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A FISCAL POLICY GAME IN A TWO-REGION MONETARY
MODEL

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1. INTRODUCTION

Engerman (1965) and Oates (1972) have examined the feasibility and efficiency of a regional stabilization policy. They both concluded on efficiency grounds that the responsibility for stabilizing the economy should be left to the national government.

This unanimous conclusion has been derived mainly from two theoretical findings. First, regions are in general, small open economies, with a high marginal propensity to import. The employment and income effects resulting from regional expenditure programs will therefore "spill over" to other regions that may already have reached the full employment level. Independent regional stabilization policies will in this way lead to an excessive increase of national aggregate demand.

Second, if regions were allowed to vary direct tax rates in order to finance their expenditure programs, the interregional mobility of capital and labour would, to a large extent, limit the efficiency of a regional stabilization program. For these reasons, regions are considered as inefficient decision units with regard to the stabilization objective.

One of the key arguments against regional stabilization policies originates in the high degree of interdependency of structurally different regions, that may give rise to conflicting policies. Although this framework calls for game theory as the appropriate tool of analysis, a game theoretic approach to regional policy is hard to find [see e.g. Schleicher (1968), De Bruyne and Van Rompuy (1977)]. Furthermore, an empirical verification of the assumed inefficiency of regional stabilization policy has, to our knowledge, only been undertaken by L'Esperance (1977). This empirical analysis suggests the feasibility of a stabilization policy for the region studied, viz. Ohio.

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At the national level, conflicting stabilization policies under decentralized control have been studied by means of small macroeconomic models using linear-quadratic differential games, e.g. by Pindyck (1976) and Kydland (1976).

It is the purpose of this paper to analyse, by means of a game theoretic approach, the conflicting behaviour of two regions, each of which pursues a stabilization policy based on two instruments, viz. debt finance and tax finance. It will be shown by means of a two-region short-run monetary model that the two regions can reach a stable solution, which will be Pareto optimal under certain conditions. Whether a national fiscal policy can do better than decentralized regional policies will ultimately depend on the specification of a national social preference function and on the efficiency of a national redistributive policy.

The two-region model and its policy multipliers will be presented in section 2. Section 3 contains the game format of the policy conflict and the equilibrium solution.

Under certain conditions, a cooperative solution may emerge. This will be discussed in section 4. Section 5 deals with possible improvements of the outcome, e.g. by organizing the stabilization policy nationally. Finally, an evaluation of the game approach to regional policy conflicts concludes the paper.

2. A TWO-REGION MONETARY MODEL

2.1. General outline

The effects of regional fiscal policy will be analyzed here by means of a Keynesian type, two-region model with money included. The two-region model is closed in the sense that no relations with the rest of the world are considered.

Each region aims at increasing regional income. The income target is identical with the employment target because of the implicitly assumed short-run proportionality between income and employment.

In order to achieve its short-run policy target, each region will increase expenditures. For this purpose it can rely on two instruments, viz. issuing bonds (perpetuities of face value \$ 1) or increasing tax revenue. The latter is restricted to an income induced tax increase and to a raise of the property tax.

Bonds, issued by regional authorities and yielding an interest payment of \$ 1 a year, are assumed to be perfect substitutes that can be transacted between regions without cost. Hence, interregional interest rate differentials cannot persist or, in other words, the money market is assumed to be national.

Each regional authority faces a budget constraint, stating that the nominal regional deficit should equal the change in the nominal stock of bonds issued by that region.

The total stock of bonds issued is distributed between the residents of the two regions and is a component of national wealth.

The national government does not appear explicitly in the model in order to keep its size operational. However, it is assumed that the sum of the regional budgets is fixed and determined by the national government, e.g. in the framework of a national monetary policy. The regional distribution of the overall budget is determined exogenously, e.g. by constitutionally specified rules of regional budget allocation. The model also allows for regional autonomy in collecting income and property tax revenue. As such, the model outlined here is a regional extension of the models analyzed by Christ (1968), Blinder and Solow (1973) and Hansen (1973). It can also be interpreted as a two-country model with fixed exchange rates and an integrated money market.

2.2. Specification of the model

The fourteen regional and three national equations are specified as follows (j is the regional subscript, $j = 1, 2$) :

$$Y_j = E_j + G_j + X_j - M_j \quad (1) \quad (2)$$

$$E_j = E_j(Y_j^d, r, \delta_j W) \quad (3) \quad (4)$$

$$Y_j^d = Y_j - T_j + \delta_j B \quad (5) \quad (6)$$

$$T_j = t_j Y_j + \tau_j \quad (7) \quad (8)$$

$$P(G_j + B_j - T_j) = \frac{\Delta B_j}{r} \quad (9) \quad (10)$$

$$X_j = M_i \quad (i \neq j) \quad (11) \quad (12)$$

$$M_j = m_j Y_j \quad (13) \quad (14)$$

$$L^d = L^d(r, Y, W) \quad (15)$$

$$H = L^d \quad (16)$$

$$W = K + \frac{H}{P} + \frac{B}{P \cdot r} \quad (17)$$

$$B = \sum B_j \quad (18)$$

$$Y = \sum Y_j \quad (19)$$

The model is specified in discrete time; the time subscript has been omitted. Equations (18) and (19) define national aggregates.

Equations (1) (2) define the goods-market equilibrium, stating that regional real income (Y_j) equals the sum of regional real private expenditure (E_j), real government expenditure (G_j), real regional exports (X_j) minus regional imports (M_j). Equations (3) (4) explain real private expenditure by real disposable income (Y_j^d), by the interest rate (r) and by the regional share of total wealth, $\delta_j W$.

Equations (5) (6) define real disposable income as the difference between real income and taxes (T_j) plus a regional share δ_j of the interest payments on the total stock of outstanding bonds B , each of face value \$ 1. Since it is assumed that each bond is a perpetuity paying \$ 1 per year, yearly interest payments are B . Equations (7) (8) specify real regional tax revenue

as the sum of a regional income tax ($t_j Y_j$) and of a property tax τ_j ¹⁾. Equations (9) (10) are the regional budget constraints stating that the nominal regional deficit, allowing for interest payments B_j , equals the change in the number of regional bonds, evaluated at their market price or $\Delta B_j / r$, where ΔB_j equals $B_j - B_{j-1}$. The price level P is assumed to be constant in the two regions and exogenously fixed at 1.0. Equations (11) (12) state the usual identity of exports of one region and imports of the other region. Exports to and imports from abroad do not appear in the model. Imports are assumed to be a linear homogeneous function of income in (13) and (14) ²⁾. The national demand for money L^d depends, according to (15), on national income Y , on the rate of interest and on real wealth W . This "national" equation can be interpreted as an aggregate of linear regional demand for money equations. Equation (16) states the money market equilibrium where H is the nominal high-powered national money stock.

