UPPER AND LOWER BOUNDS ON INFINITE TIME RUIN
PROBABILITIES IN CASE OF CONSTRAINTS ON
CLAIM SIZE DISTRIBUTIONS

by

M.J. GOOVAERTS
F. DE VYLDER

Nr. 8208

Wettelijk Depot : D/1982/2376/30.

UPPER AND LOWER BOUNDS ON INFINITE TIME RUIN PROBABILITIES IN CASE OF
CONSTRAINTS ON CLAIM SIZE DISTRIBUTIONS

M.J. GOOVAERTS
K.U. Leuven
Dekenstraat 2
B 3000 Leuven
Belgium

F. DE VYLDER
Université Catholique
de Louvain
315, Steenweg op Ninove
B 9470 Denderleeuw
Belgium

ABSTRACT

Probabilities of ruin (or non-ruin) are solutions of differential or integro-differential equations. Solving these kind of equations analytically and or numerically causes a lot of mathematical difficulties. In addition there exists a practical problem of determining, estimating or guessing the distribution of the risk. The realistic way to deal with this problem consists in deriving upper and lower bounds for the ruin probability in case of incomplete information on the distribution F . The present contribution is inspired by and generalises a result of G. Taylor who uses the concept of ordering of risks to order ruin probabilities. We show how some of the results obtained by F. De Vylder for deriving sharp bounds on the stop-loss premium in case of incomplete information can be applied to the evaluation of practical bounds on infinite time ruin probabilities.

1. INTRODUCTION

For the derivation of the equation governing the probability of ruin or of non-ruin the reader is referred to ref. [1]. The surplus of an insurance company at time $t \geq 0$ is denoted by R_t . Let $R_0 = x$ be the initial surplus, which is supposed to be known and nonnegative. The surplus $\{R_t\}$ is treated as a continuous-time stochastic process, with

$$R_t = x + ct - S_t \quad (1)$$

Here c is some constant and it is interpreted as the premiums received continuously per unit time. Thus ct is the total premiums received in $(0, t)$, S_t is the aggregate claims of this time interval. It is assumed that $\{S_t\}$ is a compound Poisson process, with Poisson parameter $\lambda > 0$, and cumulative distribution function $F(\cdot)$ of claim amounts. It is assumed that $c > \lambda p_1$ where p_1 denotes the mean claim size. Let $\psi(\cdot)$ denote the infinite time ruin probability

$$\psi(x) = \text{prob} [T < \infty] \quad (2)$$

where T denotes the time of ruin.

In [1] the following equation for the probability of ultimate ruin is derived :

$$\psi(x) = \int_0^{\infty} e^{-\lambda t} \int_0^{\infty} \psi(x+ct-y) dF(y) dt \quad (3)$$

This equation can still be cast into the form of a defective renewal equation:

$$\psi(x) = \frac{\lambda}{c} \int_0^x \psi(x-y)(1-F(y))dy + \frac{\lambda}{c} \int_x^{\infty} (1-F(y))dy \quad (4)$$

with

$$\psi(0) = \frac{\lambda}{c} p_1 \quad (5)$$

Also in ref. [1] it is shown that

$$\psi(x) \leq e^{-Rx} \quad (6)$$

where R is the so called "adjustment coefficient", determined as the positive root of the equation in r :

$$\lambda + rc = \lambda \int_{-\infty}^{+\infty} e^{ry} dF(y) \quad (7)$$

2. ORDERING OF RISKS AND INEQUALITIES FOR RUIN PROBABILITIES

Let us recall some definitions of ordering of risks and their application to the ordering of ruin probabilities (see e.g. ref. [2])

