundary layer and drive the motion of the fluid. From the visualizations (not shown here) corresponding to Fig. 2, it is found that the LSC is not stable at $Ra = 10^6$ and the flow reversals occur very frequently. In our simulations, the LSC appears to be stable for $Ra \gtrsim 3 \times 10^6$, roughly the same as the experimental investigations. In those experiments, researchers have found that the large-scale convection roll occurs at $Ra \gtrsim 2 \times 10^6$ and seems to persist even at much higher Rayleigh number by using semiquantitative techniques and quantitative measurements, although the direction of the LSC is occasional, i.e., either clockwise or anticlockwise [6]. At $Ra = 10^8$, the LSC has been well-established, rotating with the axis perpendicular to the convection roll's plane, and diagonally orientated in the cell with two counter-rotating rolls in the upper-right and lower-left corners. At $Ra = 10^{10}$, the mushroom-like plumes are well mixed by the enhanced turbulent background, and the LSC becomes more concentrated near the boundary of the cell. To overcome the problem that clockwise and anticlockwise LSC will contribute to zero velocity everywhere in the cell when performing the long time average, we introduce the global angular velocity $\Omega(t) = \langle v/(x - 0.5D) - u/(z - 0.5H) \rangle_V$ to

FIG. 2. Instantaneous (a)–(c) and mean (d)–(f) flow fields at three different Rayleigh numbers $Ra = 10^6$ (left panel), 10^8 (middle panel), and 10^{10} (right panel). The magnitudes of velocity and temperature are coded in arrows and color, respectively.

compute the mean flow fields; cf. Zhou *et al.* [30] for more details. After this transformation, the direction of the LSC is anticlockwise for all times, and all the statistics are calculated using this method in the remainder of this paper. The corresponding conditional time-averaged flow fields are shown in Figs. 2(d)–2(f). The instantaneous and mean flow fields all show the process by which the LSC evolves from a tilted and nearly elliptical shape at lower Ra to a squarish shape at higher Ra, which is similar to the observation in the 3D situation [46]. However, it should be noted that both the instantaneous and averaged flow fields presented here cannot represent the pure state or typical shape of the LSC as the flow reversals are always accompanied by the growth of the corner rolls, which is bigger for lower Ra. It is further seen that the instantaneous and time-averaged flow fields are more similar at high Ra, thus the wind may have reached a “stable” state with “damped” turbulent fluctuations [7]. It should be noted that the size of the corner flow becomes smaller as Ra increases,

which would cause a weaker influence on flow dynamics, as we will see later.

B. Statistics of turbulent fluctuations

Figure 3 provides the distribution of rms values of velocity and temperature fluctuations. The top row of panels (a)–(c) shows the horizontal rms velocity u_{rms} , the central row (d)–(f) depicts the vertical rms velocity v_{rms} , and the bottom row (g)–(i) gives the rms temperature θ_{rms} . The left, middle, and right columns are for $\text{Ra} = 10^6$, 10^8 , and 10^{10} , respectively. As mentioned above, at low Ra thermal plumes extend approximately over the entire cell, and since the plumes are more likely to travel through the bulk region, the measured magnitudes of rms velocity and temperature are large in the bulk. As Ra increases, turbulence is promoted, and the high-magnitude regions are more concentrated due to the stronger plume mixing. This can also be seen from the distribution of turbulent

FIG. 3. Root-mean-square values of horizontal velocity fluctuations u_{rms} (a)–(c), vertical velocity fluctuations v_{rms} (d)–(f), and temperature fluctuations θ_{rms} (g)–(i). Data are obtained at three different Rayleigh numbers (from left to right): $\text{Ra} = 10^6$ (left panel), 10^8 (middle panel), and 10^{10} (right panel). The data used here are the same as those in Figs. 2(d)–2(f).