Real wealth (equation 17) consists of the private stock of real capital K , of the real money stock H/P and of the real market value of the stock of bonds (B/Pr). The total stock of bonds and national income are defined in (18) and (19) ³⁾.

One may wonder why the model does not allow explicitly for adjustments in regional portfolios in response to regional balance of trade disequilibria.

These disequilibria will appear as a consequence of regional differences in the change of income and (or) in the marginal propensities to import from the other region. Suppose that region 1 would end up with a trade deficit. This implies that region 1 has an excess demand for goods whereas region 2 shows an excess supply of goods. Overall equilibrium requires that the

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- 1) It is assumed that income derived from interest payments is tax free. If interest income should be subjected to the overall income tax rate t_j , the exposition would not change substantially. In fact $\delta_j B$ could then be interpreted as the after tax regional share of interest payments.
 - 2) For simplicity, imports depend here on regional income Y_j and not on regional disposable income Y_j^d , as is frequently assumed. In addition, the import content of government expenditure is assumed the same as for private expenditure.
 - 3) The demand functions for bonds do not appear explicitly in the model. According to Walras' law, equilibrium in the goods and money market will imply equilibrium in the bond market. Regional demand functions for bonds can be derived if regional consumption and investment functions would have been stated explicitly.

trade deficit of region 1 is matched by an excess supply of financial assets (money and bonds). However, it is not clear in which proportion the deficit will cause an outflow of money and bonds from region 1 to region 2. The mix of financial assets in the adjustment process will depend on the initial regional distribution of wealth and on its composition as well as on the characteristics of the regional demand functions for money and bonds. Since the latter have not been specified, changes in regional financial portfolios cannot be traced. It will be assumed in the sequel that the regional distribution of the stock of bonds as represented by the parameters δ_j in equations (5) (6), will not be changed significantly by first period adjustments in regional portfolios. The same restrictive assumption holds for the regional distribution of the other wealth components, viz. physical capital and money, which are distributed in the same proportions as the stock of bonds.

The model contains 19 endogenous variables: $Y, Y_j, E_j, X_j, M_j, Y_j^d, T_j, B_j, r, B, L^d, W$ and 7 exogenous variables: G_j, τ_j, P, K, H ($j = 1, 2$).

The model will be written in differential form in order to calculate the reduced form and the first period policy multipliers in a comparative static analysis.

$$dY_j = dE_j + dG_j + dX_j - dM_j \quad (1') \quad (2')$$

$$dE_j = \epsilon_{yj} dY_j^d + \bar{\epsilon}_{rj} dr + \epsilon_{wj} \delta_j dW \quad (3') \quad (4')$$

$$dY_j^d = dY_j - dT_j + \delta_j dB \quad (5') \quad (6')$$

$$dT_j = t_j dY_j + Y_j dt_j + d\tau_j \quad (7') \quad (8')$$

$$dG_j + dB_j - dT_j = \frac{dB_j}{r} \quad (9') \quad (10') \quad 1)$$

$$dX_j = dM_i \quad (i \neq j) \quad (11') \quad (12')$$

1) The r.h.s. of these equations is derived as follows:

$$\frac{d(\Delta B_j)}{r} = \frac{d(B_j - B_{j-1})}{r} = \frac{dB_j}{r} \text{ if first period deviations from a long run equilibrium, in which the stock of bonds is constant, are to be evaluated (i.e. } dB_{j-1} = 0).$$

$$dM_j = m_j dY_j \quad (13') \quad (14')$$

$$dL^d = h_r dr + h_y dY + h_w dW \quad (15')$$

$$dH = dL^d \quad (16')$$

$$dW = dK + dH + \frac{dB}{r} - \frac{Bdr}{r^2} \quad (17')$$

$$dB = \sum dB_j \quad (18')$$

$$dY = \sum dY_j \quad (19')$$

Since regions are not allowed to change the income tax rate t_j , $dt_j = 0$. Furthermore, it is assumed that the stock of private real capital and the stock of high-powered money are constant, i.e. $dH = dK = 0$.

A number of parameters are restricted in the usual way, viz.:

$$\begin{aligned} 0 < \varepsilon_{yj}, \varepsilon_{wj} < 1, & \quad -\infty < \bar{\varepsilon}_{rj} < 0, & \quad 0 < t_j < 1, \\ 0 < m_j < 1, & \quad -\infty < h_r < 0, & \quad 0 < h_y, h_w < \infty \end{aligned}$$

Solving (15'), (16') and (17') for the change in the equilibrium rate of interest yields :

$$dr = \omega_y dY + \omega_b dB \quad (20)$$

where

$$\omega_y = \frac{\lambda_y}{1 - \lambda_w \rho_r}, \quad \omega_b = \frac{\lambda_w \rho_b}{1 - \lambda_w \rho_r}$$

and

$$\lambda_y = -\frac{h_y}{h_r} > 0, \quad \lambda_w = -\frac{h_w}{h_r} > 0$$

$$\rho_b = \frac{1}{r} > 0, \quad \rho_r = -\frac{B}{r^2} < 0$$

Hence, the marginal interest rate effects of income ω_y and of bonds ω_b are both positive.

2.3. First Period Policy Effects

Solving the model (1') to (19'), using (20), for dY_j yields :

$$dY_j = \varepsilon_{yj} (dY_j - t_j dY_j - d\tau_j) + \omega_y (\varepsilon_{wj} \delta_j \rho_r + \bar{\varepsilon}_{rj}) dY \\ + [\varepsilon_{yj} \delta_j + \bar{\varepsilon}_{rj} \omega_b + \varepsilon_{wj} \delta_j (\rho_b + \rho_r \omega_b)] dB + dG_j + m_i dY_i - m_j dY_j \quad (21) \quad (22)$$

Define

$$\varepsilon_{bj} = \varepsilon_{yj} \delta_j + \varepsilon_{rj} \omega_b + \varepsilon_{wj} \delta_j \rho_b \geq 0 \quad (23)$$

as the marginal expenditure effect of a bond issue, where

$$\varepsilon_{rj} = \varepsilon_{wj} \delta_j \rho_r + \bar{\varepsilon}_{rj} < 0 \quad (24)$$

The marginal expenditure effect ε_{bj} consists of three components. First, an increase in the number of bonds will entail an increase in interest payments which contribute to disposable income and hence, to private expenditure ($\varepsilon_{yj} \delta_j$).