Definition 1

A premium calculation principle defines a consistent partial ordering if for those values of β for which

$$\frac{E(e^{tX})}{E(e^{tY})^\beta} \text{ is an increasing function of } t$$

the corresponding premiums satisfy

$$\pi(X) \geq \beta \pi(Y). \quad (\text{or } Y \overset{\beta}{<} X)$$

Definition 2

Let F_X, F_Y be any distributions on the real line. Then we say that $F_X < F_Y$ if

$$i) \int_0^{\infty} x dF_X(x) < \infty$$

$$\text{ii)} \quad \int_t^{\infty} (x-t) dF_X(x) \leq \int_t^{\infty} (x-t) dF_Y(x), \quad -\infty < t < +\infty$$

$$\text{Let } \bar{F}_X(x) = 1 - F_X(x), \quad {}_1\bar{F}_X(x) = \int_x^{\infty} \bar{F}_X(x) dx$$

and in general

$${}_k\bar{F}_X(x) = \int_x^{\infty} {}_{k-1}\bar{F}_X(x) dx \quad k = 2, 3, \dots$$

Definition 3

The condition for first degree stop-loss dominance of Y over X ($X \stackrel{1}{\leq} Y$) is that

$$\bar{F}_X(x) \leq \bar{F}_Y(x)$$

The condition for second degree dominance of Y over X ($X \stackrel{2}{\leq} Y$) is that

$${}_1\bar{F}_X(x) \leq {}_1\bar{F}_Y(x)$$

The conditions for third degree stop-loss dominance of Y over X ($X \stackrel{3}{\leq} Y$) are

$$E(X) \leq E(Y), \quad {}_2\bar{F}_X(x) \leq {}_2\bar{F}_Y(x)$$

The condition for n-th degree stop-loss dominance of Y over X ($X \stackrel{n}{\leq} Y$) are

$${}_k\bar{F}_X(0) \leq {}_k\bar{F}_Y(0) \quad k = 1, 2, \dots, n-2$$

$${}_{n-1}\bar{F}_X(x) \leq {}_{n-1}\bar{F}_Y(x).$$

The following inequalities for the probabilities of ultimate ruin can be obtained :

$$\text{i)} \quad X_1 \stackrel{1}{<} X_2 \Rightarrow R_1 > R_2 \Rightarrow \exp(-R_1 x) < \exp(-R_2 x) \quad (8)$$

where R_1 and R_2 are the adjustment coefficients of X_1 and X_2 respectively.

$$\text{ii) } X_1 \stackrel{n}{\leq} X_2 \Rightarrow R_1 > R_2 \Rightarrow \exp(-R_1 x) < \exp(-R_2 x) \quad (9)$$

The proof of both inequalities follows from some of the results on ordering of risks given in ref. [1] and ref. [2].

We now recall a result obtained by G. Taylor in ref. [3]. Let us consider two claim size distribution functions $F_{X_1}(\cdot)$ and $F_{X_2}(\cdot)$ respectively. Let the corresponding non-ruin probabilities be $\phi_1(\cdot) = 1 - \psi_1(\cdot)$ and $\phi_2(\cdot) = 1 - \psi_2(\cdot)$ respectively. Now, suppose that :

$$\int_0^y [F_{X_1}(z) - F_{X_2}(z)] dz \geq 0 \quad \text{for all } y \in [0, w] \quad (10)$$

then :

$$\phi_1(w) \leq \phi_2(w) \quad (11)$$

hence :

$$\psi_1(w) \geq \psi_2(w) \quad (12)$$

Now (10) can be cast into the form

$$\int_y^\infty (1 - F_{X_1}(z)) dz \geq \int_y^\infty (1 - F_{X_2}(z)) dz \quad (13)$$

in case X_1 and X_2 have the same expectation $E(X_1) = E(X_2)$. Hence, with G. Taylor, we conclude

$$X_1 < X_2 \Rightarrow \psi_1(w) < \psi_2(w) \quad (14)$$

3. A GENERALIZATION OF TAYLOR'S RESULT

The functions considered in the sequel are defined on $\mathbb{R}^+ = [0, \infty)$. The stop-loss transform of the distribution F is the function h_F defined by

$$h_F(x) = \int_x^\infty (y-x) dF(y) = \int_x^\infty (1-F(y))dy \quad (15)$$

Notice that h_F is non-negative, decreasing (not necessarily strictly) and that $h_F(0) = p_1$ (supposed to be finite). It is convenient to display the coefficient c as $c = \lambda p_1(1+\eta)$ where $\eta > 0$ is the security loading.