FIG. 4. Turbulent kinetic energy $\langle u'^2 \rangle + \langle v'^2 \rangle$ (a)–(c) and turbulent heat flux $\langle v'\theta' \rangle$ (d)–(f) fields for $Ra = 10^6$ (left panel), 10^8 (middle panel), and 10^{10} (right panel).

kinetic energy $\langle u'^2 \rangle + \langle v'^2 \rangle$, as shown in Figs. 4(a)–4(c). Here the prime denotes the fluctuation parts of the velocity or temperature field, and the notation $\langle \cdot \rangle$ denotes time average. It has been found that larger fluctuations are closely related to plume clusters [7]. For the present 2D RB system, there are two main reasons for generating high-magnitude fluctuations: one is the plume-ejecting in the lower-right and upper-left corners where the wind-shearing is smaller compared with the central region of the isothermal plates; the other is the competition between the corner-flow rolls and the LSC, which also causes negative values of the local Nusselt number [39]. Figures 4(d)–4(f) plot the color-coded turbulent heat flux $\langle v'\theta' \rangle$. Hereafter, it is also referred to as turbulent energy production due to the fluctuation buoyancy force, i.e.,

$$P_{\theta'} = \langle v'\theta' \rangle. \quad (4)$$

Simultaneous temperature and velocity measurements in turbulent thermal convection have revealed that high values of heat flux are near the vertical sidewalls, which means that most of the heat is transported by the large-scale convection roll [19]. Similar to the main part of the heat flux $\langle v\theta \rangle$, large amplitudes of turbulent heat flux are associated with the rising and falling plumes as well. This result is also in line with the numerical and experimental Lagrangian measurements in turbulent RB convection [47]. Note that Fig. 4(f) also shows negative values of $\langle v'\theta' \rangle$, which correspond to the local countergradient heat transport in 2D thermal convection [39].

The Reynolds stress $-\langle u'_i u'_j \rangle$ represents the mean momentum transport due to turbulent fluctuations. The main components of the Reynolds stress $-\langle u'v' \rangle$ for three different Ra are illustrated in Figs. 5(a)–5(c). As shown in the figures, with increasing Rayleigh number the Reynolds stress is

increasingly concentrated near the corners, from which most of the plumes travel. As thermal plumes are coherent structures, their different velocity components naturally have an intense degree of correlation that could result in large Reynolds stress [7]. The description of Krishnamurti and Howard [5] hints that turbulent kinetic energy is transferred from turbulent fluctuations to the mean flow, i.e., the integral of turbulent energy production due to mean shear over the whole flow field is negative [48]; here, the turbulent energy production is given by

$$P = -\langle u'_i u'_j \rangle \partial \langle u_i \rangle / \partial x_j \quad (i, j = 1, 2). \quad (5)$$

However, in most turbulent systems, such as the turbulent flows in channels or pipes, boundary layers, jets, wakes, and mixing layers, the integral value of P is usually positive, and therefore the energy is extracted from the mean motion to increase the turbulent kinetic energy of the flow. The distributions of P for three different values of Ra are provided in Figs. 5(d)–5(f). It is seen that there are regions with opposite signs of the turbulent energy production, which means that locally the turbulent kinetic energy can be transferred from the mean flow to the turbulent fluctuations and vice versa. It is further found that the integral of P over the whole cell is negative except for the highest Rayleigh number $Ra = 10^{10}$, implying that the mean wind in turbulent thermal convection can be driven by the Reynolds stress, which is different from previous experimental studies [6,7]. According to the turbulent-viscosity hypothesis, the convection flow displays the negative eddy viscosity phenomenon. One possible explanation for this anomalous phenomenon is usually given by analogy with 2D turbulence, which, under certain conditions, exhibits negative eddy viscosity and other anomalous properties such as “inverse

FIG. 5. Main component of the Reynolds stress $-\langle u'v' \rangle$ (a)–(c), turbulent energy production due to mean shear $P = -\langle u'u'_j \rangle \partial \langle u_i \rangle / \partial x_j$ ($i, j = 1, 2$) (d)–(f), and total turbulent energy production $P_0 = P_{\theta'} + P$ (g)–(i) for $Ra = 10^6$ (left panel), 10^8 (middle panel), and 10^{10} (right panel).