Second, a bond issue will, through the increased demand for money lead to higher interest rates (see eq. 20) and the latter will result in capital losses for bond holders. The total interest effect is represented by $\varepsilon_{rj} \omega_b$, which is negative and consists of the interest effect $\bar{\varepsilon}_{rj}$ and of the capital gains or losses effect, $\varepsilon_{wj} \delta_j \rho_r$.

Third, larger bond holdings will have a positive wealth effect ("the Pigou effect") on private expenditure; the latter is contained in $\varepsilon_{wj} \delta_j \rho_b$.

On balance, the sign of the marginal expenditure effect ε_{bj} is indeterminate.

If the interest effect outweighs the positive wealth and interest payments effects, an increase in debt financed government expenditure will lead to a third level of "crowding out", according to the terminology of Blinder and Solow (1973).

Rearranging terms in (21) (22) and treating taxes as exogeneous, we obtain:

$$\begin{bmatrix} 1 - \epsilon_{y1} - \epsilon_{r1}\omega_y + m_1 & -(\epsilon_{r1}\omega_y + m_2) \\ -(\epsilon_{r2}\omega_y + m_1) & 1 - \epsilon_{y2} - \epsilon_{r2}\omega_y + m_2 \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \end{bmatrix} = \begin{bmatrix} \epsilon_{b1} & \epsilon_{b1} & 1 & 0 & -\epsilon_{y1} & 0 \\ \epsilon_{b2} & \epsilon_{b2} & 0 & 1 & 0 & -\epsilon_{y2} \end{bmatrix} \begin{bmatrix} dB_1 \\ dB_2 \\ dG_1 \\ dG_2 \\ dT_1 \\ dT_2 \end{bmatrix} \quad (25)$$

Finally, solving for dY_1 and dY_2 gives :

$$\begin{bmatrix} dY_1 \\ dY_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} 1 - \epsilon_{y2} - \epsilon_{r2}\omega_y + m_2 & \epsilon_{r1}\omega_y + m_2 \\ \epsilon_{r2}\omega_y + m_1 & 1 - \epsilon_{y1} - \epsilon_{r1}\omega_y + m_1 \end{bmatrix} \times \begin{bmatrix} \epsilon_{b1} & \epsilon_{b2} & 1 & 0 & -\epsilon_{y1} & 0 \\ \epsilon_{b2} & \epsilon_{b2} & 0 & 1 & 0 & -\epsilon_{y2} \end{bmatrix} \begin{bmatrix} dB_1 \\ dB_2 \\ dG_1 \\ dG_2 \\ dT_1 \\ dT_2 \end{bmatrix} \quad (26)$$

and where

$$\begin{aligned} D = & (1 - \epsilon_{y1})(1 - \epsilon_{y2}) + (1 - \epsilon_{y2})(m_1 - \epsilon_{r1}\omega_y) + (1 - \epsilon_{y1})(m_2 - \epsilon_{r2}\omega_y) \\ & - \omega_y(m_1 + m_2)(\epsilon_{r1} + \epsilon_{r2}) \end{aligned} \quad (27)$$

which, under the assumptions about the parameter values stated above, is clearly positive.

Since regional authorities have no power to issue money in order to finance a deficit, two basic policies will be considered here. An increase in government expenditures can be financed either by issuing bonds (policy 1) or by increasing taxes (policy 2). The instruments of the balanced-budget policy 2 will be restricted to an induced increase in the income tax revenue and to an increase in the property tax. A mix of the basic policies will be considered in the sequel.

The two budget constraints that correspond to these policies are :

$$\text{policy 1 : } dG_j = \left(\frac{1}{r} - 1\right)dB_j = \rho^{-1}dB_j; dT_j = 0 \quad (28)$$

$$\text{where } \rho = \frac{r}{1-r} > 0$$

$$\text{policy 2 : } dG_j = dT_j = t_j dY_j + d\tau_j; dB_j = 0 \quad (29)$$

Using (26), (28) and (29), the one period income effects of the two basic policies can now be evaluated. The multipliers will be indicated by $\partial Y_i / \partial G_{jh}$, where $i, j = 1, 2$ and h refers to the basic policy used : $h = 1$ points to a debt financed expenditure increase, $h = 2$ refers to a balanced-budget expenditure increase. Note that policy 1 implies a tax cut of magnitude $Y_j dt_j + d\tau_j$ in order to compensate for the induced increase in tax revenue $t_j dY_j$, so that $dT_j = 0$.

Policy 1 multipliers

$$\frac{\partial Y_1}{\partial G_{11}} = \frac{1}{D} [a_{11} (1 + \rho \epsilon_{b1}) + a_{12} \rho \epsilon_{b2}] \quad (30a)$$

$$\frac{\partial Y_2}{\partial G_{11}} = \frac{1}{D} [a_{21} (1 + \rho \epsilon_{b1}) + a_{22} \rho \epsilon_{b2}] \quad (30b)$$

$$\frac{\partial Y_1}{\partial G_{21}} = \frac{1}{D} [a_{11} \rho \epsilon_{b1} + a_{12} (1 + \rho \epsilon_{b2})] \quad (30c)$$

$$\frac{\partial Y_2}{\partial G_{21}} = \frac{1}{D} [a_{21} \rho \epsilon_{b1} + a_{22} (1 + \rho \epsilon_{b2})] \quad (30d)$$

Policy 2 multipliers

$$\frac{\partial Y_1}{\partial G_{12}} = \frac{1}{D} [a_{11} (1 - \epsilon_{y1})] \quad (31a)$$

$$\frac{\partial Y_2}{\partial G_{12}} = \frac{1}{D} [a_{21} (1 - \epsilon_{y1})] \quad (31b)$$

$$\frac{\partial Y_1}{\partial G_{22}} = \frac{1}{D} [a_{12} (1 - \epsilon_{y2})] \quad (31c)$$

$$\frac{\partial Y_2}{\partial G_{22}} = \frac{1}{D} [a_{22} (1 - \epsilon_{y2})] \quad (31d)$$

In the above expressions, the a_{ij} refer to the elements of the first matrix in the r.h.s. of (26) i.e.:

$$a_{ii} = 1 - \epsilon_{yj} - \epsilon_{rj}^{\omega_y} + m_j > 0 \quad (32)$$

$$a_{ij} = \epsilon_{ri}^{\omega_y} + m_j \geq 0 \quad (i \neq j) \quad (33)$$

For the evaluation of the income multipliers, the signs of the a_{ij} , ϵ_{bi} as well as of $(1 + \rho\epsilon_{bi})$ are crucial.