We remember that the convolution product $\varphi * H$ is defined by

$$(\varphi * H)(x) = \int_0^x \varphi(x-y) dH(y) \quad (16)$$

and that it is meaningful and finite whenever φ is a bounded (measurable) function on $\mathbb{R}^+$ and H a function of bounded variation on $\mathbb{R}^+$. Clearly $\varphi * H \geq 0$ if $\varphi \geq 0$ and H is increasing. The integral in (16) is over the closed interval $[0, x]$.

Remark : More generally the convolution product is defined if φ is bounded on the bounded subsets of $\mathbb{R}^+$ and H is of bounded variation on $\mathbb{R}^+$. Next lemma has an obvious extension to that case (not used in the sequel).

Lemma

If H is a strictly defective distribution function and if f is bounded, then the renewal equation

$$\xi = f + \xi * H$$

has a unique bounded solution ξ .

If $f \geq 0$, then $\xi \geq 0$. If $f \leq 0$, then $\xi \leq 0$

Proof

Let $H(\infty) = \alpha < 1$. Then by induction, $H^{*n} \leq \alpha^n (n=1,2,\dots)$. Using successively the given equation, any solution ξ must satisfy :

$$\begin{aligned} \xi &= f + \xi * H \\ &= f + (f + \xi * H) * H = f + f * H + \xi * H^{*2} \\ &= \dots \\ &= f * \left(\sum_{i=0}^{n-1} H^{*i} \right) + \xi * H^{*n} \end{aligned} \quad (17)$$

The last term of (17) is bounded by

$$\alpha^n \sup_{x \in \mathbb{R}^+} |\xi(x)|$$

and tends to zero when $n \rightarrow \infty$ if ξ is bounded. The first term of (17) is bounded by

$$\sup_{x \in \mathbb{R}^+} |f(x)| \sum_{i=0}^{n-1} \alpha^i \leq \frac{1}{1-\alpha} \sup_{x \in \mathbb{R}^+} |f(x)| < \infty$$

Clearly the unique bounded solution of the given renewal equation is

$$\xi = f * \left(\sum_{i=0}^{\infty} H^{*i} \right)$$

and then the lemma is obvious from this expression.

Theorem

The function ψ is a solution of the equations

$$\psi * (p_1 - h_F) = p_1(1+\eta)\psi - h_F \quad (18)$$

In that equation, or in any equivalent equation, replace h_F by h_0 (not necessarily a stop-loss transform) and let ψ_0 be the corresponding solution.

Then $\psi \leq \psi_0$ ($\psi \geq \psi_0$) if h_0 is decreasing, $h_0 \geq 0$, $h_0(0) = p_1$ and $h_F \leq h_0$ ($h_F \geq h_0$).

Proof

In the first integral of (4), replace $(1-F(y))dy$ by $d(p_1-h_F(y))$, use (5) and the definition of η and h_F , in order to obtain (18). Relation (18) can also be displayed as $p_1-p_1(1+\eta)\psi = (1-\psi) \star (p_1-h_F)$. (19)

By the definition of ψ_0 , we have similarly,

$$p_1-p_1(1+\eta)\psi_0 = (1-\psi_0) \star (p_1-h_0) \quad (20)$$

Subtracting, we have

$$\begin{aligned} p_1(1+\eta)(\psi_0-\psi) &= (1-\psi) \star (p_1-h_F) - (1-\psi_0) \star (p_1-h_0) \\ &= (\psi_0-\psi) \star (p_1-h_0) + (1-\psi) \star (h_0-h_F) \\ &= (\psi_0-\psi) \star (p_1-h_0) + (h_0-h_F) \star (1-\psi) \end{aligned}$$