energy cascade” [48]. However, why is there a significant transition at $Ra = 10^{10}$? To understand this, note that the turbulent production is proportional to the mean shear rate $\partial \langle u_i \rangle / \partial x_j$, and both numerical and experimental studies have shown different trends and results of $\partial \langle u_i \rangle / \partial x_j$ at higher Ra . On the one hand, the corner-flow rolls play an important role for the large-scale wind reversal in 2D RB convection, and the flow reversals could result in extremely large velocity fluctuations and even negative values of global Nu [34]. The numerical simulations and experimental investigations all show that for larger Ra , those reversals cannot be observed [24]. At $Ra = 10^8$ and $Pr = 0.1$, Burr *et al.* [6] have shown that the integral of P is positive by solving the two-dimensional Navier-Stokes equations in a square domain. Nevertheless, Sugiyama *et al.* [24] revealed that for this small Pr , the thermal energy carried by plumes can easily become lost due to large thermal diffusivity, and hence the buildup of the corner roll and the flow reversals is suppressed. Further, our recent study has revealed that for high Ra , thermal mixing

is enhanced by the increased turbulent intensity, and then the corner-flow rolls do not have a great influence in the processes of turbulent heat transport for high Ra [39]. On the other hand, in Fig. 2 we suggest that at $Ra = 10^{10}$ the LSC may have reached an “asymptotic” state with the “damped” turbulent fluctuations, i.e., velocity fluctuations become smaller at $Ra = 10^{10}$, as indicated by the color bar in Figs. 3 and 4. The two aspects illustrated above mean that the relative strength of the mean shear rate $\partial \langle u_i \rangle / \partial x_j$ is larger at $Ra = 10^{10}$, which can contribute to a positive turbulent energy production. In addition, for the system considered here, the temperature field becomes an active scalar, and buoyancy force is responsible for the mean motion of the fluid [49]. The distributions of turbulent energy production due to fluctuating buoyancy force $P_{\theta'}$ for the three different Ra are shown in Figs. 4(d)–4(f), and the corresponding maps for total turbulent energy production $P_0 = P_{\theta'} + P$ are depicted in Figs. 5(g)–5(i). The volume averages of $P_{\theta'}$, P , and P_0 at different Ra are presented in Table II. It is shown that although there exist negative values

TABLE II. Volume average of turbulent energy production due to fluctuating buoyancy force $P_{\theta'}$, turbulent energy production due to mean strain P , and total turbulent energy production $P_0 = P_{\theta'} + P$ for different Rayleigh numbers (columns from left to right).

Ra	$\langle P_{\theta'} \rangle_V (\times 10^3)$	$\langle P \rangle_V (\times 10^3)$	$\langle P_0 \rangle_V (\times 10^3)$
1×10^6	1.2780	-0.0164	1.2616
3×10^6	1.3008	-0.0024	1.2984
1×10^7	0.7813	-0.0484	0.7330
3×10^7	0.5237	-0.0529	0.4707
1×10^8	0.5121	-0.0175	0.4946
3×10^8	0.3670	-0.0039	0.3631
1×10^9	0.1887	-0.0190	0.1697
3×10^9	0.1508	-0.0137	0.1370
1×10^{10}	0.1136	0.0218	0.1354

of P_0 in the local regions, the integral of P_0 is positive over the whole cell. Thus, the contributions to total turbulent energy production are dominated by $P_{\theta'}$ in this buoyancy-driven RB convection, and the mean wind is driven both by the Reynolds stress and by the buoyancy of the plumes.