(i) If the marginal propensity to import in the other region (m_j) outweighs the income induced negative marginal interest rate effect ($\epsilon_{ri}^{\omega_y}$) on private expenditure in the region considered ($a_{ij} > 0$; $i \neq j$) and if in addition the marginal expenditure effect of a bond issue (ϵ_{bi}) is positive, both regions clearly prefer debt finance above a balanced-budget policy [compare multipliers (30a) with (31a) and (30d) with (31d)]. Moreover, each region prefers that the other region uses the same policy, i.e. debt finance [compare multipliers (30b) with (31b) and (30c) with (31c)]. These findings are straightforward and are embedded in textbook treatments of fiscal policy effects in open economies [see e.g. Peacock and Shaw (1976)].

(ii) Identical regional policy preferences will also arise if $a_{ij} > 0$ ($i \neq j$) and if $\rho\epsilon_{bi} < -1$. For the latter condition to hold, it will be necessary that ϵ_{bi} is negative, since $\rho > 0$. Comparing now the relevant multipliers, it appears that both regions prefer a balanced-budget policy above debt finance, since the latter reduces regional income.

(iii) Consider now the case where the a_{ij} are negative, i.e. if for region i the income induced negative interest effect on expenditures dominates the income induced increase in exports to region j . Looking at the multipliers (31b) and (31c) it is clear that both regions will not appreciate the other region to pursue a balanced-budget policy, since the latter decreases income in the own region. However, this finding does not necessarily imply that both regions will, on balance, prefer a debt issue.

(a) Suppose first that the ϵ_{bj} are positive. Both regions will prefer policy 1 to policy 2 if :

$$a_{ii} (1 + \rho\epsilon_{bi}) + a_{ij}\rho\epsilon_{bj} > a_{ii} (1 - \epsilon_{yi})$$

or if

$$a_{ii} (\epsilon_{yi} + \rho\epsilon_{bi}) + a_{ij}\rho\epsilon_{bj} > 0 \quad (34)$$

If, in addition, they also prefer the other region to use policy 1 to policy 2, it is required that :

$$a_{ii}\rho\epsilon_{bi} + a_{ij} (1 + \rho\epsilon_{bj}) > a_{ij} (1 - \epsilon_{yj})$$

or that :

$$a_{ii}\rho\epsilon_{bi} + a_{ij} (\epsilon_{yj} + \rho\epsilon_{bj}) > 0 \quad (35)$$

Conditions (34) and (35) can hold simultaneously, only if $\epsilon_{b1} = \epsilon_{b2}$, since $\epsilon_{yi} > 0$. So, a collective preference for floating bonds should be considered as a special case.

Alternatively, if $\epsilon_{b1} \neq \epsilon_{b2}$, e.g. if $\epsilon_{b1} > \epsilon_{b2}$, a conflict will arise since region 1 prefers a bonds issue and would like the other region to do the same, whereas region 2 is in favour of a balanced-budget policy but prefers region 1 to issue bonds.

- (b) Suppose next that the ϵ_{bi} are negative and more precisely that $\epsilon_{bi} < -1$. It can be shown in the same way as for (a) that if both regions prefer policy 1, they will also prefer the other region to use policy 2 which leads to a policy conflict.

If on the other hand, both regions prefer policy 2, it is feasible that they are also in favour of the other region's choice of policy 2, viz. if :

$$\epsilon_{y1} > \rho (\epsilon_{b1} - \epsilon_{b2}) < \epsilon_{y2} \quad (36)$$

Up to now, conditions have been examined in which the sign restrictions on the crucial parameters were symmetric for the two regions. It is clear that the set of possible cases can be extended if asymmetry with regard to sign and size restrictions are allowed for.

One such case will be treated in the sequel. In order to stress the conflict situation, asymmetry will be introduced with respect to two important parameters.

First, it is assumed that region 1 is the "richer" region in the sense that it holds the largest share of wealth :

$$\delta_1 > \delta_2 \quad (37a)$$

Second, it is assumed that ϵ_{b1} , the marginal expenditure effect of a bond issue in region 1, is positive and in absolute value larger than the negative expenditure effect in region 2 :

$$\epsilon_{b1} > 0; \epsilon_{b2} < 0 \text{ and } \epsilon_{b1} + \epsilon_{b2} > 0 \quad (37b)$$

In other words, bond financed government expenditure "crowds out" private expenditure in region 2, but not in region 1.

Third, region 1 has the largest income and wealth effects on private expenditure and the smallest interest effects, i.e. :

$$\varepsilon_{y1} > \varepsilon_{y2}; \varepsilon_{w1} > \varepsilon_{w2}$$

$$|\varepsilon_{r1}| < |\varepsilon_{r2}| ; |\bar{\varepsilon}_{r1}| < |\bar{\varepsilon}_{r2}| \quad (37c)$$

Finally, the import coefficients are identical, $m_1 = m_2$ and the a_{ij} negative or

$$m_i < |e_{rj} \omega_y| \quad (37d)$$

It follows from the above assumptions that

$$a_{11} > a_{22}$$

and that

$$|a_{21}| > |a_{12}|$$

Taking account of the above restrictions, it appears from the comparison of the multipliers (30) and (31) that region 1 is in favour of policy 1 and prefers the other region to apply the same policy.

Region 2 is inclined to use policy 2 itself if :

$$(i) \frac{\rho \varepsilon_{b2} + \varepsilon_{y2}}{\rho \varepsilon_{b1}} < - \frac{a_{21}}{a_{22}}$$

and prefers region 1 to use also policy 2 if :

$$(ii) \frac{\rho \varepsilon_{b2}}{\rho \varepsilon_{b1} + \varepsilon_{y1}} < - \frac{a_{21}}{a_{22}}$$

The latter conditions will hold unambiguously if, in addition to the restrictions (37a to d) it is assumed that

$$\rho \varepsilon_{b2} + \varepsilon_{y2} \leq 0 \quad (37e)$$

Since, condition (i) is the most restrictive one, it will furthermore be assured that :

$$\frac{\rho\epsilon_{b2} + \epsilon_{y2}}{\rho\epsilon_{b1}} > \frac{\rho\epsilon_{b2}}{\rho\epsilon_{b1} + \epsilon_{y1}} \quad (37f)$$

The above restrictions on the parameters of the model set the scene for a conflict of stabilization policies used by two regions characterized by opposite preference orderings over the policy instruments : each region prefers a different instrument and would like the other region to use his preferred instrument. Two remarks should be made in this context. First, the conflict analyzed here arises from the use of different policy instruments and not from the pursuit of conflicting objectives, which may have their roots in different regional structures.

Second, the policy effects of different instruments have been compared on a per dollar expenditure basis. In the sequel, assumptions will be made about the absolute size of the regional expenditure programs.