Then

$$\xi = \xi \star H + f$$

where $\xi = \psi_0-\psi$, $f = \frac{1}{p_1(1+\eta)} (h_0-h_F) \star (1-\psi)$

$$h = \frac{1}{p_1(1+\eta)} (p_1-h_0)$$

Suppose that h_0 is decreasing, $h_0(0) = p_1$, $h_0 \geq 0$ and $h_F \leq h_0$. Then f is bounded because h_0 , h_F , $1-\psi$ are bounded. Moreover $f \geq 0$ because $h_0-h_F \geq 0$ and $1-\psi$ is increasing. The function H is a distribution function (it is increasing and $H \geq 0$), in fact a strictly defective distribution function because $h(\infty) \leq \frac{1}{1+\eta}$. By the lemma, $\xi \geq 0$, similarly $\xi \leq 0$ if $h_F \geq h_0$.

Corollary

Let $\mathcal{F}$ be a family of claim-size distributions containing F .

If $h_0 = \sup_{G \in \mathcal{F}} h_G$ and $h_0(0) = p_1$, then $\psi \leq \psi_0$ (21)

If $h_0 = \inf_{G \in \mathcal{F}} h_G$, and $h_0(0) = p_1$, then $\psi \geq \psi_0$ (22)

Remarks :

In the following applications, $\mathcal{F}$ is a family of distributions with mean μ_1 and the relation $h_0(0) = p_1$ is satisfied automatically.

The relation for ψ_0 (corresponding to (18)) shall be used in the form

$$\psi_0(x) \left(1 - \frac{\lambda}{c} p_1\right) = \frac{\lambda}{c} \left(1 - \frac{\lambda}{c} p_1\right) h_0(x) - \frac{\lambda}{c} \int_0^x \psi_0'(x-y) h_0(y) dy \quad (23)$$

4. APPLICATION OF THE INEQUALITIES FOR RUIN PROBABILITIES TO SOME PRACTICAL SITUATIONS

In this section we consider some applications of the inequalities for ruin probabilities which can be worked out analytically. In general solving the equations (23) and (18) numerically gives rise to exactly the same kind of difficulties as encountered in the calculation of ruin probabilities (see ref. [8]). In ref. [7] a numerical method is presented for deriving numerical bounds on the infinite time ruin probabilities in case of constraints on the claim size distributions. For our present analytical applications use will be made of the Laplace transform technique for solving the equation (23)

Let $L[u] = \int_0^{\infty} e^{-sx} u(x) dx$ denote the Laplace transform of $u(\cdot)$, then from (23)

$$L[u] = \left\{ \frac{\lambda}{c} \left(1 - \frac{\lambda}{c} p_1\right) + \frac{\lambda}{c} u(0) \right\} L[b] \frac{1}{1 - \frac{\lambda}{c} p_1 + \frac{\lambda}{c} s L[b]} \quad (24)$$

where $u (= \psi_0)$ is the solution of (23) corresponding to $b (= h_0)$

Application 1

The failure rate of F_X equals c at least, $E(X) = z$, $F_X(M) = 1$. From De Vylder [6] we obtain

$$b(y) = \begin{cases} (1 - e^{-cM})^{-1} (e^{-cy} - e^{-cM})z & y < M \\ 0 & y > M \end{cases}$$

Let us consider the case $M \rightarrow \infty$, then

$$b(y) = e^{-cy} z$$

Hence :

$$L[b] = \frac{z}{s+c}$$

as well as :

$$\begin{aligned} L[u] &= \frac{\lambda}{c} \frac{z}{s+c} \frac{1}{1 - \frac{\lambda}{c} p_1 + \frac{\lambda}{c} \frac{sz}{s+c}} \\ &= \frac{\frac{\lambda}{c} z}{z \frac{\lambda}{c} + 1 - \frac{\lambda}{c} p_1} \frac{1}{s + \frac{c(1 - \frac{\lambda}{c} p_1)}{\frac{\lambda}{c} z + 1 - \frac{\lambda}{c} p_1}} \end{aligned}$$

Finally inversion gives

$$U(x) = \frac{\frac{\lambda}{c} z}{\frac{\lambda}{c} + 1 - \frac{\lambda}{c} p_1} e^{-\frac{c(1 - \frac{\lambda}{c} p_1)}{\frac{\lambda}{c} z + 1 - \frac{\lambda}{c} p_1} x}$$

Compare the result obtained by H. Gerber [1] for distributions with monotonic failure rate.