C. Boundary layer

We now examine the BL velocity and temperature profiles. Because the velocity and temperature fluctuations are the main subject of the present paper, we focus here mainly on the rms velocity and temperature fields near the conducting plates. In Fig. 6, the profiles of u_{rms} (a), v_{rms} (b), and θ_{rms} (c) for $\text{Ra} = 10^8$ are plotted, which have summits close to the plates and then decay as the distance from the plates increases. Here, the profiles are obtained by averaging the rms velocity and temperature fluctuations along the x [Figs. 6(a) and 6(c)] or z direction [Fig. 6(b)], i.e., the profiles of u_{rms} , v_{rms} , and θ_{rms} are either horizontally or vertically averaged. The definition of the corresponding BL thicknesses, i.e., horizontal velocity BL thickness δ_u , vertical velocity BL thickness δ_v , and temperature BL thickness δ_θ , is also illustrated in the figures using linear fitting of the profiles near the plates, as proposed by Xin and Xia [50]. Figures 7(a)–7(c) present the normalized profiles $u_{\text{rms}}/u_{\text{rms}}^{\text{max}}$ versus z/δ_u (a), $v_{\text{rms}}/v_{\text{rms}}^{\text{max}}$ versus x/δ_v (b), and $\theta_{\text{rms}}/\theta_{\text{rms}}^{\text{max}}$ versus z/δ_θ (c) at five different values of Ra, where $u_{\text{rms}}^{\text{max}}$, $v_{\text{rms}}^{\text{max}}$, and $\theta_{\text{rms}}^{\text{max}}$ are the first maximum rms values in the corresponding profiles. It is seen that the shapes of the v_{rms} and θ_{rms} profiles do not significantly change with Ra, whereas some scatter can be observed for the u_{rms} profiles

[see Fig. 7(a)]. Note that the observed scatter is located in the crucial region of the BL profiles, i.e., the region that can be used to validate different BL theories and where typically the largest deviations for different types of BLs occur. The BL rms profiles exhibit some kind of self-similarity in the present parameter range, indicating that the BL thicknesses defined here are the relevant length scales for the rms velocity (temperature) fields. In addition, when the normalized profiles of u_{rms} and v_{rms} are plotted in the same figure, as shown in Figs. 7(d)–7(f), one finds reasonable collapse in the vicinity of plates for all measured Ra.

Figure 8(a) shows the Ra dependence of viscous BL thicknesses normalized by the cell height H . The solid circles represent δ_u/H and the upward triangles represent δ_v/H . The dashed line marks the best power-law fits, with $\delta_u/H \sim \text{Ra}^{-0.26 \pm 0.03}$ and $\delta_v/H \sim \text{Ra}^{-0.26 \pm 0.02}$. The result of δ_v/H versus Ra is different from that of the experimental studies by Qiu and Xia in a cubic RB convection cell in water [16,17], which shows $\delta_v/H \sim \text{Ra}^{-0.38 \pm 0.03}$. Figure 8(b) plots the Re dependence of δ_u/H and δ_v/H with the same symbols, and the best power-law fits yield $\delta_u/H \sim \text{Re}^{-0.43 \pm 0.04}$ and $\delta_v/H \sim \text{Re}^{-0.43 \pm 0.04}$, which consist of 3D numerical simulations [51]. The results show that the scalings can be affected by the cell geometry and by the Prandtl number. A possible explanation for this could be attributed to the different large-scale flow modes, which are sensitive to the geometry of the convection cell and are of great importance to the dynamics in the BLs [52]. The horizontal and vertical viscous BL thicknesses and their Ra scalings are in line with the scaling exponent -0.22 given by the Grossmann-Lohse (GL) theory, in which the viscous BL thicknesses are defined through the slope criterion of the mean profiles [18,53]. The normalized thermal BL thicknesses δ_θ/H as a function of Ra and Re are, respectively, given in Figs. 8(c) and 8(d). The best power-law fits to the corresponding data give $\delta_\theta/H \sim \text{Ra}^{-0.30 \pm 0.01}$ and $\delta_\theta/H \sim \text{Re}^{-0.49 \pm 0.01}$. Given the alternative thermal BL thickness defined as $\delta_{\text{th}} = H/(2\text{Nu})$, the unifying GL theory [18,53] assumes that δ_{th} obeys a laminar-like Prandtl-Blasius-Pohlhausen scaling [54]:

$$\delta_{\text{th}}/H \sim \text{Re}^{-1/2} f(\text{Pr}). \quad (6)$$

Our present results suggest that δ_θ follows the same scaling behavior as the theoretical prediction for δ_{th} but with a different magnitude. We also note that, although the viscous and thermal BL thicknesses are thinner when compared with those obtained using the slope method based on the mean

FIG. 6. Profiles of u_{rms} (a), v_{rms} (b), and θ_{rms} (c) near the plates for $\text{Ra} = 10^8$; the solid lines illustrate the definition of boundary layer thickness.