3. A GAME FORMAT AND COMPETITIVE SOLUTIONS

3.1. The Game Format

The conflict situation described above can be characterized as a two-person non-zero sum game. Each player, i.e. each region, can use a convex combination of two strategies in order to maximize his criterion function, i.e. regional income. The two strategies correspond to the basic policies discussed in section 2, viz. policy 1 (bond issue) and policy 2 (balanced-budget). In the foregoing, it was assumed that each region uses either of these policies. However, a combination of the basic policies is also feasible.

Let μ_i denote the share of the budget of the i -th region financed by policy 1 and $1 - \mu_i$ the share of the budget financed by policy 2, where

$$0 \leq \mu_i \leq 1 \quad (38)$$

A pure strategy corresponds to zero-one values of μ_i , whereas a mixed strategy implies that $0 < \mu_i < 1$.

Since each region's income depends on its own policies as well as on those used by the other region, a marginal increase of income of the i -th region can be written as :

$$dY_i = \mu_i \frac{\partial Y_i}{\partial G_{i1}} dG_{i1} + (1-\mu_i) \frac{\partial Y_i}{\partial G_{i2}} dG_{i2} + \mu_j \frac{\partial Y_i}{\partial G_{j1}} dG_{j1} + (1-\mu_j) \frac{\partial Y_i}{\partial G_{j2}} dG_{j2} \quad (39)$$

The marginal increase of regional income resulting from the use of mixed or pure strategies corresponds to the payoff for one player. All possible pairs of payoffs (dY_i, dY_j) , calculated as in (39) are contained in the attainable payoff set.

The payoffs for region 1, that result from pure strategies, are denoted by o_{hk} ($h, k = 1, 2$) and derived from (39) :

$$o_{hk} = \frac{\partial Y_1}{\partial G_{1h}} dG_{1h} + \frac{\partial Y_1}{\partial G_{2k}} dG_{2k} \quad (h, k = 1, 2) \quad (40)$$

The subscripts h, k refer to the policy number (1,2) used by regions 1,2; e.g. o_{12} indicates the pay-off resulting from policy 1 by region 1 and policy 2 by region 2.

In the same way, the payoffs for region 2 can be represented by p_{hk} :

$$p_{hk} = \frac{\partial Y_2}{\partial G_{1h}} dG_{1h} + \frac{\partial Y_2}{\partial G_{2k}} dG_{2k} \quad (h, k = 1, 2) \quad (41)$$

The payoff matrices O, P that appear in table 1 and 2 contain the pure-strategy payoffs o_{hk}, p_{hk} . The latter have been calculated from the first period multipliers (30), (31) assuming that each region increases its expenditures by a (marginal) amount dG_{ih} and omitting the common proportionality factor D^{-1} .

A careful inspection of the payoff matrices reveals dominance features of rows and columns.

In the payoff matrix O for region 1, the first row dominates the second, whereas the first column dominates the second.

1) Solutions to the game problems are invariant to positive linear transformations of the payoffs [see e.g. Luce and Raiffa (1957), p. 72-73].

Table 1 - Payoff matrix 0 for region 1

Strategies region 2 region 1	1	2
1	$[a_{11}(1+\rho\epsilon_{b1}) + a_{12}\rho\epsilon_{b2}]dG_1 + [a_{11}\rho\epsilon_{b1} + a_{12}(1+\rho\epsilon_{b2})]dG_2$	$[a_{11}(1+\rho\epsilon_{b1}) + a_{12}\rho\epsilon_{b2}]dG_1 + a_{12}(1-\epsilon_{y2})dG_2$
2	$a_{11}(1-\epsilon_{y1})dG_1 + [a_{11}\rho\epsilon_{b1} + a_{12}(1+\rho\epsilon_{b2})]dG_2$	$a_{11}(1-\epsilon_{y1})dG_1 + a_{12}(1-\epsilon_{y2})dG_2$

Table 2 - Payoff matrix P for region 2

<div>Strategies region 2</div> <div>region 1</div>	<div>1</div> <div>2</div>	
	1	2
1	$[a_{22}(1+\rho\epsilon_{b2}) + a_{21}\rho\epsilon_{b1}] dG_2 + [a_{22}\rho\epsilon_{b2} + a_{21}(1+\rho\epsilon_{b1})] dG_1$	$[a_{22}\rho\epsilon_{b2} + a_{21}(1+\rho\epsilon_{b1})] dG_1 + a_{22}(1-\epsilon_{y2})dG_2$
2	$[a_{22}(1+\rho\epsilon_{b2}) + a_{21}\rho\epsilon_{b1}] dG_2 + a_{21}(1-\epsilon_{y1})dG_1$	$a_{22}(1-\epsilon_{y2})dG_2 + a_{21}(1-\epsilon_{y1})dG_1$

The row difference h equals :

$$h = o_{1k} - o_{2k} = [a_{11}(\epsilon_{y1} + \rho\epsilon_{b1}) + a_{12}\rho\epsilon_{b2}]dG_1 > 0 \quad (k = 1, 2) \quad (42)$$

whereas the column difference l is :

$$l = o_{h1} - o_{h2} = [a_{11}\rho\epsilon_{b1} + a_{12}(\rho\epsilon_{b2} + \epsilon_{y2})]dG_2 > 0 \quad (h = 1, 2) \quad (43)$$

In the matrix P for region 2, the second row and column dominate the first one :

$$\text{row difference} : m = p_{2k} - p_{1k} = -[a_{21}(\epsilon_{y1} + \rho\epsilon_{b1}) + a_{22}\rho\epsilon_{b2}]dG_1 > 0 \quad (44) \\ (k = 1, 2)$$

$$\text{column difference} : k = p_{h2} - p_{h1} = -[a_{22}(\epsilon_{y2} + \rho\epsilon_{b2}) + a_{21}\rho\epsilon_{b1}]dG_2 > 0 \quad (45) \\ (h = 1, 2)$$

The constant-difference feature of the rows and columns in the payoff matrices is not accidental. It can be shown that it results from the use of a linear model combined with a linear criterion function for each region.

The dominance features of the payoff matrices reveal the dominance of pure strategies. Because of row dominance, region 1 will prefer policy 1 whatever policy region 2 may choose, whereas column dominance will lead region 2 to prefer strategy 2. If region 1 applies its preferred strategy, the best it can hope for is that region 1 also uses the same strategy (policy 1) in view of the column dominance in 0. Row dominance in P implies that region 1 would like region 2 to stick to the same strategy (policy 2).