Application 2

Let us assume that F is concentrated on $[0, \infty)$ and that F_X is a mixture of the Pareto distributions

$$H_\theta(x) = [1 - \frac{1}{(1+x)^\theta}] 1_{0 \leq x} \quad (c \leq \theta \leq d)$$

where $1 < c < d \leq \infty$

From De Vylder [6] we obtain

$$b(y) = (d-c)^{-1} (z(d-1)-d) \frac{1}{(1+y)^{c-1}} + (d-c)^{-1} (c-z(c-1)) \frac{1}{(1+y)^{d-1}}$$

with $d(d-1)^{-1} \leq z \leq c(c-1)^{-1}$

Consequently :

$$L[b] = (d-c)^{-1}(z(d-1)-d) \int_0^{\infty} \frac{e^{-sy}}{(1+y)^{c-1}} dy + (d-c)^{-1}(c-z(c-1)) \int_0^{\infty} \frac{e^{-sy}}{(1+y)^{d-1}} dy$$

But, introducing an integral representation of $\frac{1}{(1+y)^{\alpha-1}}$ we get for $\alpha > 1$:

$$\int_0^{\infty} \frac{e^{-sy}}{(1+y)^{\alpha-1}} dy = \frac{1}{\Gamma(\alpha-1)} \int_0^{\infty} \frac{e^{-k} k^{\alpha-2}}{s+k} dk$$

Hence :

$$L[b] = (d-c)^{-1}(z(d-1)-d) \frac{1}{\Gamma(c-1)} \int_0^{\infty} \frac{e^{-k} k^{c-2}}{s+k} dk + (d-c)^{-1}(c-z(c-1)) \frac{1}{\Gamma(d-1)} \int_0^{\infty} \frac{e^{-k} k^{d-2}}{s+k} dk$$

It is clear now that $L[b]$ represents an analytical function in the whole s -plane out along the negative real axis (see fig. 1).

Fig. 1.

Let us consider next the properties of the function

$$\varphi_{\alpha}(s) = \int_0^{\infty} \frac{e^{-k} k^{\alpha-2}}{s+k} dk \quad \alpha > 2$$

for complex values $s = x + iy$.

Let $\varphi_{\alpha}^{+}(-x)$ denote the value of the function $\varphi_{\alpha}(\cdot)$ along $\mathbb{C}D$, then

$$\varphi_{\alpha}^{+}(-x) = \lim_{\varepsilon \rightarrow 0} \int_0^{\infty} \frac{e^{-k} k^{\alpha-2}}{-x+i\varepsilon+k} dk$$

By means of one partial integration we obtain :

$$\varphi_{\alpha}^{+}(-x) = \lim_{\varepsilon \rightarrow 0} - \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-1} \right) \text{Log}(-x+i\varepsilon+k) dk$$

or

$$\varphi_{\alpha}^{+}(-x) = \lim_{\varepsilon \rightarrow 0} \left[- \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-2} \right) \left[\ln \sqrt{(k-x)^2 + \varepsilon^2} + i \text{Arctg} \frac{\varepsilon}{k-x} \right] \right]$$

But

$$\lim_{\varepsilon \rightarrow 0} \text{Arctg} \frac{\varepsilon}{k-x} = \begin{cases} \pi & k < x \\ 0 & k \geq x \end{cases}$$

Consequently the following result is obtained :

$$\varphi_{\alpha}^{+}(-x) = - \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-2} \right) \ln|k-x| dk - i\pi \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-1} \right) H(x-k) dk$$

where $H(\cdot)$ as usual denotes the Heaviside function.