FIG. 7. The normalized profiles for u_{rms} (a), v_{rms} (b), and θ_{rms} (c) at five different values of Ra . Normalized profiles compared between u_{rms} and v_{rms} for $Ra = 10^6$ (d), 10^8 (e), and 10^{10} (f).

velocity and temperature profiles, their Ra dependences do not show significant deviation [14,15,51]. In addition, the scaling exponents of δ_v and δ_θ with respect to Rayleigh and Reynolds numbers do not change in an obvious manner when the BL thicknesses are defined as the distance between the wall and the position at which the rms velocity (temperature) fluctuation reaches its maximum. For reference, Table III summarizes the

obtained scaling exponents for the BL thicknesses from several experimental and numerical studies.

Usually, the local Reynolds number is adopted when considering the scaling relationship between the BL thickness and the Rayleigh number Ra [14]. To do this, we calculate the Reynolds numbers, based on the maximum rms velocities, $Re_{u_{\text{rms}}^{\text{max}}} = u_{\text{rms}}^{\text{max}} H/\nu$ and $Re_{v_{\text{rms}}^{\text{max}}} = v_{\text{rms}}^{\text{max}} H/\nu$, from the rms

FIG. 8. Ra and Re dependence of δ_u , δ_v , and δ_θ . The dashed lines correspond to the least-squares fit to the data and the scaling exponents are summarized in Table III.

TABLE III. Summary of the fitting results for the normalized viscous and thermal BL thicknesses. The fitting parameters are defined through the power laws: $\delta_u/H \sim \text{Ra}^{\gamma_u(\text{Ra})}$, $\delta_u/H \sim \text{Re}^{\gamma_u(\text{Re})}$, $\delta_v/H \sim \text{Ra}^{\gamma_v(\text{Ra})}$, $\delta_v/H \sim \text{Re}^{\gamma_v(\text{Re})}$, $\delta_\theta/H \sim \text{Ra}^{\gamma_\theta(\text{Ra})}$, and $\delta_\theta/H \sim \text{Ra}^{\gamma_\theta(\text{Re})}$. The control parameters and cell geometry are also listed. The sources are as follows: (a) Xin and Xia [50], (b) Qiu and Xia [16,17], (c) Sun *et al.* [14], (d) Zhou and Xia [15], (e) Scheel *et al.* [51], and (f) the present work.

Source	Ra	Pr	Γ	Geometry	$\gamma_u(\text{Ra})$	$\gamma_u(\text{Re})$	$\gamma_v(\text{Ra})$	$\gamma_v(\text{Re})$	$\gamma_\theta(\text{Ra})$	$\gamma_\theta(\text{Re})$
a	10^7 – 10^{11}	~ 7	0.5–4.4	Cylindrical (3D)	–0.25					
b	10^8 – 10^{10}	~ 7	1	Cubic (3D)	–0.25		–0.38			
c	10^9 – 10^{10}	4.3	1	Rectangular (3D)	–0.37					
d	10^{10} – 10^{12}	4.3	1	Rectangular (3D)					–0.31	
e	10^5 – 10^9	0.7	1	Cylindrical (3D)	–0.23	–0.45			–0.27	
f	10^6 – 10^{10}	5.3	1	Square (2D)	–0.26	–0.43	–0.26	–0.43	–0.30	–0.49