Whether a stable situation for both players is feasible depends on the existence of an equilibrium pair of strategies. The latter is defined as a strategy pair, corresponding to a payoff pair (o^x, p^x) , that offers no incentives for any player to use a different strategy if the opponent plays his equilibrium strategy. Non-zero, non-cooperative games are said to be solvable

in the sense of Nash if an equilibrium pair exists [see Luce and Raiffa (1957), p. 106].

It appears from the payoff matrices O and P , written in a simplified format below, that $(1, 2)$ is the equilibrium pair of strategies, i.e. when region 1 uses policy 1 and region 2 applies policy 2.

$$O = \begin{bmatrix} o_{11} & o_{11}^{-1} \\ o_{11}^{-h} & o_{11}^{-h-1} \end{bmatrix} \quad P = \begin{bmatrix} p_{11} & p_{11}^{+k} \\ p_{11}^{+m} & p_{11}^{+m+k} \end{bmatrix} \quad (46)$$

The corresponding payoff pair (o^x, p^x) is : $(o_{11}^{-1}, p_{11}^{+k})$.

The competitive solution to the stabilization policy game leads region 1 to issue bonds and region 2 to raise taxes in order to finance their expenditure programs. The question arises whether the equilibrium strategies yield a Pareto-optimal allocation of payoffs, i.e. an allocation that cannot be improved for any region by the choice of strategies, different from the equilibrium pair, without resulting in a less preferred allocation for the other region.

This point can be clarified by combining the two payoff matrices O and P into a matrix Q , the elements of which are the payoff pairs (o_{hk}, p_{hk}) , $(h, k = 1, 2)$:

$$Q = \begin{bmatrix} (o_{11}, p_{11}) & (o_{11}^{-1}, p_{11}^{+k}) \\ (o_{11}^{-h}, p_{11}^{+m}) & (o_{11}^{-h-1}, p_{11}^{+m+k}) \end{bmatrix} \quad (47)$$

Moving from the equilibrium payoff pair q_{12} to the pairs q_{11} and q_{22} entails losses for one region. The allocation of payoffs corresponding to q_{21} may imply an improvement for at least one region if $1 > h$ and $m \geq k$, or if $1 \geq h$ and $m > k$.

Hence if $1 < h$ and $m < k$, the equilibrium pair of strategies implies a Pareto optimal allocation of payoffs in pure strategies.

A graphical representation of the payoff set will illustrate the conditions for the equilibrium to be Pareto optimal.

The feasible pairs of payoffs correspond to the coordinates in the dY_1, dY_2 plane of points contained in a parallelogram ACDB, where the coordinates of the cornerpoints are the pairs q_{hk} of Q resulting from the use of pure strategies ¹⁾. Payoffs resulting from mixed strategies are convex combinations of the pure strategy payoffs; the points corresponding to the former lie on line segments joining the cornerpoints.

The 4 possible combinations of $h \geq 1, k \geq m$ correspond to 4 different payoff configurations that appear in figures 1a to 1c, assuming positive payoffs for both regions.

In the four figures, the equilibrium pair of strategies leads to the payoff pair, represented by point B.

Hence, only in figure 1a, where $h > 1$ and $k > m$, will the strategy pair (1,2) be an equilibrium pair and at the same time imply a Pareto optimal allocation of payoffs. In figure 1b, there are payoff pairs, i.e. on the line segments AC and CD that are Pareto better than the one in B. Also in figures 1c and 1d can allocations be realized that are Pareto better than those in the equilibrium point B.

It will be shown that the restrictions imposed on the parameters of the model will only allow for the configurations in figure 1a and 1c.

Using (42), (43), (44), (46) and remembering that

$$dG = dG_1 + dG_2 \quad (48)$$

i.e. that the total regional budget is fixed by the national authority, we have :

¹⁾ A straight line configuration would result if $1/k = h/m$. However, this condition is not fulfilled here.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1d

$$h - 1 = (\alpha_1 + \beta_1) dG_1 - \beta_1 dG \quad (49)$$

$$k - m = (\beta_2 - \alpha_2) dG_1 - \beta_2 dG \quad (50)$$

where

$$\alpha_1 = a_{11}(\epsilon_{y1} + \rho\epsilon_{b1}) + a_{12}\rho\epsilon_{b2}; \quad \alpha_1 dG_1 = h \quad (51a)$$

$$\alpha_2 = a_{22}\rho\epsilon_{b2} + a_{21}(\epsilon_{y1} + \rho\epsilon_{b1}); \quad -\alpha_2 dG_1 = m \quad (51b)$$

$$\beta_1 = a_{11}\rho\epsilon_{b1} + a_{12}(\epsilon_{y2} + \rho\epsilon_{b2}); \quad \beta_1 dG_2 = 1 \quad (51c)$$

$$\beta_2 = a_{22}(\epsilon_{y2} + \rho\epsilon_{b2}) + a_{21}\rho\epsilon_{b1}; \quad -\beta_2 dG_2 = k \quad (51d)$$

and

$$\alpha_1, \beta_1 > 0, \quad \alpha_2, \beta_2 < 0,$$

The upward sloping straight lines, corresponding to (49) and (50) appear in fig. 2. The slope of the $h-1$ line, viz. $\alpha_1 + \beta_1$, is greater than the slope of the $k-m$ line, viz. $\beta_2 - \alpha_2$ since:

$$(\alpha_1 + \beta_1) - (\beta_2 - \alpha_2) = 2a_{11}\rho\epsilon_{b1} + 2a_{12}\rho\epsilon_{b2} + \epsilon_{y1}(a_{11} + a_{21}) - \epsilon_{y2}(a_{22} - a_{12}) > 0$$

using the parameter restrictions (37).

The two lines intersect for $dG_1 = dG_1^*$, where from (49) and (50) :

$$dG_1^* = \frac{(\beta_1 - \beta_2)}{(\beta_1 - \beta_2) + (\alpha_1 + \alpha_2)} dG \quad (52)$$

Now

$$\beta_1 - \beta_2 = (\epsilon_{y2} + \rho\epsilon_{b2})(a_{12} - a_{22}) + \rho\epsilon_{b1}(a_{11} - a_{21}) > 0$$

and

$$\alpha_1 + \alpha_2 = (\epsilon_{y1} + \rho\epsilon_{b1})(a_{11} + a_{21}) + \rho\epsilon_{b2}(a_{12} + a_{22}) > 0$$

Hence,

$$0 < dG_1^* < dG$$

Fig. 2 - The h-1 and k-m lines

It appears from fig. 2 that for $0 \leq dG_1 \leq dG$, $k > m$. Furthermore, $h \geq 1$ according to $dG_1 \geq \bar{dG}_1$, where

$$\bar{dG}_1 = \frac{\beta_1}{\alpha_1 + \beta_1} dG < \frac{1}{2} dG \text{ since } \alpha_1 > \beta_1$$

It may be concluded from the foregoing analysis that, depending on the size of the regional budget, the Nash equilibrium will be Pareto optimal or not.