Finally the following expression is obtained for $\varphi_{\alpha}^{+}(-x)$:

$$\varphi_{\alpha}^{+}(-x) = - \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-2} \right) \ln|k-x| dk - i\pi e^{-x} x^{\alpha-2}$$

and of course

$$\varphi_{\alpha}^{-}(-x) = - \int_0^{\infty} \left(\frac{d}{dk} e^{-k} k^{\alpha-2} \right) \ln|k-x| dk + i\pi e^{-x} x^{\alpha-2}$$

On the other hand, for large values of s the following asymptotic result is obtained :

$$\varphi_{\alpha}(s) = \frac{1}{s} \Gamma(\alpha-1) + O\left(\frac{1}{s^c}\right)$$

while for all values of $s(=x+iy)$ lying outside the cut

$$\begin{aligned} s\varphi_{\alpha}(s) &= (x+iy) \int_0^{\infty} \frac{e^{-k} k^{\alpha-2}}{(x+k)^2+y^2} \frac{x+k-iy}{(x+k)^2+y^2} dk \\ &= \int_0^{\infty} \frac{e^{-k} k^{\alpha-2}}{(x+k)^2+y^2} \frac{x(x+k)+y^2}{(x+k)^2+y^2} dk + iy \int_0^{\infty} \frac{e^{-k} k^{\alpha-1}}{(x+k)^2+y^2} dk \end{aligned}$$

Hence

$$\operatorname{Im}\{s\varphi_{\alpha}(s)\} = \begin{cases} y \int_0^{\infty} \frac{e^{-k} k^{\alpha-1}}{(x+k)^2+y^2} dk & \text{outside the cut} \\ \pi e^{-x} x^{\alpha-1} & \text{on the cut} \end{cases}$$

From which follows that $\operatorname{Im}\{s\varphi_{\alpha}(s)\} > 0$ for all $s \neq 0$.

Let

$$\begin{aligned} K_1(x) &= (d-c)^{-1} (z(d-1)-d) \frac{1}{\Gamma(c-1)} \int_0^{\infty} \left(-\frac{d}{dk} e^{-k} k^{c-2}\right) \ln|k-x| dk \\ &\quad + (d-c)^{-1} (c-z(c-1)) \frac{1}{\Gamma(d-1)} \int_0^{\infty} \left(-\frac{d}{dk} e^{-k} k^{d-2}\right) \ln|k-x| dk \end{aligned}$$

and

$$\begin{aligned} K_2(x) &= (d-c)^{-1} (z(d-1)-d) \frac{1}{\Gamma(c-1)} \pi e^{-x} x^{c-2} \\ &\quad + (d-c)^{-1} (c-z(c-1)) \frac{1}{\Gamma(d-1)} \pi e^{-x} x^{d-2} \end{aligned}$$

From the preceeding properties of $\varphi_{\alpha}(s)$ we deduce the following once for $L(b;s] = L[b]$.

$$L[b;-x]^+ = K_1(x) - i K_2(x)$$

$$L[b;-x]^- = K_1(x) + i K_2(x)$$

$$L[b;s] = \frac{z-1}{s} + O\left(\frac{1}{s^2}\right)$$

$$\operatorname{Im} L[b; -x] > 0 \text{ for } x \neq 0$$

$$L[b, 0] = \Gamma(\alpha - 2)$$

We now have to consider

$$\frac{\lambda}{c} L^{-1} \left\{ \frac{L[b, s]}{1 - \frac{\lambda}{c} p_1 + \frac{\lambda}{c} s L[b, s]} \right\}$$

or :

$$u(x) = \frac{\lambda}{c} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{xs} \frac{L[b, s]}{1 - \frac{\lambda}{c} p_1 + \frac{\lambda}{c} s L[b, s]} ds$$