profiles. Figure 9(a) shows the Ra dependence of $\text{Re}_{u_{\text{rms}}}^{\text{max}}$ and $\text{Re}_{v_{\text{rms}}}^{\text{max}}$; the leftward triangles represent $\text{Re}_{u_{\text{rms}}}^{\text{max}}$ and the diamonds represent $\text{Re}_{v_{\text{rms}}}^{\text{max}}$. The best power-law fits to the corresponding data give $\text{Re}_{u_{\text{rms}}}^{\text{max}} \sim \text{Ra}^{0.47 \pm 0.06}$ and $\text{Re}_{v_{\text{rms}}}^{\text{max}} \sim \text{Ra}^{0.48 \pm 0.05}$. For reference, the dashed line with a slope of 0.5 is plotted in Fig. 9(a), as suggested by Xia and his co-workers [14,16,17,50]. The viscous BL thicknesses δ_u/H and δ_v/H versus $\text{Re}_{u_{\text{rms}}}^{\text{max}}$ and $\text{Re}_{v_{\text{rms}}}^{\text{max}}$ are provided in log-log plots of Figs. 9(b) and 9(c), respectively. The best power-law fits to the numerical data yield $\delta_u/H \sim \text{Re}_{u_{\text{rms}}}^{\text{max} - 0.52 \pm 0.11}$ and $\delta_v/H \sim \text{Re}_{v_{\text{rms}}}^{\text{max} - 0.53 \pm 0.08}$, which also suggests a Prandtl-Blasius-like scaling from the rms perspective for the present flow fields.

D. Turbulent dissipation rates in the boundary layers

The kinetic ε_u and thermal ε_θ energy dissipation rates are of significance in turbulent RB convection: they are closely related to heat-transport processes in this system, and the two exact relations between dissipation rates and Nusselt number are the backbone of the popular GL theory as well

[1,18,53]. There have been many experimental [55,56] and numerical [57–60] studies focusing on the statistical properties of dissipation rates in 3D turbulent RB convection. In addition, the statistics of ε_u and ε_θ are comprehensively studied in 2D turbulent thermal convection for different Pr and Ra in our former work [44]. In this work, we intend to investigate PDFs of the turbulent kinetic $\varepsilon_{u'}$ and thermal $\varepsilon_{\theta'}$ energy dissipation rates. The definitions of the two quantities are given by

$$\varepsilon_{u'}(x, z, t) = \frac{1}{2} \nu \sum_{i,j} \left[\frac{\partial u'_i(x, z, t)}{\partial x_j} + \frac{\partial u'_j(x, z, t)}{\partial x_i} \right]^2 \quad (7)$$

and

$$\varepsilon_{\theta'}(x, z, t) = \kappa \sum_i \left[\frac{\partial \theta'(x, z, t)}{\partial x_i} \right]^2. \quad (8)$$

Apart from their close relation to the heat transport, $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$ are also considered to play a substantial role in determining the statistical characteristics of local velocity and temperature fluctuations. The turbulent thermal dissipation rate $\varepsilon_{\theta'}$ was

FIG. 9. (a) The rms-velocity-based Reynolds numbers as a function of Ra. The dashed line has a slope of 0.5. (b) The dependence of δ_u/H on $\text{Re}_{u_{\text{rms}}}^{\text{max}}$ and (c) the dependence of δ_v/H on $\text{Re}_{v_{\text{rms}}}^{\text{max}}$. The dashed lines in (b) and (c) are the power-law fits to the corresponding data, which yield $\delta_u/H \sim \text{Re}_{u_{\text{rms}}}^{\text{max} - 0.52 \pm 0.11}$ and $\delta_v/H \sim \text{Re}_{v_{\text{rms}}}^{\text{max} - 0.53 \pm 0.08}$, respectively.

FIG. 10. PDFs of turbulent dissipation rates $\varepsilon_{u'}$ (a) and $\varepsilon_{\theta'}$ (b) normalized, respectively, by their rms values $(\varepsilon_{u'})_{\text{rms}} = \sqrt{\langle \varepsilon_{u'}^2 \rangle_{\text{BL},t}}$ and $(\varepsilon_{\theta'})_{\text{rms}} = \sqrt{\langle \varepsilon_{\theta'}^2 \rangle_{\text{BL},t}}$ (here the notation $\langle \cdot \rangle_{\text{BL},t}$ denotes the average over BL parts and over time) for five different Rayleigh numbers. The solid lines are the best fits of stretched exponentials, as given by Eq. (9). The data are calculated within the respective viscous and thermal BLs and over a sequence of statistically independent snapshots.