More specifically, if $dG_1 < \bar{dG}_1$, then $h < 1$ and the Nash equilibrium will not be Pareto optimal (see fig. 1c). If, on the other hand, $dG_1 \geq \bar{dG}_1$, then $h \geq 1$ and the Nash equilibrium will also be Pareto optimal (see fig. 1a).

The choice of the size of the regional budget does not belong to the strategy set in this model. It will be considered as exogenously determined, e.g. by constitutionally defined distribution keys, as it is the case in many countries.

If in our example, the "richer" region 1 is allowed to spend at least $\bar{dG}_1$ which is below half of the total budget, the policy conflict will have a stable and Pareto optimal solution. If, for some reason or other, the "poorer" region 2 is allowed to spend more than $dG - \bar{dG}_1$, which is more than half of the total budget, allocations that are Pareto better than those in the equilibrium point exist.

4. A COOPERATIVE SOLUTION

4.1. The Nash bargaining model

If the equilibrium, that has been reached in the competitive case is not Pareto optimal, there are opportunities for cooperation. In this context, cooperation means that the players jointly try to increase their payoffs by agreeing on a pair of strategies. The strategies pair agreed upon is binding for both players; it implies a deliberate restriction on each player's strategy set.

Nash (1950, 1953) has offered a solution to the cooperative two-person non-zero-sum game¹⁾. The set of points for which each player obtains at least the equilibrium payoff, i.e. $o \geq o^x$, $p \geq p^x$ is called the negotiation set and indicated as $\mathcal{H}$ in fig. 1c.

Let Δz_i denote the difference between any payoff in $\mathcal{H}$ and the equilibrium payoff, i.e.

$$\Delta z_1 = o - o^x; \quad \Delta z_2 = p - p^x$$

The Nash solution to the cooperative game is a payoff pair $(\bar{o}, \bar{p})$ that satisfies the following condition :

$$(\bar{o} - o^x)(\bar{p} - p^x) = \underset{o, p \in \mathcal{H}}{\text{Max}} \Delta z_1 \cdot \Delta z_2 = \underset{o, p \in \mathcal{H}}{\text{Max}} (o - o^x)(p - p^x)$$

1) If four axioms are satisfied, Nash has shown a unique solution to exist for the two-person cooperative game. The four axioms are satisfied in the game problem considered here. See also Luce and Raiffa (1957), p. 126-127 and Friedman (1977), p. 239.

The solution $(\bar{o}, \bar{p})$ is found where a rectangular hyperbola, defined on $\mathcal{H}$ and represented by $\Delta z_1 \cdot \Delta z_2 = \text{constant}$, is tangent to the upper boundary of $\mathcal{H}$, i.e. the line segment EF in fig. 1c.

For payoffs on the parallelogram contour, the Nash product $\Delta z_1 \cdot \Delta z_2$ can be formulated as (using the payoff table 1 and 2) :

$$\Delta z_1 \Delta z_2 = [-(1-\mu_1)h + \mu_2 l] [(1-\mu_1)m - \mu_2 k]$$

Since the set of solutions is restricted to the upper boundary of $\mathcal{H}$, i.e. to the line segment EF with slope $-1/k$ and where $\mu_1 = 0$, the maximization problem can be reduced to :

$$\begin{aligned} \text{Max}_{o, p \in \mathcal{H}} \Delta z_1 \Delta z_2 &= \text{Max}_{o, p \in \mathcal{H}} (o - o^x)(p - p^x) = \text{Max}_{o, p \in \mathcal{H}} (-h + \mu_2 l)(m - \mu_2 k) \end{aligned}$$

$$\text{s.t. } \frac{do}{dp} = -\frac{1}{k}$$

Solving the first order condition for μ_2 , we obtain :

$$\mu_2 = \frac{lm + hk}{2lk}, \quad \text{where } 0 < \mu_2 < 1 \quad (53)$$

Substituting μ_2 in Δz_1 and Δz_2 gives :

$$\Delta z_1 = \frac{lm - hk}{2k}$$

$$\Delta z_2 = \frac{lm - hk}{2l}$$

Now

$$lm - hk = D [(\epsilon_{y1} + \rho \epsilon_{b1}) (\epsilon_{y2} + \rho \epsilon_{b2}) - \rho^2 \epsilon_{b1} \epsilon_{b2}]$$

where D is the positive determinant, defined in (27). The restrictions (37) ensure furthermore that $lm - hk > 0$, i.e. $\Delta z_1 > 0$, $\Delta z_2 > 0$.

It is also obvious that $\Delta z_1 \cong \Delta z_2$ depending on $k \leq 1$. The restrictions (37) imply that $1 > k$, hence $\Delta z_1 > \Delta z_2$.

So, if the competitive equilibrium is not Pareto optimal, the two regions can gain by bargaining, i.e. by agreeing on a pair of strategies. In the model discussed here, region 1 will use the balanced budget policy 2 ($\mu_1 = 0$) whereas region 2 applies a mixture of a debt issue and a tax increase in the proportions μ_2 and $1-\mu_2$. In the outcome, region 1 will gain more from the bargain than region 2.

4.2. A Numerical example

Assume the following parameter values :

$$\begin{aligned}
 \epsilon_{y1} &= .80 & \epsilon_{w2} &= .10 \\
 \epsilon_{y2} &= .40 & \bar{\epsilon}_{r1} &= -500 \\
 m_1 = m_2 &= .40 & \bar{\epsilon}_{r2} &= -800 \\
 \omega_y &= .001 & B &= 0; \rho_r = 0 \\
 \omega_b &= .005 & r &= .10 \\
 \epsilon_{w1} &= .80 & \rho_b &= 10; \rho = .11 \\
 \partial_1 &= .90; \partial_2 &= .10
 \end{aligned}$$

Hence

$$\begin{aligned}
 \epsilon_{b1} &= 5.44 & a_{11} &= 1.80 \\
 \epsilon_{b2} &= -3.86 & a_{22} &= 1.10 \\
 \epsilon_{y1} + \rho \epsilon_{b1} &= 1.40 & a_{12} &= -.10 \\
 \epsilon_{y2} + \rho \epsilon_{b2} &= -.02 & a_{21} &= -.40
 \end{aligned}$$

The total increase in government expenditure is assumed to be 100, and distributed as follows : $dG_1 = 10$; $dG_2 = 90$.