Because of the properties of $L[b, s]$ it is possible to deform the path of integration as is indicated in fig. 1. An appropriate limit transition then results in :

$$u(x) = \frac{\lambda}{c} \frac{i}{2\pi} \int_0^{\infty} e^{-xk} \frac{L[b, -k]^+}{1 - \frac{\lambda}{c} p_1 - \frac{\lambda}{c} L[b, -k]^+} dk$$

$$- \frac{\lambda}{c} \frac{i}{2\pi} \int_0^{\infty} e^{-xk} \frac{L[b, -k]^-}{1 - \frac{\lambda}{c} p_1 - \frac{\lambda}{c} k L[b, -k]^-} dk$$

Consequently $u(x)$ can be cast into the form :

$$u(x) = \frac{\lambda}{c} \frac{1}{\pi} \operatorname{Im} \int_0^{\infty} e^{-xk} \frac{-L[b, -k]^+}{1 - \frac{\lambda}{c} p_1 - \frac{\lambda}{c} k L[b, -k]^+} dk$$

Such that finally

$$u(x) = \frac{\lambda}{c} \cdot \frac{1}{\pi} \cdot \int_0^{\infty} e^{-xk} \frac{(1 - \frac{\lambda}{c} p_1) \cdot K_2(k)}{(1 - \frac{\lambda}{c} p_1 - \frac{\lambda}{c} k K_1(k))^2 + (\frac{\lambda}{c} k K_2(k))^2} dk$$

This is an upper bound (which is reached for some claim size distribution) for a claim size distribution which is a mixture of a Pareto distribution, with given mean z .

The domain of the weighting distribution is $[c, d]$ with $c > 2$. As a particular case we let $d \rightarrow \infty$. Then the functions K_1 and K_2 transform into :

$$K_1(x) = \frac{z-1}{\Gamma(c-1)} \int_0^{\infty} \left(-\frac{d}{dk} e^{-k} k^{c-2} \right) \ln|k-x| dk$$

$$K_2(x) = \frac{z-1}{\Gamma(c-1)} \pi e^{-x} x^{c-2}$$

Application 3

Let us assume next that F_X is a mixture of exponential distributions with density

$$f_X(x) = \int_0^{\infty} w^{-1} e^{-\frac{x}{w}} dF_W(w)$$

with $E(W) = z$.

This is the class of completely monotonic densities.

From ref. [9] we obtain

$$b(y) = a \frac{b-z}{b-a} e^{-\frac{t}{a}} + b \frac{z-a}{b-a} e^{-\frac{t}{b}}$$

In the present case the upper bound for the ruin probability can be calculated according to one of the methods explained in H. Gerber book (p.115).

REFERENCES

- [1] Hans U. GERBER, An Introduction to Mathematical Risk Theory, (eds.) Huebner Foundation Monograph, (1979), pp. 1-164.
- [2] M.J. GOOVAERTS, F. DE VYLDER, J. HAEZENDONCK, Ordering of risks : a review, Insurance : Mathematics and Economics 1,2(1982), pp. 131-163.
- [3] G.C. TAYLOR, Use of differential and integral inequalities to bound ruin and queiung probabilities, (1976), Scandinavian Actuarial Journal, pp. 197-208.
- [4] F. DE VYLDER, M.J. GOOVAERTS, Analytical best upper bounds on stop-loss premiums, Insurance : Mathematics and Economics 1,3 (1982), pp. 197-212.
- [5] F. DE VYLDER, M.J. GOOVAERTS, Upper and lower bounds on stop-loss premiums in case of known expectation and variance of the risk variable, Mitt. der Ver. Schw. Vers. Math., (1982), pp. 149-164.
- [6] F. DE VYLDER, Duality Theory of bounds on integrals with applications to stop-loss premiums, (subm), S.A.J.(1982).
- [7] M. VAN WOUWE, F. DE VYLDER, M. GOOVAERTS, "Numerical bounds on infinite time ruin probabilities in case of constraints on claim size distributions" (subm), (1982).
- [8] H.L. SEAL "Survival Probabilities : The Goal of Risk Theory". (eds.) Wiley, London (1978).
- [9] M.J. GOOVAERTS and F. DE VYLDER "Upper bounds on stop-loss premiums for scale and power mixtures of distributions" (subm) (1982).