first obtained from direct measurements of the temperature gradient vector in a cylindrical cell filled with water by He *et al.* [55] and He and Tong [56]. They concluded that $\varepsilon_{\theta'}$ is mainly generated by thermal plumes, it dominates in the bulk region of the convection cell, and it decreases with increasing Ra. Following their idea, Emran and Schumacher [59] studied the small-scale statistics of temperature and its derivatives in turbulent thermal convection via the DNS method. Their results showed that the tails of the PDFs of $\varepsilon_{\theta'}$ in the bulk region can be depicted by a stretched exponential function and always deviate from Gaussian. Here, we are mainly concerned with the two quantities $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$ within the viscous and thermal BLs, respectively.

The PDFs of turbulent dissipation rates calculated over at least 1000 snapshots equally sampled in a total of 1000 turnover time at different Ra are presented in Fig. 10, where $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$ are normalized, respectively, by their rms values $(\varepsilon_{u'})_{\text{rms}}$ and $(\varepsilon_{\theta'})_{\text{rms}}$. The PDFs of $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$ both display longer tails with increasing Ra, indicating more intense dissipation that may be attributed to the plume emissions from the thermal BLs. The tails of the PDFs become more extended with increasing Ra, which is different from the experimental results obtained in a cylindrical convection cell by He *et al.* [55]. They revealed that the normalized PDFs calculated in the cell center at different Ra can all be brought into agreement. In correspondence with the cases of kinetic and thermal energy dissipation rates in turbulent flows [55,61], the tails of the measured distribution can be described by a stretched exponential function:

$$h(\chi) \approx e^{-c\chi^d}, \quad (9)$$

where χ can be $\varepsilon_{u'}/(\varepsilon_{u'})_{\text{rms}}$ or $\varepsilon_{\theta'}/(\varepsilon_{\theta'})_{\text{rms}}$, and the fitting parameters c and d are given in Table IV. This function form was first introduced to describe the tails of passive scalar dissipation in the limit of large Peclet and Prandtl numbers in 1D and 2D cases [62] and then extended to arbitrary space dimensions [63]. In the present work, we use this function to fit the PDF tails of both $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$. One sees that the value of d decreases monotonically with increasing Ra for both $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$, suggesting an increasing degree of small-scale intermittency within the BLs. This is in agreement with the

numerical results obtained in 3D cylindrical cells [59]. Note that our present fitted d for $\varepsilon_{\theta'}$ obeys the threshold $d > 1/3$ analytically derived for a passive scalar in the Navier-Stokes flow [59,64].

For the derivation of the refined similarity hypotheses, Kolmogorov conjectured that the dissipation rates are log-normally distributed, i.e., their logarithmic fields have a Gaussian distribution [65]. In the thermally driven flow considered here, one may expect that the warm and cold plumes emitted from the thermal BLs near the conducting plates make the BL region especially interesting for the investigation of deviations of the statistical properties of turbulent fluctuations. To shed some light on the study of turbulent dissipation in BLs, we adopt log-normal distributions to characterize the differences. Figure 11 illustrates the PDFs of $\ln(\varepsilon_{u'})$ and $\ln(\varepsilon_{\theta'})$ where the same data sets have been performed in Fig. 10; the PDFs are normalized using the mean and standard deviation values and plotted on a logarithmic scale. The results for all Ra have some common features, i.e., they are asymmetric and they have negative skewness compared with log-normal distribution. As Ra increases, the tails of the PDFs skew to a higher standardized variable due to the stronger intermittency of the flow fields. Similar behavior was also observed by He and Tong [56] in cylindrical convection cells filled with water, and their measurements showed that the PDFs of $\ln(\varepsilon_{\theta'})$ within the temperature BLs have an intense degree of asymmetry compared with those in the cell center, where the plumes are

TABLE IV. The fitting parameters of Eq. (9).