The critical value $\overline{dG}_1$ is in this case 29, so that $dG_1 < \overline{dG}_1$.

Furthermore :

$$h = 29.20$$

$$l = 97.20$$

$$k = 23.40$$

$$m = 10.20$$

Since $h < l$ and $k > m$, the competitive equilibrium will not be Pareto optimal.

$$Q = \begin{bmatrix} 121,47 & 24,70 \\ 92,57 & -5,80 \end{bmatrix}$$

and $o^x = 24$, $p^x = 70$.

The Nash solution to the bargaining problem yields :

$$\mu_2 = .37, \quad \mu_1 = 0$$

$$\Delta z_1 = 6.58$$

$$\Delta z_2 = .78$$

Hence, the final allocation of payoffs $(\bar{o}, \bar{p})$ is:

$$(30.58, 70.78)$$

5. REALLOCATIONS OF THE BUDGET

The question arises whether, in comparison to the competitive solution, a larger total (i.e. national) payoff may be reached, given the total budget increase dG . This kind of problem will be most relevant if efficient solutions to the stabilization problem are aimed at.

Write Δz_1 and Δz_2 as :

$$\Delta z_1 = -(1-\mu_1)\alpha_1 dG_1 + \mu_2\beta_1 dG_2$$

$$\Delta z_2 = -(1-\mu_1)\alpha_2 dG_1 + \mu_2\beta_2 dG_2$$

where, as before, $\alpha_1, \beta_1 > 0$; $\alpha_2, \beta_2 < 0$. We obtain for the total increase in income Δz :

$$\Delta z = \Delta z_1 + \Delta z_2 = -(1-\mu_1)(\alpha_1 + \alpha_2)dG_1 + \mu_2(\beta_1 + \beta_2)dG_2$$

$$\text{where } \alpha_1 + \alpha_2 = (\epsilon_{y1} + \rho\epsilon_{b1})(a_{11} + a_{21}) + \rho\epsilon_{b2}(a_{12} + a_{22}) > 0$$

$$\text{and } \beta_1 + \beta_2 = \rho\epsilon_{b1}(a_{11} + a_{21}) + (\epsilon_{y2} + \rho\epsilon_{b2})(a_{22} + a_{12}) > 0$$

Clearly, for any given allocation of regional budgets (dG_1, dG_2), a larger total payoff can be obtained if $\mu_1 = \mu_2 = 1$, i.e. if both regions agree to finance their budget by issuing bonds. Such a move, away from the equilibrium point, cannot be considered as "rational" for both parties, since for the strategy pair (1,1) : $\Delta z_1 = 1 > 0$ but $\Delta z_2 = -k < 0$. Hence region 2, as the losing party, will only consent to move to point A of the payoff set, if it receives from the winning party (region 1) a compensation such that after compensation $\Delta z_2 \geq 0$.

The compensation transferred to region 2 should be paid out of the gains realized by region 1, such that after compensation $\Delta z_1 \geq 0$.

The distribution of the total gain in A :

$$\Delta z = (\beta_1 + \beta_2)dG_2$$

between the two parties cannot be determined within the model presented here. Exogenous factors, such as social and political power, should be called for in order to establish a distribution key that is accepted by the two parties,

[see e.g. Zusman (1976)]. The way in which the compensation materializes can take various forms. The transfer to region 2 can occur as a lump sum resulting from the tax revenue in region 1. It could also take the form of a public good supplied by region 1 to its residents as well as to those of region 2¹⁾.

Once the move to A as well as the distribution key for the gains has been agreed upon, the parties may wonder whether a reallocation of their budgets may increase the total payoff. Denote the latter by z_A . It follows from the payoff tables 1 and 2 that :

$$z_A = \pi_1 dG_1 + \pi_2 dG_2$$

where

$$\pi_1 = (a_{22} + a_{12})\rho\epsilon_{b2} + (a_{11} + a_{21})(1 + \rho\epsilon_{b1})$$

$$\pi_2 = (a_{22} + a_{12})(1 + \rho\epsilon_{b2}) + (a_{11} + a_{21})\rho\epsilon_{b1}$$

Due to the restrictions (37) imposed on the model it appears that

$$\pi_1 \geq \pi_2 \quad \text{if} \quad \epsilon_{y1} \geq \epsilon_{y2}.$$

Since it has been assumed that $\epsilon_{y1} > \epsilon_{y2}$, i.e. that region 1 has the largest marginal income effect on private expenditure, the total payoff in A may be increased by reallocating the total regional budget in favour of region 1. A central government may obtain the same increase in the total payoff, i.e. in national income, by concentrating its expenditures in the region with the largest marginal expenditure effect and by applying the most efficient policy mix. The latter will correspond to the policy mix in A, i.e. a bond financed expenditure program in both regions.

Whether in the outcome, the regional allocation of payoffs will, from a distributive point of view, be preferable to the one obtained in a decentralized setting, depends on the relative success of a national redistributive

1) For the treatment of regional spillovers from public goods within a system of fiscal federalism, see e.g. Breton (1970), Sandler and Shelton (1972).

policy [see e.g. Mera (1967)]. If, contrary to the identical linear regional welfare functions assumed here, the latter differ significantly, there is no clear evidence on a priori grounds that a centralized policy will be better in efficiency and distributive terms.

A final remark is related to the coordination aspect of regional decentralized policies. Following Corden (1974), a distinction should be made between the mutual adjustment of regional policies and the deliberate agreement to coordinate policies. Apparently, there is no need for a central authority to impose coordination in an institutional way if the competitive equilibrium is Pareto optimal (section 3.1.). If the competitive equilibrium is not Pareto optimal, the regions will coordinate their policies in Corden's interpretation, merely on rational grounds.

6. CONCLUDING REMARKS

It has been shown in this paper that decentralized regional stabilization policies can lead to a stable solution which may imply the use of regionally different policy instruments.

Whether the competitive solution is Pareto optimal or not depends on the regional distribution of the budgets. If this distribution is such that the most efficiently stabilizing region receives the largest share, the outcome will be Pareto optimal.

If, for some reason or other, the distribution key for the regional budgets, favours the less efficiently stabilizing region, a Nash solution to the cooperative model has been proved to exist and to yield a Pareto better allocation of payoffs than the equilibrium pair.

The limitations of the game approach and of the model analyzed here may seem severe. Indeed, linear welfare functions and a linear model have been assumed throughout. The framework of analysis is static and does not take account of a leading player, such as the national government, which makes first moves.

However, a simplified approach may have some advantages by the fact that it allows to focus on the role of key parameters that are familiar in macro-economic models.

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