Ra	$c(\varepsilon_{u'})$	$d(\varepsilon_{u'})$	$c(\varepsilon_{\theta'})$	$d(\varepsilon_{\theta'})$
1×10^6	1.622	0.7463	1.407	0.8889
3×10^6	2.133	0.5857	2.136	0.6036
1×10^7	2.265	0.5608	1.649	0.7615
3×10^7	2.202	0.5589	1.729	0.7232
1×10^8	2.424	0.5243	2.123	0.6028
3×10^8	2.494	0.4989	1.900	0.5790
1×10^9	2.778	0.4435	2.200	0.5605
3×10^9	3.347	0.3713	2.319	0.5296
1×10^{10}	3.468	0.3657	2.589	0.4827

FIG. 11. PDFs of $[\ln(\varepsilon_{u'}) - \mu]/\sigma$ and $[\ln(\varepsilon_{\theta'}) - \mu]/\sigma$ for five different values of Ra . Here μ and σ are the mean and standard deviation values of $\ln(\varepsilon_{u'})$ or $\ln(\varepsilon_{\theta'})$.

well mixed and the flow is more homogeneous and isotropic. In addition, it is interesting to find that the measured PDFs obey approximately a Bramwell-Holdsworth-Pinton (BHP) distribution, which is

$$\Pi(y) = K(e^{y-e^y})^a, \quad y = b(w - s), \quad (10)$$

where the parameters are $a = \pi/2$, $b = 0.938$, $s = 0.374$, and $K = 2.14$. This formula was first introduced to characterize rare fluctuations in a confined turbulent flow and critical phenomena [66,67]. The results shown here thus suggest some kind of universal properties of dissipation with respect to different turbulence systems.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have presented results of various properties of the velocity and temperature fluctuations in 2D turbulent RB convection. These were obtained by numerically solving the dimensionless governing equations in vorticity-stream function form in a square domain with Ra increasing from 10^6 to 10^{10} and Pr fixed at 5.3. The instantaneous and time-averaged flow fields are also investigated, which shows that as Ra increases, the shape of the LSC becomes more squarelike and the corner flow rolls have a weaker impact on flow dynamics.

It is found that the large amplitudes of velocity and temperature fluctuations are associated with plume emissions and the competition between the LSC and the corner flows. In addition, the present results reveal that turbulent kinetic energy moves in the “opposite” direction, i.e., from fluctuations to the mean flow, implying that the mean wind in turbulent thermal convection can be driven by the Reynolds stress, in an averaged sense. Thus, apart from the buoyancy of thermal plumes, the Reynolds stress is the reason for the generation of the LSC as well, which is different from the conclusions in 3D experimental studies [6,7]. It should be noted that,

although 2D and 3D convection share some similarities, 2D turbulence has its own feature, such as the inverse energy cascade and negative eddy viscosity phenomenon [48], hence the results observed here cannot readily be translated to 3D convection.

By using the slope method, the boundary layer thicknesses (δ_u , δ_v , and δ_θ) based on the rms profiles are obtained. The analyses reveals that the shape of the normalized rms profiles depend slightly on Ra and exhibit self-similarity near the plates. More interestingly, the normalized profiles of the rms horizontal velocity near the conducting plates and those of the rms vertical velocity near the sidewalls have a similar shape, and thus they can yield the same thickness for a fixed Ra . In addition, the viscous BL thicknesses δ_u and δ_v both scale as $\sim Ra^{-0.26 \pm 0.03}$ and $\sim Re^{-0.43 \pm 0.04}$. Although the thermal BL thicknesses δ_θ and δ_{th} defined by two different ways show different magnitudes, they share similar scalings with respect to Ra and Re . The Reynolds numbers based on the local maximum rms velocity are also calculated, and the results show a Prandtl-Blasius-like scaling relation.

With regard to turbulent dissipation rates $\varepsilon_{u'}$ and $\varepsilon_{\theta'}$ within the BL regions, we find that their PDF tails can be well described by a stretched exponential function. These tails become more extended with increasing Ra due to an increasing degree of small-scale intermittency within the BLs. Furthermore, when the PDFs are plotted on a logarithmic scale, they obey a BHP distribution, which was first introduced to depict rare fluctuations in a confined turbulent flow and critical phenomena [66,67].

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